Lecture 14

Last time we defined a new vector field: The electric current density

$$\boxed{\bar{J}(\bar{x},t) = q\, n(x,t)\, \bar{V}_D(x,t)}$$

q = charge of charge carrier

$n(\bar{x},t)$ = # charge carriers / volume

$\bar{V}_D$ = drift velocity of charge

The current through an area A is related to the current density by

$$\boxed{I = \int_A \bar{J}(\bar{x},t) \cdot \underbrace{\hat{n}(\bar{x}t)}_{\text{unit normal to the surface}}\, dA}$$

I measures the amount of positive charge that passes through the surface per unit time.

The force on a charge is

$$\bar{F} = q\bar{E} - c\bar{V}$$

where $c\bar{V}$ represents the effect of collisions on the charge – it opposes the motion of the charge and is proportional to the velocity when the velocity reaches a steady state

$$\bar{F} = 0 \Rightarrow \bar{V}_D = \frac{q}{c}\bar{E}$$

This implies (for most materials) that the drift velocity is proportional to the electric field.

This gives

$$\boxed{\bar{J}(\bar{x},t) = q\,n(\bar{x},t)\frac{q}{c}\bar{E} = \sigma(\bar{x},t)\,\bar{E}(\bar{x}t)}$$

where $\sigma(\bar{x},t)$ is called the conductivity of the medium (it depends on the medium because the medium causes the collisions)

$$\boxed{\rho(x,t) = \frac{1}{\sigma(x,t)}}$$

is called the resistivity

For the current

$$I = \int_A \bar{J}\cdot\hat{n}\,dA = \int \frac{1}{\rho} E\cdot n\,dA$$

For a wire of length L and cross sectional area A

$$EL = V$$

potential between the 2 ends of

the wire. Assuming $\bar{E}, \rho$ constant

$$I = \frac{1}{\rho}\frac{V}{L}A$$

$$V = \left(\rho\frac{L}{A}\right)I$$

The quantity $\left(\rho\frac{L}{A}\right) = R$ is called the resistance. It is measured in ohms.

$$1 \text{ ohm} = \frac{1 \text{ volt}}{1 \text{ ampere}}$$

The above equation is written as

$$\boxed{V = IR} \qquad \boxed{R = \rho\frac{L}{A}}$$

This relation is called ohm's law. It states that the current is proportional to the voltage

Note that the resistance increases proportional to the length of the wire, and decreases as the area is increased.

experimentally

$$R = \frac{V}{I}$$

R is constant for materials obeying ohms law, but not all materials have this property.

In general, since raising the temperature results in more collisions per unit time; $\rho(x t)$ depends on temperature

for small temperature changes

$$\rho(T) \approx \rho(T_0) + \rho(T_0)\alpha(T-T_0)$$

$$\rho(T_0)(1+\alpha(T-T_0))$$

α has units of inverse temperature. It is called the temperature coefficient of resistivity. It depends on the material - see table on p 754 of text

Note that both insulators and conductors differ primarily in their resistivity, which is much higher for insulators

Recall

$$\boxed{V_D = \frac{qE}{c}}$$

The physical picture is that there are collisions that reset the velocity, the mean time between collisions is called τ

$$V_D = a\tau = \frac{F}{m}\tau = \frac{qE}{m}\tau$$

comparing to the above we see

$$\boxed{\frac{1}{c} = \frac{\tau}{m}}$$

we remark that the velocity the electron has 2 components - a drift component that is directed, and a thermal component that is random.

Power = change in potential energy / time

$$dU = V\,dQ = V\frac{dQ}{dt}dt = VI\,dt =$$

$$\boxed{P = \frac{dU}{dt} = IV}$$

For a given current

$$P = IV = I^2 R$$

For a given voltage

$$P = IV = \frac{V^2}{R}$$

The unit of power is 1 watt

$$1 \text{ Watt} = \frac{1 \text{ Joule}}{1 \text{ second}}$$

example: what is the resistance of a 100 watt light bulb

$$P = 100\,W = \frac{V^2}{R} = \frac{(110\,V)^2}{R}$$

$$R = \frac{(110)^2}{100}\,\Omega = 121\,\Omega$$

While the resistivity is a property of the material - the resistance also depends on the geometry

recall

$$R = \rho \frac{L}{A}$$

30 meter extension cord has 3x the resistance of a 10 meter cord

while we think of resistors as devices, almost any electrical device that draws power acts as a resistor

fan

hair dryer

oven

light bulb

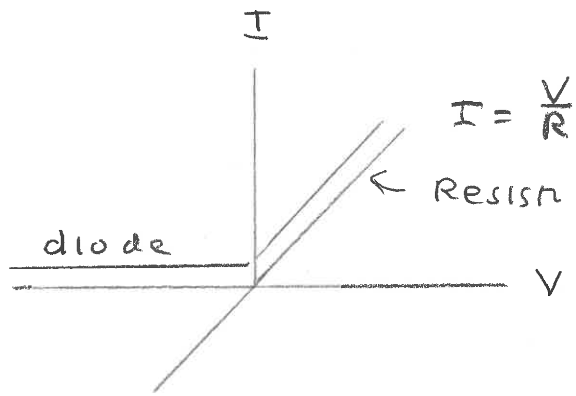
motor

radio

heater

The dissipated power goes into work or heat, or radiation

Experimentally a resistor obeying ohms law has a linear relation involving



$I = \frac{V}{R}$ $\quad \frac{1}{R} = \text{slope}$

There are devices that dont obey ohms law -

some devices not have a linear relation

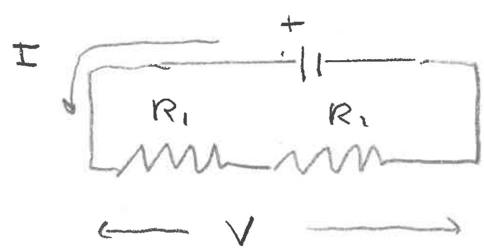
diodes only allow current in one direction

superconductors - have 0 resistance these normally have to be very cold - currents can persist for years in a superconductor

For resistors obeying ohms law we can treat circuits like we do with capacitors, but the mathematics is different

(1) current must be conserved

(2) potential change around closed loop is 0.



resistors in series

* current conserved

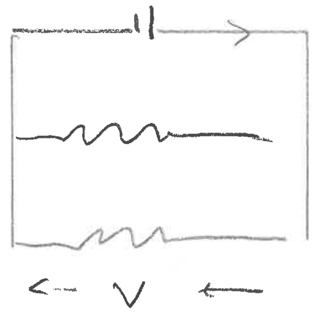
$$I = I_1 = I_2$$

$$V = V_1 + V_2 = IR_1 + IR_2 = IR$$

cancel I

$$\boxed{R = R_1 + R_2 \quad \text{resistors in series}}$$

resistors in parallel



* In this case current conservation gives

$$I = I_1 + I_2$$

the potential across both resistors is the same

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\boxed{R = \frac{R_1 R_2}{R_1 + R_2}}$$

consider 2 Identical resistors in series - voltage V

$$P = \frac{V^2}{R} \rightarrow \frac{V^2}{2R}$$

power reduced by a factor of 2

for the same 2 resistors in series

$$R = \frac{RR}{R+R} = \frac{R}{2}$$

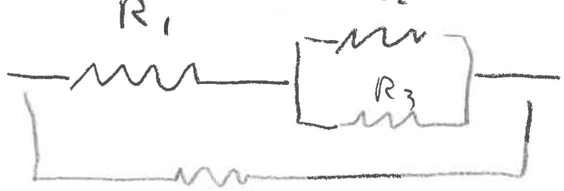
$$P = \frac{V^2}{R} \rightarrow 2\frac{V^2}{R}$$

we see in parallel the power is doubled.

for the previous circuits the battery also has resistance

$$P = \frac{V^2}{R} \rightarrow \frac{V^2}{R+R_B}$$ (reduced by resistance due to battery)

Like capacitors – these relations can be combined to find the resistance of complex combinations of resistors



① replace R_2, R_3 by $R_{23} = \frac{R_2 R_3}{R_2+R_3}$ (parallel)

② replace R_1 R_{23} by $R_{1,23} = R_1 + \frac{R_2 R_3}{R_2+R_3} =$

$$\frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2+R_3}$$

③ replace circuit by

$$R = \frac{R_4 R_{123}}{R_4 + R_{123}} = \frac{\frac{R_4}{R_2+R_3}(R_1R_2 + R_1R_3 + R_2R_3)}{R_4 + R_1 + \frac{R_2R_3}{R_2+R_3}}$$

$$\boxed{R = \frac{R_4(R_1R_2 + R_1R_3 + R_2R_3)}{R_2R_4 + R_3R_4 + R_1R_2 + R_1R_3 + R_2R_3}}$$

* correction from last time computing drift velocity - not all electrons in copper are free to move - only the valence or conduction electrons

Copper

conduction electrons per m^3

$8.49 \times 10^{28}\ (m)^{-3} = n$

$$J = qn\bar{v}_D = qn\frac{qE}{c} = \frac{q^2 n E}{m}\tau = \sigma E$$

$$\rho = \frac{1}{\sigma} = \frac{m}{q^2 n \tau}$$

$$\tau = \frac{m}{q^2 n \rho} = \frac{9.1\times 10^{-31}}{(1.6\times10^{-19})^2(8.49\times10^{28})(1.69\times10^{-8}\,\Omega\cdot m)}$$

$$\simeq 2.5\times 10^{-14}\ s$$

$$I = JA = qnV_D A$$

$$V_D = \frac{I}{A}\frac{1}{qn} = \frac{1A}{2\times10^{-6}}\frac{1}{(1.6\times10^{-19})(8.49\times10^{28})} = 3.68\times10^{-5}$$