8)

there are two important observations

(1) the current through all devices in this circuit are identical

(2) the charges on all capacitors are identical because there is no <u>net</u> charge in the devices connecting the capacitors

using the loop equations

$$0 = \sum L_n \frac{dI_n}{dt} + \sum R_n I_n + \sum \frac{Q_n}{C_n}$$

$$= \frac{dI}{dt}\left(\sum L_n\right) + I\left(\sum R_n\right) + Q\left(\sum \frac{1}{C_n}\right)$$

this behaves like an RLC circuit with

$$L = \sum_n L_n \qquad R = \sum_n R_n \qquad \frac{1}{C} = \sum_n \frac{1}{C_n}$$

#21) $C = 64 \times 10^{-6} F$

$$I(t) = (1.6A) \sin(2500t + .680)$$

a) $2500t + .680 = \frac{\pi}{2}$

$$t = \left(\frac{\pi}{2} - .680\right)/2500 = 3.56 \times 10^{-4} s$$

b) $\omega = \frac{1}{\sqrt{LC}}$ $\quad L = \frac{1}{C\omega^2} = \frac{1}{(64 \times 10^{-6} F)(2500 s^{-1})^2} = 2.5 \times 10^{-3} H$

$=$

c) $\mathcal{E} = \frac{1}{2} L I^2_{max} = \frac{1}{2}(1.6A)^2 \times (2.5 \times 10^{-3}) = 3.2 \times 10^{-3} J$

25) $q(t) = q(0)\, e^{-Rt/2L} \cos(\omega' t + \phi)$

$\omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$

$L = 220 \times 10^{-3}\ \text{H}$

$C = 12 \times 10^{-6}\ \text{F}$

The time for 50 cycles is

$\omega' t = 50 \times 2\pi$

$t = 50 \times 2\pi \Big/ \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$

In that time

$\left|\frac{q(t)}{q(0)}\right| = e^{-Rt/2L} = .99$

$\ln .99 = -Rt/2L$

$\ln .99 = -\left(\frac{R}{2L}\right) 50 \times 2\pi \frac{1}{\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}}$

$(\ln .99)^2 \left(\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right) = \left(\frac{R}{2L}\right)^2 (50 \times 2\pi)^2$

$(\ln .99)^2 \frac{1}{LC} = \left(\frac{R}{2L}\right)^2 \left((50 \times 2\pi)^2 - \underbrace{(\ln .99)^2}_{\text{small compared to } (50 \times 2\pi)^2}\right)$

$\frac{R}{2L} \approx \frac{|\ln .99|}{\sqrt{LC}} \frac{1}{50 \times 2\pi}$

$R = \frac{|\ln .99|}{50\pi} \sqrt{\frac{L}{C}} = \frac{|\ln .99|}{50 \cdot \pi} \sqrt{\frac{220 \times 10^{-3}\ \text{H}}{12 \times 10^{-6}\ \text{F}}}$

$= 8.16 \times 10^{-3}\ \Omega$

27) $\mathcal{U} = \frac{1}{2} q^2/c$

$$= \frac{1}{2} \frac{q(o)^2}{c} e^{-Rt/L} \cos^2(\omega' t + \phi)$$

In one cycle $\omega \Delta t = 2\pi$ $\quad \Delta t = 2\pi/\omega'$

$$\Delta \mathcal{U} = \frac{1}{2} \frac{q(o)^2}{c} \left(e^{-Rt/L} - e^{-R(t+\Delta t)/L} \right) \cos^2(\omega' t + \phi)$$

$$= \frac{1}{2} \frac{q(o)^2}{c} e^{-Rt/L} \left(1 - e^{-R\Delta t/L} \right) \cos^2(\omega' t + \phi)$$

$$\frac{\Delta U}{U} = \left(1 - e^{-R\Delta t/L} \right) \simeq \frac{R\Delta t}{L} = \frac{2\pi R}{L\omega'} \simeq \frac{2\pi R}{L\omega}$$

(If the loss is small per cycle then $\omega' \simeq \omega$)

35) $I = \frac{\mathcal{E}_0}{Z} \cos(\omega t + \phi)$ $\qquad L = 88 \times 10^{-3} H$

$\mathcal{E} = \mathcal{E}_0 \cos(\omega t)$ $\qquad C = .94 \times 10^{-6} F$

$\qquad \omega = 2\pi \times 930\ s^{-1}$

$$\frac{X_C - X_L}{R} = \tan\phi$$

$$R = \cot\phi \left(\frac{1}{C\omega} - L\omega \right)$$

$$= \cot(75°) \left(\frac{1}{.94 \times 10^{-6} \times 2\pi \times 930} - 88 \times 10^{-3} \times 2\pi \times 930 \right)$$

$$= 8.9 \times 10^{1}$$

This circuit is inductive — we should have $\phi = -75°$

53) $R = 12\,\Omega$ $X_L = 1.3\,\Omega$ $\mathcal{E}_{rms} = 120\,V$

a) $Z = \sqrt{R^2 + X_L^2} = 12.07\,\Omega$

b) $P = I_{rms}^2 R = \left(\frac{\mathcal{E}_{rms}}{Z}\right)^2 R = \mathcal{E}^2 \frac{R}{Z^2}$

$= (120)^2 \frac{12}{12^2 + 1.3^2}$

$= 1.18 \times 10^3$ watts

56) The power supplied to the light bulb is

$P = \mathcal{E}^2 \frac{1}{Z} \cdot \frac{R}{Z} = \mathcal{E}^2 \frac{R}{L^2 + R^2}$

to find L max

a) $\frac{1}{5} P = \frac{1}{5} \frac{\mathcal{E}^2}{R} = \mathcal{E}^2 \frac{R}{L^2 + R^2} \Rightarrow$

$1 = \frac{5R^2}{L^2 + R^2} \Rightarrow L^2 = 4R^2$ $L = 2R$

$\therefore L = 2\left(\frac{\mathcal{E}^2}{1000\,W}\right) = \frac{2 \times (120)^2}{1000} = 28.8\,H$

d, b) yes – but additional energy would be lost through the other resistor

c) $I(R_1 + R_2) = \mathcal{E}$ $I = \frac{\mathcal{E}}{R_1 + R_2}$

$P_1 = \frac{1}{5} \frac{\mathcal{E}^2}{R} = \frac{\mathcal{E}^2}{(R_1 + R)^2} \cdot R$

$1 = \frac{5R^2}{(R_1 + R)^2} \Rightarrow R_1 + R = \sqrt{5} R$ $R_1 = (\sqrt{5} - 1) R$

$$R_1 = (\sqrt{5} - 1) \times \frac{\mathcal{E}^2}{1000\,\omega}$$

$$= (\sqrt{5} - 1) \frac{(120)^2}{1000} =$$

$$= 17.79\ \Omega$$