21.21 $\rho = b/r$

$$Q = \int_{r_1}^{r_2} r^2 dr \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi \left(\frac{b}{r}\right)$$

$$= \int_{r_1}^{r_2} br \, dr \int_0^{\pi} \sin\theta \, d\theta \cdot 2\pi$$

$$= \int_{r_1}^{r_2} br \, dr \cdot (-\cos(\pi) - (-\cos(0)) \, 2\pi$$

$$= 4\pi b \left(\frac{r_2^2}{2} - \frac{r_1^2}{2}\right)$$

$$Q = 2\pi b \, (r_2 - r_1)(r_2 + r_1)$$

$$2\pi \left(3 \times 10^{-6} \tfrac{c}{m^2}\right)(6 - 4 \times 10^{-2} m)(6 + 4 \times 10^{-2} m)$$

$$1.20\pi \times 10^{-10} \; c$$

21.42

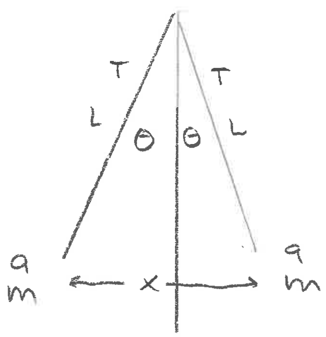
		step 1	step 2	step 3
QW		QA/2	(QA/2 − 32e)/2	$\boxed{\left(\frac{QA}{2} - 32e\right)\frac{1}{4} + \frac{48e}{4} = 18e}$
QA		QA/2	QA/2	QA/2
QB	(−32e)	(−32e)	(QA/2 − 32e)/2	(QA/2 − 32e)/2
QC	(+48e)	(48e)	(48e)	$\left(\frac{QA}{2} - 32e\right)\frac{1}{4} + \frac{48e}{4}$

The equation in the box is

$$\frac{QA}{8} = 18e - 12e + 8e = -2$$

$\boxed{QA = -16e}$ (at each state the total change on 2 spheres is shared equally

21.42



The system is in equilibrium so the forces must balance (for each charge)

$$F_x = -T\sin\theta + \frac{kq^2}{x^2} = 0$$

$$F_y = T\cos\theta - mg = 0$$

we also have

$$\sin\theta = \frac{x}{2L}$$

① eliminate T

$$\frac{\sin\theta}{\cos\theta} = \frac{kq^2}{x^2}\cdot\frac{1}{mg}$$

② eliminate θ $\quad \sin\theta = \frac{x}{2L} \quad \cos\theta = \left(1-\left(\frac{x}{2L}\right)^2\right)^{1/2}$

$$\frac{x/2L}{\left(1-\left(\frac{x}{2L}\right)^2\right)^{1/2}} = \frac{kq^2}{x^2mg}$$

assume $\frac{x}{2L} \ll 1 \longrightarrow$

$$(x/2L) \approx \frac{kq^2}{x^2mg} \rightarrow x^3 = \frac{2Lkq^2}{mg} = \frac{Lq^2}{2\pi\epsilon_0 mg}$$

$$\boxed{x \simeq \left(\frac{Lq^2}{2\pi\epsilon_0 mg}\right)^{1/3} \text{ for } \frac{x}{2L} \ll 1}$$

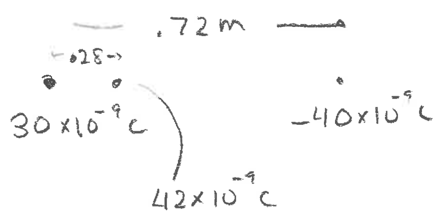
$$q^2 = \frac{2\pi x^3 \epsilon m g}{L}$$

$$q = \sqrt{\frac{2\pi x^3 \epsilon_0 m g}{L}}$$

$$= \left(\frac{2\pi (5\times10^{-2}\,m)^3 (8.85\times10^{-12}\,\frac{C^2}{m^2 N})(10\times10^{-3}\,k)(9.8\,\frac{m}{s^2})}{.12\,m} \right)^{\frac{1}{2}}$$

check $\frac{x}{2L} = \frac{5\,cm}{240\,cm} = \frac{1}{120} << 1$

21.63



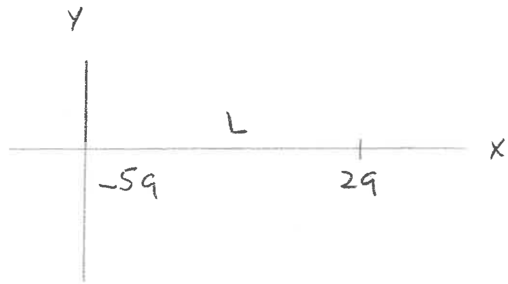
note both forces are in the positive x direction

$$ma = F = k\frac{q_1 q}{r_1^2} + k\frac{q_2 q}{r_2^2}$$

$$m = \frac{kq}{a}\left(\frac{|q_1|}{r_1^2} + \frac{|q_2|}{r_2^2} \right)$$

$$= \frac{8.99\times10^9\,\frac{Nm^2}{C^2}}{100\times10^3\,m/s^2} \times 42\times10^{-9}\,C \left(\frac{30\times10^{-9}\,C}{(.28\,m)^2} + \frac{40\times10^{-9}\,C}{(.72-.28)^2} \right)$$

21.67



The equilibrium conditions are

$$0 = F_x = k(-5q)q \frac{x}{(x^2+y^2)^{3/2}} + k(2q)q \frac{x-L}{((x-L)^2+y^2)^{3/2}}$$

$$0 = F_y = k(-5q)q \frac{y}{(x^2+y^2)^{3/2}} + k(2q)q \frac{y}{((x-L)^2+y^2)^{3/2}}$$

taking ratios

$$\frac{y}{x} = \frac{y}{x-L} \Rightarrow xy - yL = xy \Rightarrow yL = 0$$

so F_y must vanish - for $y=0$ the first equation becomes

$$\frac{5}{x^2} = \frac{2}{(x-L)^2}$$

taking square roots

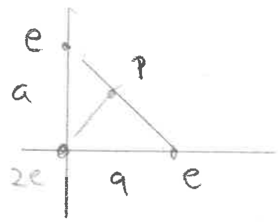
$$\frac{\sqrt{5}}{x} = \frac{\sqrt{2}}{x-L} \qquad \sqrt{5}(x-L) = \sqrt{2}\,x$$

$$(\sqrt{5}-\sqrt{2})x = \sqrt{5}L$$

$$x = \frac{\sqrt{5}}{\sqrt{5}-\sqrt{2}} L = 2.72 L$$

This makes sense because x must be larger that L

22.15



$a = 6 \times 10^{-m}$

$$\bar{E}(P) = ke\left(\frac{\left(\frac{a}{2}\ \frac{a}{2}\right) - (0\ a)}{\left(\frac{a^2}{4} + \frac{a^2}{4}\right)^{3/2}}\right) +$$

$$ke\,\frac{\left(\frac{a}{2}\ \frac{a}{2}\right) - (a, 0)}{\left(\frac{a^2}{4} + \frac{a^2}{4}\right)^{3/2}} +$$

$$+\ 2ke\,\frac{\left(\frac{a}{2}, \frac{a}{2}\right) - (0, 0)}{\left(\frac{a^2}{4} + \frac{a^2}{4}\right)^{3/2}} =$$

$$= \frac{ke}{(a/\sqrt{2})^3}\left(2\frac{a}{2}, 2\frac{a}{2}\right)$$

$$\bar{E}(P) = \frac{ke}{a^2}\,2\sqrt{2}\,(1, 1)$$

We see all of the field at this point is due to the charge at the origin