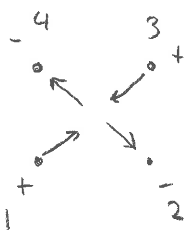
Lecture 15

Exam review

Average 88
standard dev 54
High 180

A > 120
B > 80
C > 40

① use symmetry

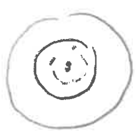


a,b) components cancel in x y plane
1–3 cancel – $F_x = F_y = 0$
2–4 cancel

c) on z axis 1–3 give opposite force to 2-4
net force is 0 $F_z = 0$

d) no work done because z component of field vanishes

②



a) gauss' law $\vec{E} = \frac{q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}$ $r < r_1$

b) $\vec{E} = 0$ in conductor $r_1 < r < r_2$

c) gauss law $\vec{E} = \frac{-3q+q}{4\pi\epsilon}\frac{\hat{r}}{r^2} = -\frac{2q}{4\pi\epsilon}\frac{\hat{r}}{r^2}$

d) charge on inner surface must cancel charge at center to get no field in conductor

$\sigma = \frac{-q}{4\pi r_1^2}$

e) charge conservation ⇒ $-2q$ on outer surface

$\sigma = -\frac{2q}{4\pi r_2^2}$

③ $V = \alpha(yx^2 - xy^2)$

a) $E_x = -\frac{\partial V}{\partial x} = -\alpha(2xy - y^2) = -\alpha(2ab - b^2)$

b) $E_y = -\frac{\partial V}{\partial y} = -\alpha(x^2 - 2xy) = -\alpha(a^2 - 2ab)$

c) work $= q\left[(\alpha(ba^2 - ab^2)) - 0\right]$

only depends on potential difference.

d) same $q\alpha(ba^2 - ab^2) = W$

④ a) $\frac{\sigma}{2\epsilon_0}$ each plate

$\bar{E} = \bar{E}_1 + \bar{E}_2 = \frac{\sigma}{2\epsilon_0} - \left(-\frac{\sigma}{2\epsilon_0}\right) = \frac{\sigma}{\epsilon_0}$

b) $\bar{E}$ constant so $V = Ed = \frac{\sigma d}{\epsilon_0}$

c) $\bar{E}$ independent of $d \Rightarrow E = \frac{\sigma}{\epsilon_0}$

d) $V \propto d$ so V doubles $V \rightarrow \frac{\sigma \cdot 2d}{\epsilon_0}$

e) work done by plate 1 moving plate 2 a distance d.

$(E_1)(Q_2)(d) = \left(\frac{\sigma}{2\epsilon_0}\right)(\sigma A)\, d = W$

$\frac{W}{A} = \frac{1}{2}\frac{\sigma^2 d}{\epsilon_0}$

Circuits

EMF: In order to cause currents in circuits work has to be done to move electric charges to higher potential energy.

This normally accomplished with a device - battery, generator, photocell, --- The effect of these devices is convert some other kind of energy into electric power.

The EMF (Electromotive force) is the work done by the device per unit charge.

$$\frac{W}{\text{charge}} = \frac{\text{energy}}{\text{charge}} = \text{voltage}$$

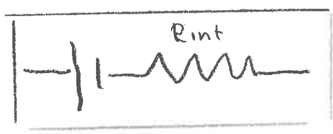
$$\mathcal{E} = \text{EMF} = \frac{dW}{dq}$$

The symbol

is used to denote an EMF device

The EMF of this device is the increase in electric potential V in moving from the - side of the device to the + side while the notation suggest the soure of EMF is a battery – it could be and EMF device

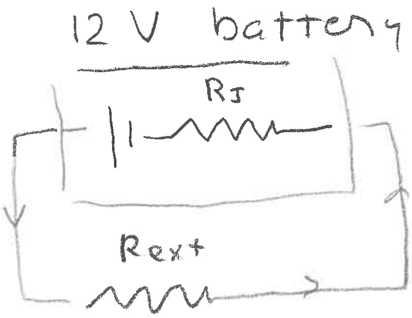
A ideal EMF device has no internal resistance; however real EMF devices do have a resistance. They can sometimes be expressed as



where R_{int} is the internal resistance of the device

* if there is no current through the device, $V = IR_{INT} = 0$, To the change in potential is due entirely to the EMF

* if there is a current I through to device the potential drop is lowered by $\boxed{V = EMF - IR_{INT}}$



In this case the current through the circuit is I, the change in potential going around the circuit is

$$EMF - IR_I - IR_E = 0$$

$$I = \frac{EMF}{R_I + R_E}$$

the potential difference across the external resistor is

$$V = IR_{ext} = EMF\left(\frac{R_E}{R_E + R_I}\right) = EMF\,\frac{1}{1 + R_I/R_E}$$

this is lower than the EMF, but it approaches the EMF as $R_I/R_E \to 0$

this means when you start your engine the starter motor does not fell the 12 volts of the battery

The power dissipated through an external resistance also depends on the internal resistance of the EMF source.

$$P = \underbrace{IV}_{\left(\frac{dq}{dt}V\right)} = \underbrace{I^2R = V^2/R}_{\text{uses ohms law}} \qquad V = IR$$

In this case V is the voltage drop across the device

$$P = \left(EMF\left(\frac{R}{R+R_I}\right)\right)^2 / R$$

$$= (EMF)^2 \frac{R}{(R+R_I)^2}$$

$$= \underbrace{\frac{(EMF)^2}{R}}_{\text{power with no internal resistance}} \cdot \underbrace{\frac{R^2}{(R+R_I)^2}}_{\text{reduction in power due to internal resistance of EMF device.}}$$

Note that energy is conserved - just that some of the power is dissipated (typically as heat) in the EMF device

This is different than the power generated by the EMF device

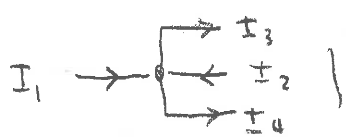
$$EMF \times I = P$$

where I is the current

when there are more complex circuts there are 2 key concepst that are used

① charge is conserved

This means that the sum of all currents in to a point must equal the sum of all currents out of a point



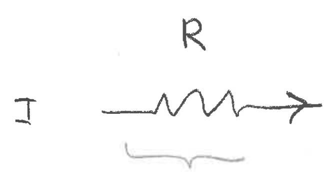
$$\boxed{I_1 + I_2 = I_3 + I_4}$$

② the work done in moving a charge q around any closed loop is 0 (because the force is conservative)

If we divide by the charge, this means that the change in potential around any closed loop is 0

other step

① guess the direction of the current (transport of positive charge). draw an arrow in that direction



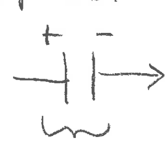
potential drops going in direction of arrow

$$V = -IR$$

potential increases in moving opposite to direction of arrow.

$$V = +IR$$

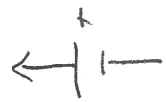
for capacitor



$V = -\frac{Q}{C}$ going from + to −

$V = +\frac{Q}{C}$ going from − to +

EMF source

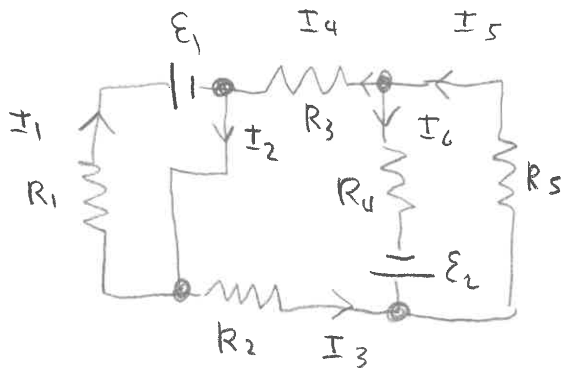


potential increases by EMF moving following arrow

potential decreases by EMF if moving is opposite to arrow.

For circuits with resistors you will get a set of equations for the currents, if they come out negative that means that your guess for the current direction was wrong.

example

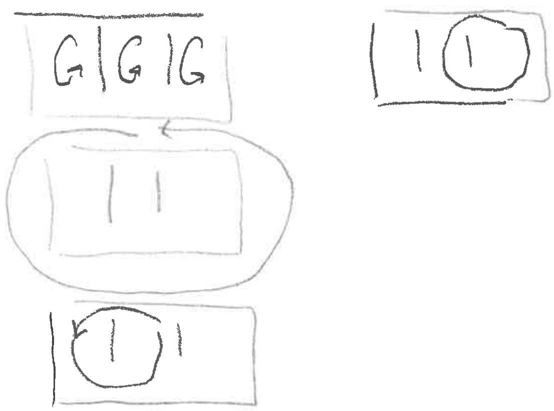


current conservation:

$$I_1 + I_4 - I_2 = 0$$
$$I_2 - I_3 - I_1 = 0$$
$$I_3 - I_6 + I_5 = 0$$
$$I_5 - I_6 - I_4 = 0$$

These equation indicate current conservation at the dots.

we also can find loop

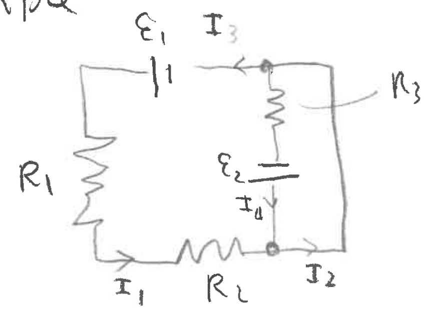


$$\mathcal{E}_1 + I_1 R_1 = 0$$

$$-I_4 R_3 - R_2 I_3 + \mathcal{E}_2 + R_4 I_6 = 0$$

$$-I_5 R_5 - I_6 R_4 + \mathcal{E}_2 = 0$$

This gives a set of 7 equations in 6 unknowns – they are not all independent. We can also general additional equations by considering additional loops example



① step 1 draw arrow.

2 junction $I_2 = I_3 + I_4$, $I_4 + I_1 = I_2$

② step 2 need at least 2 loops

$\boxed{1}$ $\mathcal{E}_1 - I_1R_1 - I_1R_2 = 0$

$\boxed{10}$ $-R_3I_4 + \mathcal{E}_2 = 0$

the unknowns are the current

* $I_1(R_1+R_2) = \mathcal{E}_1$

$I_1 = \frac{\mathcal{E}_1}{R_1+R_2}$

* $I_4 = \frac{\mathcal{E}_2}{R_3}$

$I_2 = I_4 + I_1 = \frac{\mathcal{E}_1}{R_1+R_2} + \frac{\mathcal{E}_2}{R_3}$

$I_3 = I_2 - I_4 = \frac{\mathcal{E}_1}{R_1+R_2} + \frac{\mathcal{E}_2}{R_3} - \frac{\mathcal{E}_2}{R_3} = \frac{\mathcal{E}_1}{R_1+R_2}$

we could have got the last equation using $\boxed{01}$.

general considerations

① resistors in series

$$V = \sum I R_i = I \sum R_i = IR$$

$$R = \sum_{i=1}^{N} R_i$$

② resistors in parallel

$$I = \sum I_i = \sum \frac{V}{R_i} = \frac{V}{R}$$

$$\frac{1}{R} = \sum_{i=1}^{N} \frac{1}{R_i}$$

③ capacitors in series same Q

$$V = \sum V_i \qquad Q = CV$$

$$\frac{Q}{C} = \sum \frac{Q}{C_i} = Q \sum \frac{1}{C_i}$$

$$\frac{1}{C} = \sum_{i=1}^{N} \frac{1}{C_i}$$

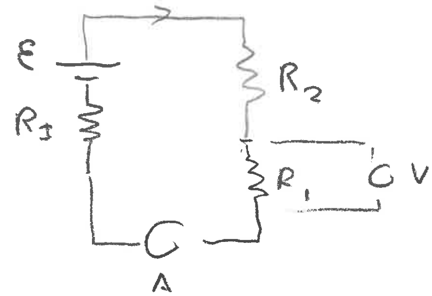
④ capacitors in parallel same V

$$Q = \sum Q_i$$

$$CV = \sum C_i V = V \sum C_i$$

$$C = \sum_{i=1}^{N} C_i$$

Ammeter + Voltmeter



① consider a meter that reads voltage and has an internal resistance R_V

$$R_{eff} = \frac{R_1 R_V}{R_1 + R_V}$$

$$I = \frac{V}{R_{eff}} = \frac{V}{R_1 R_V}(R_1 + R_V)$$

$$V = I R_{eff} \qquad I_S = \frac{1}{R_{eff}} = \frac{1}{R_1}\left(1 + \frac{R_1}{R_V}\right) \; ; \quad V = I_V \times R_V$$

these have approximately the same voltage for a given current provided $R_V >> R_1$

3 since V meter $\propto I_V$ so the voltage across R_1 is approximately proportional to the current through R_V

② next consider a meter that reads current

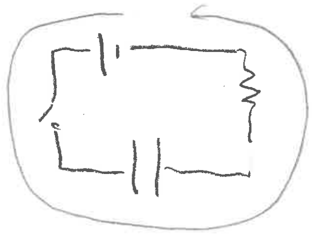
$$I = (R_I V + R_A V) = (R_I + R_A) V \qquad I = R_I V$$

$$= R_I \left(1 + \frac{R_A}{R_I}\right) V$$

this looks like $R_I V$ when $R_A << R_I$

RC circuits

when there is a current in a circuit that contains a capacitor it takes time to fully charge the capacitor



$$0 = \mathcal{E} - \frac{Q}{C} - IR$$

ensures that the potential at any point in the circuit is fixed

since $I = \frac{dQ}{dt}$

we can write this as

$$\boxed{\frac{dQ}{dt} R + Q\frac{1}{C} = \mathcal{E}}$$

we write this as

$$\frac{dQ}{dt} = \frac{1}{R}\left(\mathcal{E} - \frac{Q}{C}\right)$$ (this is a differential equation)

$$\frac{dQ}{\mathcal{E} - \frac{Q}{C}} = \frac{dt}{R}$$

multiply by $-C$ in the denominator

$$\frac{dQ}{Q-\mathcal{E}C} = -\frac{dt}{RC}$$

integrate $Q(t_0) = Q_0$ $\quad Q(t) = Q$

$$\int_{Q(t_0)}^{Q(t)} \frac{dQ}{Q-\mathcal{E}C} = \int_{t_0}^{t} -\frac{dt}{RC}$$

Let $u = Q - \mathcal{E}C$ $\quad dQ = du$

$$\int_{Q(t_0)-\mathcal{E}C}^{Q(t)-\mathcal{E}C} \frac{du}{u} = -\frac{1}{RC}(t-t_0)$$

$$\ln\left(\frac{Q(t)-\mathcal{E}C}{Q(t_0)-\mathcal{E}C}\right) = -\frac{1}{RC}(t-t_0)$$

$$e^{\ln\left(\frac{Q(t)-\mathcal{E}C}{Q(t_0)-\mathcal{E}C}\right)} = e^{-\frac{1}{RC}(t-t_0)}$$

$$(Q(t)-\mathcal{E}C) = (Q(t_0)-\mathcal{E}C)\,e^{-\frac{1}{RC}(t-t_0)}$$

If $Q(t_0) = 0$

$$\boxed{\begin{aligned} Q(t) &= \mathcal{E}C\left(1-e^{-\frac{1}{RC}(t-t_0)}\right) \\ I = \frac{dQ}{dt} &= \mathcal{E}C\cdot\frac{1}{RC}\,e^{-\frac{1}{RC}(t-t_0)} = \frac{\mathcal{E}}{R}\,e^{-t/RC} \end{aligned}}$$

the quantity t/RC must be dimensionless. this means $RC = \tau$ has units of time

after time τ the charge on the capacitor is

$$q = C\mathcal{E}(1 - e^{-1}) = .63\, C\mathcal{E}$$

as t gets $\gg \tau$ q approaches

$$q \to C\mathcal{E} \quad (\text{large } t)$$

since the current $\to 0$ $\mathcal{E} \to V$

$$\boxed{Q = CV}$$

after the current $\to 0$

To discharge a charged capacitor we remove the EMF $\Rightarrow$

$$\frac{dQ}{dt} R = -\frac{Q}{C}$$

$$\frac{dQ}{Q} = -\frac{dt}{RC}$$

integrating both side

$$\int_{Q(0)}^{Q(t)} \frac{dQ}{Q} = \ln\left(\frac{Q(t)}{Q(0)}\right) = -\int_{t_0}^{t} \frac{dt}{RC} = -\frac{(t - t_0)}{RC}$$

Taking exponenti

$$e^{\ln \frac{Q(t)}{Q(t_i)}} = e^{-(t-t_i)/RC}$$

$$\boxed{Q(t) = Q(t_i)\, e^{-(t-t_i)/RC}}$$

which determines how a charged capacitor discharges