Assignment 10 - due November 16

1. Verify the following properties of Poisson brackets:

\[ \{ F, GH \} = \{ F, G \} H + G \{ F, H \} \]

\[ \{ F, \{ G, H \} \} + \{ G, \{ H, F \} \} + \{ H, \{ F, G \} \} = 0 \]

Let \( D_F G = \{ F, G \} \). Show

\[ e^{D_F} \{ G, H \} = \{ e^{D_F} G, e^{D_F} H \} \]

2. A real \( 2N \times 2N \) matrix \( M \) is symplectic if and only if

\[ MJM^T = J \]

\[ J = \begin{pmatrix} 0 & I_{N \times N} \\ -I_{N \times N} & 0 \end{pmatrix} \]

Show \( M^T \) and \( M^{-1} \) are symplectic.

Show that the product of symplectic matrices are symplectic.

Show that if \( \lambda \) is an eigenvalue of a symplectic matrix \( M \) then so is \( \lambda^* \), \( 1/\lambda \), and \( 1/\lambda^* \).

3. Consider the following one, two, and three forms

\[ dx \quad dx \wedge dy \quad dy \wedge dz \quad dx \wedge dy \wedge dz \]

Replace \( x, y, z \) by spherical polar coordinates and evaluate these forms in terms of the spherical polar coordinates.

4. Consider the one form

\[ F = f_1(x, y) dx + f_2(x, y) dy \]

What are the conditions on \( f_1 \) and \( f_2 \) for this one form to be exact, i.e. \( F = dG \) for some \( G \)?

5. Apply the generating function

\[ F(Q, p) = \sinh(p)Q^2 \]

Find the canonical transform from \( (q, p) \) to \( (Q, P) \). Apply this canonical transformation to

\[ H = \frac{p^2}{2m} + \frac{kq^2}{2} \]

Find the transformed Hamiltonian, the equations of motion, and the solution to the equations of motion.

6. Let \( H = \frac{p^2}{2m} + gq \) Calculate

\[ p(t) = e^{Dt}p(0) \quad \text{and} \quad q(t) = e^{Dt}q(0) \]

Verify that your calculated expressions are solutions of Hamilton’s equations.