1. Consider a differential equation of the form

\[ f''(x) + p(x)f'(x) + q(x)f(x) = 0 \]

In class we showed that if \( f(x) \) is a solution to this equation that we could find a second solution of the form \( g(x) = h(x)f(x) \). Following the lecture solve for \( h(x) \) and show that \( g(x) \) is independent of \( f(x) \) by showing the Wronskian:

\[ \det \begin{pmatrix} f(x) & f'(x) \\ g(x) & g'(x) \end{pmatrix} \neq 0 \]

2. Use the Feynman Hellmann theorem to compute the expectation value of the kinetic and potential energy of a one electron atom in a \( n,l,m \) state.

3. Apply the power series method to a three dimensional harmonic oscillator Hamiltonian

\[ H = -\frac{\hbar^2}{2\mu} \nabla^2 + \frac{1}{2}kr^2 \]

Find the asymptotic form of the solutions (large \( r \) and small \( r \)) and include this behavior in the solution. Find the recursion relating different terms in the series.

4. Consider a central force problem with Hamiltonian

\[ H = -\frac{\hbar^2}{2\mu} \nabla^2 + \lambda r^\alpha \]

Let \( |\psi\rangle \) be an eigenstate of \( H \). Use the Heisenberg equations of motion to show that for any operator \( O \)

\[ \frac{d}{dt} \langle \psi| O |\psi\rangle = 0 \]

Consider the case \( O = r \cdot k \). Show that for the Hamiltonian above this relates the expectation values of the potential and kinetic energies.

5. Show that

\[ L_n^\alpha(x) := \sum_{k=0}^{n} (-1)^k \frac{(n + \alpha)!}{k!(n - k)!(\alpha + k)!}x^k \]

is a solution to

\[ xL_n^\alpha(x)'' + (\alpha + 1 - x)L_n^\alpha(x)' + nL_n^\alpha(x) = 0 \]

6. Use the Feynman Hellmann theorem to compute \( \langle n|x^2|n\rangle \) for a 1 dimensional harmonic oscillator with Hamiltonian

\[ H = \frac{p^2}{2m} + \frac{1}{2}kx^2 \]