1. Assume that an electron is in an eigenstate of $| + x \rangle$ of $s_x$ with eigenvalue $\hbar/2$.
   
a. What is the result of applying the time reversal operator to this state?
   
b. What is the result of applying the time reversal operator to this state twice?
   
c. Consider the state $| \chi \rangle = \frac{i}{\sqrt{2}}(| + x \rangle - | - x \rangle)$. What is the result of applying the time reversal operator to this state.

2. Consider a one-dimensional harmonic oscillator Hamiltonian for a particle with mass $m$ and spring constant $k$.
   
a. What is the classical momentum as a function of $x$ and the harmonic oscillator energy?
   
b. What is the condition that determines the oscillator binding energies in the WKB approximation.
   
c. Find the approximate binding energies.

3. Consider the scattering of two nucleons of mass $m$ interacting via an attractive Yukawa interaction
   
   $$V(r) = -\lambda e^{-\alpha r}/r.$$ 

   a. Find the scattering amplitude for this reaction in the first Born approximation.
   
b. Find the differential cross section (in the Born approximation) as a function of angle in the center of momentum frame.
   
c. Use the result of part b.) to find the differential cross section (in the Born approximation) for electron-positron scattering with the Coulomb potential.
   
d. How does the result of part c.) change for electron-electron scattering.

4. Consider two particles of equal mass $m = 2\mu$ interacting via a delta-shell potential of range $R$ and strength $\lambda$:
   
   $$\langle r|V|r' \rangle = -\lambda \delta(r - r')\delta(r - R)$$
a. Solve the Lippmann-Schwinger equation for each partial wave
\[
\langle r|k^- \rangle_l = \langle r|k \rangle_l + \int_0^\infty g_l(r, r', k)V(r')r'^2 dr' \langle r'|k^- \rangle_l
\]
\[
\langle r|k \rangle_l = \frac{4\pi i^l j_l(kr/\hbar)/(2\pi\hbar)^{3/2}}{}
\]
\[
g_l(r, r', k) = -\frac{2i\mu k}{\hbar^3} \int_0^\infty j_l(kr'/\hbar)\hbar^l(kr'/\hbar)
\]
b. What is the form of this wave function for \( r > R \)?
c. What is the scattering amplitude for each \( l \)?
d. What is the full scattering amplitude as a function of the angle between the initial and final momenta?
e. Estimate the number of partial waves needed to calculate the scattering amplitude in d.) for incident momentum \( k \) in the center of mass system.