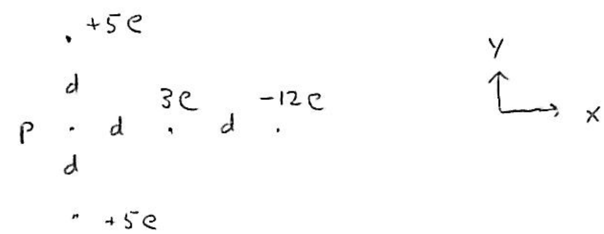
Homework # 3

8



because changes 1,2 are equal and the same distance from P - the y component of the electric field at P is zero

For the x component $(q = |e|)$

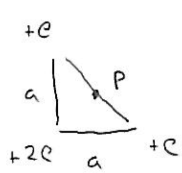
$$E_x = +k\,3q\,\frac{1}{d^2} - 12kq\,\frac{1}{4d^2}$$

$$= k\,3q\,\frac{1}{d^2} - 3kq\,\frac{1}{d^2}$$

$$= 0$$

It follow that the net electric field at P is zero.

15



As in the previous problem the field due to the +e changes at the point P cancel.

The field at P is due entirely to

the particle with charge 2e. Since it is positive it is parallel to the line from the charge 2e to the point P. The magnitude of the field is

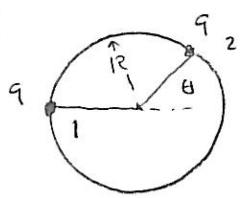
$$E = k(2e)\frac{1}{(a/\sqrt{2})^2} = \frac{4ke}{a^2} =$$

$$= \frac{(8.99\times10^9\ Nm^2/C^2)\cdot 4\cdot(1.602\times10^{-19}/C)}{(6\times10^{-6}m)^2}$$

$$= \frac{8.99\cdot 4\cdot 1.602}{36}\times 10^2\ \frac{N}{C}$$

$$= 1.6\times10^2\ N/C$$

16

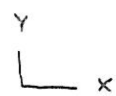


$R = .5\,m$

$q_1 = +2\times10^{-6}\,C$

$|\bar{E}(0)| = 2\times10^5\ N/C$

$q_2 = +6\times10^{-6}$



$$\bar{E}(0) = \bar{E}_1(0) + \bar{E}_2(0)$$

$$= kq_1\frac{1}{R^2}\hat{x} - kq_2\frac{1}{R^2}\cos\theta\,\hat{x} - kq_2\frac{1}{R^2}\sin\theta\,\hat{y}$$

$$= \frac{k}{R^2}\left((q_1 - q_2\cos\theta)\hat{x} - q_2\sin\theta\,\hat{y}\right)$$

$$|E(0)|^2 = \frac{k^2}{R^4}\left(q_1^2 + q_2^2 - 2q_1q_2\cos\theta\right)$$

$$\cos\theta = \frac{q_1^2 + q_2^2 - \frac{R^4}{k^2}|\bar{E}(0)|^2}{2q_1q_2}$$

$$\cos\theta = \frac{(2\times10^{-6}c)^2 + (6\times10^{-6}c)^2 - \frac{(.5m)^4}{(8.99\times10^9\,N\,m^2/c^2)^2\,(2\times10^5 N/c)^2}}{2\,(2\times10^{-6}c)(6\times10^{-6}c)}$$

$$= \frac{4+36-30.93}{24} = .378$$

$\theta = 67.8°$, 292.2 or $\pm 67.8°$

21. Note in this problem we need to keep corrections beyond a point dipole.

By symmetry the field at P will be parallel to the orientations of the dipoles

$$E_x(x,0,0) = kq\left(\frac{1}{(x+d)^2} - \frac{2}{x^2} + \frac{1}{(x-d)^2}\right)$$

$$= kq\,\frac{1}{x^2}\left(\frac{1}{(1+d/x)^2} - 1 + \frac{1}{(1-d/x)^2}\right)$$

recall $\left(\frac{1}{1-\epsilon}\right)^2 = (1+\epsilon+\epsilon^2+\dots)(1+\epsilon+\epsilon^2+\dots)$

$$= 1 + 2\epsilon + 3\epsilon^2 + \dots$$

similarly

$$\frac{1}{(1+\epsilon)^2} = 1 - 2\epsilon + 3\epsilon^2 + \dots$$

For $\epsilon = \frac{d}{x}$ we get

$$E_x(0,0,x) = \frac{kq}{x^2}\left(1 - 2\frac{d}{x} + 3\frac{d^2}{x^2} + \cdots - 2 + 1 + 2\frac{d}{x} + 3\frac{d^2}{x^2} + \cdots\right)$$

$$= 6\frac{kq}{x^2}\left(\frac{d^2}{x^2}\right) + \cdots$$

$$= \frac{6}{4\pi\epsilon_0}\frac{q}{x^2}\frac{d^2}{x^2} + \cdots$$

$$= \frac{3}{4\pi\epsilon_0}\frac{(2qd^2)}{x^4} \qquad Q \equiv 2qd^2$$

$$= \frac{3}{4\pi\epsilon_0}\frac{Q}{x^4}$$

$\underline{\underline{23}}$ Recall for a ring of charge

$$E_z(0,0,z) = k\left(\frac{q}{2\pi R}\right)\frac{2\pi R z}{(R^2+z^2)^{3/2}} = \frac{kqz}{(R^2+z^2)^{3/2}}$$

For this problem we put the rings in the y-z plane. The field at P is in the x direction

$$E_x(0,0,0) = kq_1\frac{R\hat{x}}{(R^2+R^2)^{3/2}} + kq_2\frac{-(d-R)\hat{x}}{(R^2+(d-R)^2)^{3/2}}$$

For this to vanish

$$\frac{q_1}{q_2} = \frac{(d-R)}{(R^2+(d-R)^2)^{3/2}} \cdot \frac{2\sqrt{2}R^2}{1}$$

For $d = 3R$ this becomes

$$\frac{q_1}{q_2} = \frac{2R}{(5R^2)^{3/2}} \frac{2\sqrt{2}R^2}{1} = 5.06$$

$\underline{\underline{48}}$ Since $\bar{F} = m\bar{a} = q\bar{E}$

$$a = \frac{q}{m} \bar{E}$$

For a uniformly charged disc

$$E = 2\pi k \sigma \left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$$

$$a = \frac{2\pi k q \sigma}{m} \left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$$

$$\frac{2\pi k \sigma}{m_e} = \frac{6.28 \times 8.99 \times 10^{9} \frac{Nm^2}{C^2} 4 \times 10^{-6} \frac{C}{m^2}}{9.1 \times 10^{-31} k}$$

$$= 2.48 \times 10^{35} m/s^2$$

for $z = R$

$$a = \frac{2\pi k q_e \sigma}{m_e} \left(1 - \frac{R}{\sqrt{2}R}\right)$$

$$= \frac{2\pi k \sigma q_e}{m_e} \left(1 - \frac{1}{\sqrt{2}}\right) = 7.26 \times 10^{34} m/s^2$$

for $z = R/100$

$$a = \frac{2\pi k \sigma q_e}{m_e} \left(1 - \frac{1}{100} \frac{1}{\sqrt{1 + (\frac{1}{100})^2}}\right)$$

$$= 2.46 \times 10^{35} m/s^2$$

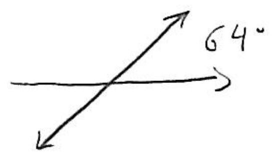
for $z = R/100$

$$a = \frac{2\pi k \sigma q_e}{m_e} \left(1 - \frac{1}{1000} \frac{1}{\sqrt{1 + (\frac{1}{1000})^2}}\right) = 2.48 \times 10^{35}$$

The last 2 numbers are close because close to the disc the z component of this force is primarily from charges close to the center of the disk.

59.)

$$p = 3.02 \times 10^{-25} \text{ C}\cdot\text{m}$$



$$\bar{E} = 46 \text{ N/C}$$

The work required is the change in potential energy

$$\Delta W = -\bar{E}\cdot\bar{p}_{64} + \bar{E}\cdot\bar{p}_{64+180}$$

$$= 2|E||p|\cos(64°)$$

$$= 1.21 \times 10^{-23} \text{ N}\cdot\text{m}$$