Lecture 13

So far we have only considered static electric charges - i.e. charges that do not move.

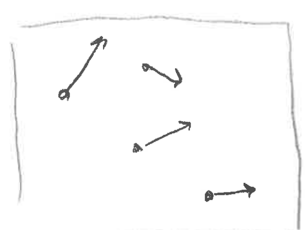
The next step is to consider charges that move

While is is possible to consider the motion of single charges, in many situations there are too many single charges to keep track of. In order to treat the motion of many charges we introduce a new vector field $\vec{J}(\vec{r},t)$ called the current density

$$\boxed{\bar{J}(\bar{r},t) = \rho(\bar{r},t)\,\bar{V}_n(r,t)}$$

where $\rho(\bar{r},t)$ is the number of charges per unit volume at $\bar{r}$ at time t.

$\bar{V}(r,t)$ is called the drift velocity. It is the average velocity of moving charges in a small region near $\bar{r}$ t (by definition it is in the direction of moving positive charge. It points in the opposite direction for electron



add these velocity vectors. divide by the number of particles. (-) if the particles are electrons

For charge carries with charge q

$$\boxed{\bar{J} = q n_q \bar{v}_D}$$

where n_q is the number of particles with charge q per unit volume and $\bar{v}_D$ is the drift velocity for those particles

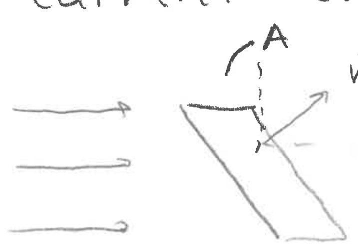
example

assume we have 10^{18} electrons per cubic centimeter moving with drift velocity $.01\,m/s$ in the x direction Then

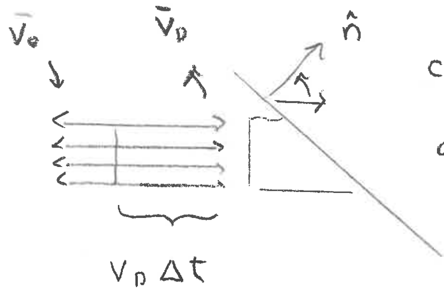
$$\boxed{\bar{J} = (q_e)\,\frac{10^{18}}{(10^{-2})^3} \times .01\,(-\hat{x})}$$

because the electrons have negative charge

For this current consider



how much charge passes through A per second ?



$\cos\theta = \hat{n}\cdot\hat{v}_D$

$dA_\perp = dA\, \hat{n}\cdot\hat{v}_D$

electrons $\longleftarrow$

The total positive transported through dA is

$$\Delta Q = \underbrace{(V_D \Delta t)(dA_\perp)}_{\text{volume}}(q_e)(\underset{\uparrow}{n})$$

electrons / volume

$$= (V_D \Delta t)(dA\, \hat{n}\cdot\hat{v}_D)(n)(q_e)$$

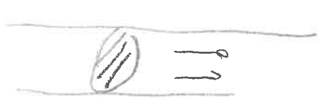
$$= (V_D \hat{v}_D)\cdot(\hat{n}\, dA)\, n q\, \Delta t$$

$$= \bar{V}_D \cdot \hat{n}\, dA\; n q\, \Delta t$$

$$\frac{\Delta Q}{\Delta t} = (n q \bar{V}_D)\cdot(\hat{n}\, dA)$$

$$\boxed{\frac{dQ}{dt} = \bar{J}\cdot\hat{n}\, dA}$$

In this example n and $\bar{V}_D$ are constant. In general they may depend on $(\bar{x}, t)$



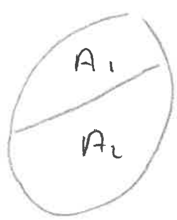
$$\frac{dQ}{dt} = \int_S \bar{J}\cdot\hat{n}\, dA$$ = is called the electric current through the area A

The unit of current is

$$\boxed{1 \text{ Ampere} = \frac{1 \text{ coulomb}}{\text{second}}}$$

Recall - this is how the coulomb is defined.

If we break the area up into 2 parts $A = A_1 \cup A_2$



$$\int_A \bar{J} \cdot \hat{n}\, dA = \int_{A_1} \bar{J} \cdot \hat{n}\, dA + \int_{A_2} \bar{J} \cdot \hat{n}\, dA$$

$$I = I_1 + I_2$$

This is nothing more than saying change can neither be created or destroyed.

What is the drift velocity of electrons in a copper wire of cross sectional area $1(mm)^2$ at a current of 1 Ampere?

(Assume n is uniform)

$$I = JA = (n V_D e)(1mm)^2$$

$$V_D = \frac{1 \text{ Ampere}}{(1mm)^2 (q_e)(n_{cu})}$$

$q_e = 1.6 \times 10^{-19}\ C$

$n_{cu} = \left(\frac{atoms}{mole}\right)\left(\frac{moles}{kg}\right)\left(\frac{kg}{m^3}\right)$

$(6.02 \times 10^{23})(63.54 \times 10^3)(8.96 \times 10^{-3} \times 10^6)$

Appendix F

$\left(\frac{mole}{g}\ \frac{g}{kg}\right)\left(\frac{g}{cm^3}\ \frac{k}{g}\ \frac{cm^3}{m^3}\right)$

$= 3.42 \times 10^{32}$

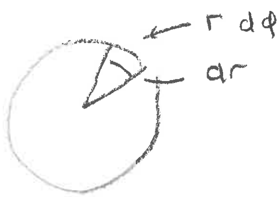
$v_D = \frac{1}{10^{-6}(1.6 \times 10^{-19})(3.42 \times 10^{32})} = 1.8 \times 10^{-8}\ m/s$

which shows that the drift velocity through a copper wire at 1 amper is still very slow.

In general the current density will not be uniform.

$\bar{J}(\bar{r}\, t) = j(r)\,\hat{x}$

$I = \int_0^R dr \int_0^{2\pi} r\, d\phi\, (j(r))\, \hat{x}\cdot\hat{x} = 2\pi \int r\, dr\, j(r)$



If $j(r) = Cr$

$I = 2\pi \int r\, dr\, (Cr) = \frac{2\pi}{3} C R^3$

consider a collection of electrons in a constant electric field in the $\hat{x}$ direction

$$\bar{E} = E\hat{x}$$

the force on an electron is

$$\bar{F} = q_e \bar{E} = q_e E \hat{x}$$

(where q_e is negative). If the electron is in a medium, it will collide. the number of collisions per second will be approximatey proportional to the velocity. This suggest that the net force is

$$\bar{F}_{net} = q_e E \hat{x} - c\vec{V}_D$$

the electron will accelerate until the force vanishes

$$\bar{V}_D = \frac{q_e \bar{E}}{c}$$

using this in the definition of of the current

$$\bar{J} = q n V_D = q n \left(\frac{q_e \bar{E}}{c} \right)$$

$$= q n \left(\frac{q_e}{c} \right) \bar{E}$$

this shows that the current density is proportional to the electric field strength

The relation

$$V = IR$$

is called Ohm's law. The derivation followed from the assumption that the effect of collisions was due to a damping force proportional to the drift velocity

Going back to the definition

$$\bar{J} = qn\bar{V}_D$$
$$= qn\left(\frac{q}{c}\bar{E}\right)$$

which show that the current density is proportional to the electric field

This is usually expressed as

$$\bar{E} = \rho\bar{J}$$

where ρ is called the <u>resistivity</u> - which in this case only depends on the material. This relation can equivalently be expressed as

$$\bar{J} = \sigma\bar{E}$$

This is written as

$$\bar{J} = \sigma \bar{E}$$

where σ is called the conductivity of the material. This can also be written

$$\bar{E} = \frac{1}{\sigma}\bar{J} = \rho \bar{J}$$

$\rho = \frac{1}{\sigma}$ is called the resistivity of the material.

Next consider a wire of cross sectional area A and length ℓ. Assume the ends are maintained at a potential V

$$\bar{E} \cdot \bar{L} = V = \text{potential difference}$$

$$I = \int \bar{J} \cdot \hat{n}\, dA = \int \sigma \bar{E} \cdot \hat{n}\, dA = \sigma \frac{V}{L} A = \left(\sigma \frac{A}{L}\right) V$$

This shows that the current is proportional to the potential difference

$$V = IR \;;\; R = \frac{1}{\sigma}\frac{L}{A} = \rho \frac{L}{A}$$

R is called the resistance

The unit of resistance is called

$$1 \text{ ohm} = \frac{1 \text{ Volt}}{1 \text{ Amp}} = \frac{\text{Joule}}{\text{Coulomb}} \cdot \frac{\text{Second}}{\text{Coulomb}}$$

The relation

$$V = IR$$

is called ohms law

Note for a wire

$$R = \rho \frac{\ell}{A}$$

which means the resistance increases with length, and decreases with increasing cross sectional area.

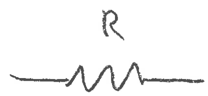
The resistivity generally increases with temperature.

$$\rho = \rho_0 + \rho_0 \alpha (T - T_0)$$

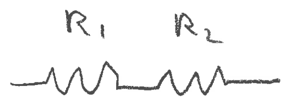
$$\rho_0 = \rho(T_0)$$

α is called the temperature coefficient of resistivity.

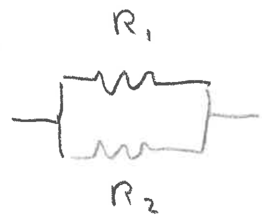
Circuits involving resistors



notation for resistors

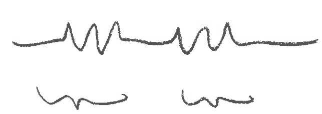


resistors in series



resistors in parallel

Resistors in series:



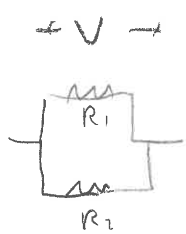
$I_1 = I_2 = I$ (charge is conserved)

$V_1 = IR_1 \quad V_2 = IR_2$

$V = V_1 + V_2 = I(R_1 + R_2) = IR$

$$\boxed{\therefore R = R_1 + R_2}$$ (resistors in series)

Resistors in parallel



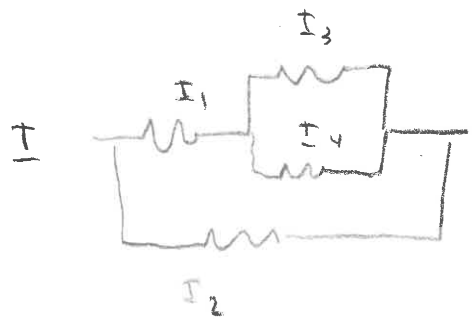
$I = I_1 + I_2$ charge conservation

$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$

$\therefore \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$$\boxed{R = \frac{R_1 R_2}{R_1 + R_2}}$$ (resistors in parallel)

These rules are the opposite of the corresponding rules for capacitors



To calculate the total resistance

$$R_{34} = \frac{R_3 R_4}{R_3 + R_4} \qquad R_{134} = R_1 + \frac{R_3 R_4}{R_3 + R_4}$$

$$\frac{1}{R} = \frac{1}{R_2} + \frac{1}{R_1 + \frac{R_3 R_4}{R_3 + R_4}} =$$

$$R = \frac{R_2 \left(R_1 + \frac{R_3 R_4}{R_3 + R_4} \right)}{R_1 + R_2 + \frac{R_3 R_4}{R_3 + R_4}} = \frac{R_2 (R_1 R_3 + R_1 R_4 + R_3 R_4)}{R_1 R_3 + R_2 R_4 + R_3 R_4}$$

$$V = IR$$

$$I_2 = \frac{V}{R_2}$$

$$I_1 = \frac{V}{R_{134}} = \frac{V}{R_1 + \frac{R_3 R_4}{R_3 + R_4}}$$

$$V_1 = I_1 R$$

$$I_3 = \frac{V - V_1}{R_3}$$

$$I_4 = \frac{V - V_1}{R_4}$$