1. Show that if $R$ is a real $3 \times 3$ matrix that preserves the length of all vectors,

$$ r' = Rr \quad r' \cdot r' = r \cdot r, $$

that $R$ must satisfy $R^T R = I$ where $R^T$ is the transpose of $R$. Matrices with this property are called orthogonal matrices.

2. Show that if $R_1$ and $R_2$ are orthogonal matrices that the matrix product $R_3 = R_2 R_1$ is also an orthogonal matrix.

3. Show that the product of 2 Galilean transformations

$$ G = \begin{pmatrix} R & v_0 & r_0 \\ 0 & 1 & t_0 \\ 0 & 0 & 1 \end{pmatrix} $$

is another Galilean transformation. This means that multiple applications of these transformations do not lead to a new type of transformation.

4. Show that every Galilean transformation has an inverse that is also a Galilean transformation. Write down the inverse transformation.

5. An isolated mechanical system consists of three point particles that are initially at rest in an inertial coordinate system. Show that they remain in the initial plane for all time (assume for this problem that inertial coordinate systems are related by Galilean transformations).

6. A car is moving at constant speed $v$ around a circular track of radius $R$. Find a transformation relating an earth fixed rectangular coordinate system to a car fixed rectangular coordinate system. Find the inertial forces on a particle in the car-fixed coordinate system.