HW Solutions

53 $L = 6.3 \times 10^{-6} H$

$R = 1.2 \times 10^{3}\ \Omega$

$\mathcal{E} = 14 V$

$$\mathcal{E}(t) = L\frac{dI}{dt} + IR \Rightarrow I(t) = \frac{\mathcal{E}}{R}(1 - e^{-\frac{tR}{L}})$$

a) $$.8\frac{\mathcal{E}}{R} = \frac{\mathcal{E}}{R}(1 - e^{-tR/L})$$

$$.2 = e^{-tR/L}$$

$$\ln\left(\frac{1}{5}\right) = -\frac{tR}{L} = -\ln 5$$

$$t = \frac{L}{R}\ln 5 = \frac{6.3\times10^{-6}H}{1.2\times10^{3}\Omega}\ln 5 = 8.45\times10^{-9}$$

b) $$I(t) = \frac{\mathcal{E}}{R}(1 - e^{-1}) = \frac{\mathcal{E}}{R}\left(\frac{e-1}{e}\right)$$

$$= \frac{14V}{1.2\times10^{3}\Omega}\left(\frac{e-1}{e}\right) = 7.4\times10^{-3} A$$

57a $R = 15\Omega$ $L = 5H$ $\mathcal{E} = 10V$

when the switch is closed the current through the fuse is the same as the current through the inductor

$$\mathcal{E} = L\frac{dI}{dt} \Rightarrow \frac{dI}{dt} = \frac{\mathcal{E}}{L} \quad I(t) = \frac{\mathcal{E}}{L}t$$

$$t = \frac{L\cdot I_{max}}{\mathcal{E}} = \frac{(5H)(3A)}{(10V)} = 1.5 \text{ sec}$$

62

$L = 2.0\,H \qquad \mathcal{E} = 100\,V$

$R = 10\,\Omega$

$$I(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-tR/L}\right)$$

(a)

$$\frac{dU_B}{dt} = \frac{d}{dt}\left(\frac{1}{2}LI^2\right) = \frac{1}{2}\,2LI\frac{dI}{dt}$$

$$= L\frac{\mathcal{E}}{R}\left(1 - e^{-tR/L}\right)\frac{\mathcal{E}}{R}\frac{R}{L}e^{-tR/L}$$

$$= \frac{\mathcal{E}^2}{R}\left(e^{-tR/L} - e^{-2tR/L}\right)$$

$$= \frac{(100\,V)^2}{10\,\Omega}\left(e^{-\frac{10}{2\cdot 10}} - e^{-\frac{2\cdot 10}{2\cdot 10}}\right)$$

$$= 2.39 \times 10^2\,W$$

(b)

$$\frac{dU_R}{dt} = I^2R = \frac{\mathcal{E}^2}{R}\left(1 - e^{-tR/L}\right)^2$$

$$= \frac{\mathcal{E}^2}{R}\left(1 - 2e^{-tR/L} + e^{-2tR/L}\right)$$

$$= \frac{(100\,V)^2}{10}\left(1 - 2e^{-\frac{10}{2\cdot 10}} + e^{-\frac{2\cdot 10}{2\cdot 10}}\right)$$

$$= 1.55 \times 10^2\,W$$

(c)

$$\frac{dU}{dt} = \mathcal{E}I = \frac{\mathcal{E}^2}{R}\left(1 - e^{-tR/L}\right)$$

$$= \frac{(100\,V)^2}{10}\left(1 - e^{-\frac{10}{2\cdot 10}}\right)$$

$$= 3.94 \times 10^2\,W$$

Note $a + b = c$ as energy is conserved

77

a) $\mathcal{E} = \underbrace{-L_1\frac{dI_1}{dt} - M\frac{dI_2}{dt}}_{\substack{\text{EMF in coil}\\ \text{1 due to self}\\ \text{and mutual}\\ \text{inductance}}} \underbrace{- L_2\frac{dI_2}{dt} - M\frac{dI_1}{dt}}_{\substack{\text{EMF in coil}\\ \text{2 due to self}\\ \text{and mutual}\\ \text{inductance}}}$

since $I_1 = I_2 \Rightarrow$

$$\mathcal{E} = -\underbrace{(L_1 + M + L_2 + M)}_{\substack{\text{effective inductance}\\ \text{of the 2 coil system}}}\frac{dI}{dt}$$

b) 

this would change the direction of the flux and induced field in the second coil

It changes the direction of the mutual inductance EMF

$$\mathcal{E} \to \underbrace{(L_1 - M + L_2 - M)}_{\text{effective } L}\frac{dI}{dt}$$

4) $U = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}L\frac{d^2Q}{dt^2}$

a) $C = \frac{Q^2}{2U} = \frac{(1.6 \times 10^{-6}C)^2}{2 \times 140 \times 10^{-6}} = 9.14 \times 10^{-9} F$

9) $L = 50 \times 10^{-3}$ $C = 4 \times 10^{-6} F$

maximum current $\Rightarrow \frac{1}{4}$ period
will be maximum charge

$\frac{1}{4}T = \frac{1}{4f} = \frac{2\pi}{4\omega} = \frac{\pi}{2}\sqrt{LC}$

$\frac{1}{4}T = \frac{\pi}{2}\sqrt{(50 \times 10^{-3})(4 \times 10^{-6})} = 7.02 \times 10^{-4} s$

10) $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$

$L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2 (10 \times 10^3)^2 (6.7 \times 10^{-6} F)} = 3.78 \times 10^{-3} H$