Assignment 11 - due November 30

1. Determine if the transformation

\[ Q = \ln\left(\frac{1}{q} \sin(p)\right) \quad P = q \cot(p) \]

is a canonical transformation.

2. Find the generating function for the identity (i.e. that takes \( p = P \) and \( q = Q \)).

3. Consider the Lagrangian for a particle of mass \( m \) in an electromagnetic field

\[ L(q, \dot{q}, t) = \frac{1}{2} m \dot{q}^2 - e \phi(q, t) + \frac{e}{c} q \cdot A(q, t) \]

where \( \phi(q, t) \) and \( A(q, t) \) are the scalar and vector potentials of the electromagnetic field. Find the canonical momentum, the Hamiltonian, and Hamilton’s equations of motion for the canonical variables.

4. Find the general form of a \( 2 \times 2 \) symplectic matrix.

5. The Schrödinger equation for a particle of mass \( m \) in a potential \( V(x) \) is:

\[ i\hbar \frac{\partial \psi(x, t)}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi(x, t) + V(x) \psi(x, t). \]

Assume the quantum mechanical wave function is expressed in terms of a real amplitude and phase \( \psi(x, t) = A(x, t)e^{iS(x, t)/\hbar} \). Show that in the limit that \( \hbar \to 0 \) the equation for the phase becomes the Hamilton-Jacobi equation.

6. Solve the Hamilton Jacobi equation for a particle of mass \( m \) is a uniform gravitational field with gravitational acceleration \( g \).