1. **Newtonian principle of determinacy**

\[ \tilde{x}(t) = \tilde{x}(t, t_0, \tilde{r}_0, \tilde{v}_0) \]

Leads to 2nd order equation

\[ \frac{d^2 \tilde{x}}{dt^2} = \frac{d^2 \tilde{r}}{dt^2} (t, t_1, \tilde{r}(t), \frac{d \tilde{r}}{dt}) \]

Acceptable acceleration function must admit global solutions.

2. **Separate particle from force**

\[ m_1 \tilde{A}_1 = m_2 \tilde{A}_2 = \tilde{F} \quad (2^{nd} \text{ Law}) \]

In preferred coordinate systems.

3. \( \tilde{F} = 0 \quad \tilde{A}_1 = 0 = \frac{d \tilde{x}}{dt} = 0 \quad (1^{st} \text{ Law}) \)

Particle moves with constant velocity (inertial cs).

4. **Galilean transformation**

\[
\begin{pmatrix}
R & V & a \\
0 & 1 & t \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
x \\
t
1
\end{pmatrix}
\]

\[ R^TR = 1 \]

Relates different inertial cs.

5. **Principle of Galilean relativity**

Equations of motion have same form in all inertial coordinate systems.
requires

1. $F_i$ has explicit time dependence
2. $F_i$ depends on coordinate systems
3. $F_i$ depends on velocity differences
4. $RF(\Delta r, \Delta v) = F(R\Delta r, R\Delta v)$

5. Newton's 2nd law in non-inertial coordinate systems

$\ddot{\mathbf{r}}_i = \ddot{\mathbf{r}}_i(x, t)$

A general feature - inertial forces are proportional to the masses

6. Conservative forces

$$F_i = -\frac{\partial V}{\partial \mathbf{r}_i}.$$  

$V$ single valued

$W_{AB}$ independent of path

$$\oint F_i \cdot ds = 0$$

7. V - differences

$$\sum F_i = 0 \quad \bar{F}_{ix} + \bar{F}_{ix} = 0 \quad (3^{rd} \text{ law})$$
6 Systems

\[ M = \sum m_i \text{ total mass} \]

\[ \ddot{R} = \sum \frac{m_i \dot{r}_i}{M} \text{ center of mass coordinate} \]

\[ M \frac{d\ddot{R}}{dt} = F_{\text{ext}} \]

(Justifies using idealized point particles)

10 Conservation Laws

0 Conservation Law

\[ \sum m_i \ddot{r}_i + \frac{dV}{dt} = 0 \quad \dot{r} \]

\[ \frac{d}{dt} \left( \frac{1}{2} \sum m_i \dot{r}_i^2 + V \right) = 0 \quad \text{energy conservation} \]

\[ \mathbf{p} = M \ddot{R} \quad M \ddot{R} = 0 \quad \text{(no external forces)} \]

\[ \mathbf{p} \text{ conserved} \]

\[ \mathbf{L} = \dot{R} \times \mathbf{p} \]

\[ \dot{\mathbf{L}} = \dot{\mathbf{R}} \times \dot{\mathbf{p}} + \dot{\mathbf{R}} \times \ddot{\mathbf{p}} = \ddot{\mathbf{R}} \times F_{\text{ext}} \]

\[ \text{external torques} \]

\[ \text{no external torques} \quad \mathbf{L} \text{ conserved} \]

11 Holonomic Constraints

\[ h(\ddot{r}_1, \ldots, \ddot{r}_n, t) = 0 \]

Generalized coordinates

\[ \ddot{r}_i = \ddot{r}_i(q_1, \ldots, q_n) \]

\[ \text{dealing with forces of constraint} \]
D'Alembert's principle -
principle of virtual work

Forces of constraint do no work

\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) = \dot{Q}_i + \sum F_i \frac{\partial \dot{q}_i}{\partial q_i} = \text{generalized forces} \]

Conservative forces

\[ L = T - V \]

\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = 0 \quad \text{(Lagrangian equations)} \]

Non-conservative forces

\[ \dot{Q}_i = \frac{\partial P}{\partial \dot{q}_i} \quad P = -\frac{\partial}{\partial t} \frac{q_i}{m} \left( \dot{q}_i, \dot{f}_i \right)^n \]

Finding forces of constraint

\[ \mathbf{0} = \sum \frac{\partial \mathbf{h}}{\partial \dot{q}_i} \delta q_i \]

\[ \sum \left[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} - \sum F_i \frac{\partial \dot{q}_i}{\partial q_i} \right] \delta q_i = 0 \]

(1) Choose \( \lambda_i \) so first we \( \delta L \) vanish

(2) Given these \( \lambda_i \) we remain to finding

\[ \delta q_i \text{ in } \sum \]

\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = \dot{Q}_i = \sum \lambda_i \frac{\partial q_i}{\partial \dot{q}_i} \quad \text{(generalized forces of constraint)} \]

\[ L + \sum \lambda_i \mathbf{h}_i \] gives the above
(6) Functional calculus

\[ F[y'] \]

\[ SF[y_0, s y'] = \frac{d}{d\lambda} F[y_0 + \lambda s y'] \bigg|_{\lambda=0} \]

\( y, y_0 \) have some boundary condition

\( F[y_0] + SF[y_0, s y'] \) best linear approximation to \( F[y] \) at \( y = y_0 \)

Stationary \( y_0 \) \( \Rightarrow \) \( SF[y_0, s y'] = 0 \) all \( s \)

(6) Hamilton's principle - principle of stationary action

\[ A = \int_{t_1}^{t_2} L(\dot{y}(t), \ddot{y}(t), t) \, dt \]

\[ SA[y_0, s \ddot{y}] = 0 \]

\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \ddot{y}} \right) - \frac{\partial L}{\partial y} = 0 \quad q(t_1) = q_1, \quad q(t_2) = q_2 \]

(6) Second variation - condition for local minimum

\[ S^2 A[y_0, s y'] > 0 \quad \text{all } s \]

\[ \frac{d^2 A[y_0 + \lambda s y']}{d\lambda^2} \bigg|_{\lambda=0} \]
check
\[ p \{ s_{\phi} \} = s^2 \mathbf{A} \{ s_{\phi} \} \]
\[ s \{ s_{\phi} \} = 1 - \lambda s \{ \phi - c \} = 0 \]

gives eigenvalue problem - \( \infty \)
Hermitean matrix
Eigenvalues > 0 \( \Rightarrow \) local minimum

Homogeneous equation \( \lambda = 0 \)
Solution vanishes at \( t = \) conjugate point

6. Noether's Theorem

Action invariant under
\[ q_i(t') \rightarrow q_i(t') = q_i(t) + s q_i(q_i,t) \]
\[ t \rightarrow t' = t + s t(t) \]

then
\[ (L - \sum \frac{\partial L}{\partial q_i(q_i)} s t + \frac{\partial L}{\partial \dot{q}_i(q_i)} s \dot{q}_i) \]

is conserved

6. Fields

\[ L(\phi, \partial \phi) \] Lagrangian density
\[ L = \int L(\phi, \partial \phi) \, d^3x \]
\[ A = \int A(\phi, \partial \phi) \, d^3xdt \]
\[ \delta A / \delta \phi = 0 \]
\[ \frac{\partial}{\partial x^\mu}(\frac{\partial L}{\partial (\partial \phi)}) - \frac{\partial L}{\partial \phi} = 0 \quad \text{BC on 4 boundary} \]
(3) **Noether's Theorem for Fields**

\[ \phi_i(x,t) \rightarrow \phi_i'(x',t') = \phi_i(x,t) + \Delta \phi_i(x,t) \]
\[ x_t \rightarrow x'_t = x_t + (\delta x, \delta t) \]

**Action Invariant** 

\[ \mathcal{L} = \sum \phi^a \left( \frac{\partial \phi^a}{\partial x^\mu} \right) \frac{\partial x^\mu}{\partial x'^\nu} \phi^b \left( \frac{\partial \phi^b}{\partial x'^\nu} \right) - \frac{1}{2} \sum \frac{\partial^2 \phi^a}{\partial x^\mu \partial x^\nu} \delta^{ab} \]

\[ \partial_{\mu} J^\mu = 0 \quad \mathcal{Q} = \int J^\mu \delta^3 x \text{ conserved} \]

(2) **Small Oscillations about Static Equilibrium**

\[ \mathcal{Q} \quad \frac{\partial \mathcal{V}}{\partial q_i} (q_i) = 0 \]

stable \( \frac{\partial \mathcal{V}}{\partial q_i \partial q_i} (q_i) > 0 \) \ positive eigenvalue

gen coord \( \xi_i = q_i - q_{i_0} \)

assume \( \xi_i, \xi_i \) small (initial condition)

\[ T = \frac{1}{2} \sum \Sigma_{ij} \xi_i \xi_j \quad S \quad \frac{1}{2} \sum \Sigma_{ij} (q_{i_0} \xi_i \xi_j) \]

\[ \sum T_{ij} \dot{\xi}_i + \sum V_{ij} \dot{\xi}_j = 0 \quad (\text{Newton's Law}) \]

\[ \xi_i(t) = \xi_i(0) e^{i \omega t} \]

\[ \text{det} \left( -\omega^2 T + V \right) = 0 \]

gives frequencies and eigenvectors