HW #5 Due Friday, October 9

1.) Show for

\[ L_z = -i \frac{\partial}{\partial \phi}, \quad L_{\pm} = \pm e^{\pm i \phi} \left( \frac{\partial}{\partial \theta} \pm i \cot(\theta) \frac{\partial}{\partial \phi} \right) \]

that

\[ \int_0^\pi \sin(\theta) d\theta \int_0^{2\pi} d\phi (L_{\pm} f)^*(\theta, \phi) g(\theta, \phi) = \int_0^\pi \sin(\theta) d\theta \int_0^{2\pi} d\phi f^*(\theta, \phi) (L_{\mp} g)(\theta, \phi) \]

and

\[ \int_0^\pi \sin(\theta) d\theta \int_0^{2\pi} d\phi (L_z f)^*(\theta, \phi) g(\theta, \phi) = \int_0^\pi \sin(\theta) d\theta \int_0^{2\pi} d\phi f^*(\theta, \phi) (L_z g)(\theta, \phi) \]

2.) For

\[ L^2 = - \frac{1}{\sin(\theta)} \frac{\partial}{\partial \theta} \left( \sin(\theta) \frac{\partial}{\partial \theta} \right) - \frac{1}{\sin^2(\theta)} \frac{\partial^2}{\partial \phi^2} \]

and \( L_{\pm} \) and \( L_z \) as defined in (1) show:

\[ L_{\pm} L_{\mp} = L^2 - L_z (L_z \mp 1) \]

and

\[ [L^2, L_{\pm}] = [L^2, L_z] = 0 \]

3.) Prove equation (3.87) using the power series (3.82).

4.) 3.1

5.) 3.2a, 3.2b