Lecture 17

why are homes wired in parallel

① resistance of a 100 watt light bulb

$$P = I^2 R = V^2/R$$

$$R = V^2/P = (110)^2/(100) = 121\,\Omega$$

② power output of 2 121 Ω resistors in parallel

$$R' = \frac{RR}{R+R} = \frac{R}{2}$$

$$P = V^2/R' = \frac{(110)^2}{121/2} = 200 \text{ watts}$$

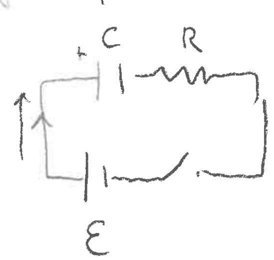
③ power output of 2 121 Ω resistors in series

$$R' = R + R = 2R$$

$$P = V^2/R' = \frac{V^2}{2R} = \frac{1}{2} 100 = 50 \text{ watts}$$

The combined output of 2 100 watt light bulbs in series is 50 WATTS

charging and discharging capacitors



loop equation

$$\boxed{\varepsilon - \frac{Q}{C} - IR = 0}$$

note $I = \frac{dQ}{dt}$

using this in the loop equation gives

$$\boxed{\mathcal{E} - \frac{Q}{C} - R\frac{dQ}{dt} = 0}$$

To solve this we need to specify the <u>initial</u> <u>conditions</u>.

* Assume that the capacitor is uncharged at time $t=0$
* We use the differential equation to find $Q(t)$ at later times

To solve the equation write it in the form

$$\frac{dQ}{dt} = \frac{1}{R}\left(\mathcal{E} - \frac{Q}{C}\right) = -\frac{1}{R}\left(\frac{Q}{C} - \mathcal{E}\right)$$

we write this as

$$\frac{dQ}{\frac{Q}{C} - \mathcal{E}} = -\frac{dt}{R}$$

multiply by C

$$\frac{dQ}{Q - \mathcal{E}C} = -\frac{dt}{RC}$$

To incorporate the initial condition integrate

$$\int_{Q(0)}^{Q(t)} \frac{dQ}{Q-\mathcal{E}C} = -\int_0^t \frac{dt}{RC}$$

let $u = Q - \mathcal{E}C \qquad du = dQ$

$u(t) = Q(t) - \mathcal{E}C$

$u(0) = Q(0) - \mathcal{E}C$

$$\int_{Q(0)-\mathcal{E}C}^{Q(t)-\mathcal{E}C} \frac{du}{u} = -\frac{t}{RC} = \ln(Q(t)-\mathcal{E}C) - \ln(Q(0)-\mathcal{E}C)$$

use $\ln a - \ln b = \ln a/b$

$$-\frac{t}{RC} = \ln\left(\frac{Q(t)-\mathcal{E}C}{Q(0)-\mathcal{E}C}\right) = \ln\left(\frac{\mathcal{E}C-Q(t)}{\mathcal{E}C-Q(0)}\right)$$

taking exp

$$e^{-t/RC} = \frac{\mathcal{E}C-Q(t)}{\mathcal{E}C-Q(0)}$$

$$\boxed{Q(t) = \mathcal{E}C - (\mathcal{E}C-Q(0))e^{-t/RC}}$$

If we set $Q(0)=0$ this becomes

$$\boxed{Q(t) = \mathcal{E}C(1-e^{-t/RC})}$$

This has the right properties

① when $t=0$ $Q(0)=0$

② as t gets very large $Q(t)$ approaches $\mathcal{E}C = VC$, which the charge on a capacitor with a voltage $\mathcal{E}$.

③ RC has units of seconds. – It is called the time constant of the RC circuit

④ Large t be $t \gg \tau$.

How long does it take for a capacitor to reach 99% of its full charge.

$$Q(t) = \mathcal{E}C(1-e^{-t/RC})$$

$$Q(t) = (.99)\mathcal{E}C$$

$$.99\,\mathcal{E}C = \mathcal{E}C - \mathcal{E}C e^{-t/RC}$$

$$e^{-t/RC} = .01$$

Taking lns

$$-\frac{t}{RC} = \ln .01$$

$$t = -RC\ln\left(\frac{1}{100}\right) = RC\ln 100$$

$$= \tau \ln 100$$

$$= 4.6\,\tau$$

If we have an expression for $Q(t)$ we can use it to find the current

$$Q(t) = \mathcal{E}C(1-e^{-t/RC})$$

$$I(t) = \frac{dQ}{dt} = 0 - \mathcal{E}C\, e^{-t/RC}\left(-\frac{1}{RC}\right)$$

$$I(t) = \frac{\mathcal{E}}{R}\, e^{-t/RC}$$

here $\frac{\mathcal{E}}{R}$ is the steady state current through a resistor with resistance R. At $t=0$ the current is at this maximum value. When t gets large it heads toward 0 as the charge on the capacitor builds up

The same ideas can be used to treat the discharge of a capacitor. In this case there is no emf in the circuit

$$0 = -IR - \frac{Q}{C}$$

which becomes

$$-\frac{dQ}{dt}R - \frac{Q}{C} = 0$$

$$\frac{dQ}{dt} = -\frac{Q}{RC}$$

This becomes

$$\frac{dQ}{Q} = -\frac{dt}{RC}$$

In this case at time 0 we assume that the capacitor is fully charged with charge Q

$$\int_{Q}^{Q(t)} \frac{dQ}{Q} = -\frac{1}{RC}\int_0^t dt$$

Integrating

$$\ln Q(t) - \ln Q = -\frac{t}{RC}$$

$$\frac{Q(t)}{Q(0)} = e^{-t/RC}$$

$$\boxed{Q(t) = Q(0)\, e^{-t/RC}}$$

In this case the time constant is the same as in the case of charging

The time to loose half the charge is

$$\frac{1}{2}Q = Q\, e^{-t/RC}$$

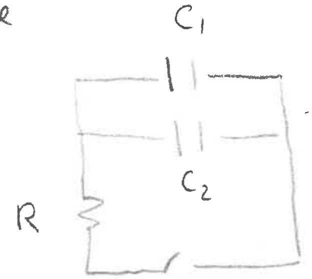
$$\ln(\tfrac{1}{2}) = -\ln 2 = -t/RC$$

$$t = RC\ln 2 = \tau \ln 2 = .69\,\tau$$

making R or C larger increases the time constant – it increases the charge time and discharge time

S_1 closed S_2 open – slow charge
S_2 closed S_1 open . fast discharge

example



In this case the effective capacitance is
$$C' = C_1 + C_2$$

charging
$$Q(t) = \mathcal{E}C'(1 - e^{-t/RC})$$

we can figure out the charge on each capacitor
$$\frac{Q_1}{C_1} = V = \frac{Q_2}{C_2}$$

$$Q = Q_1 + Q_2 = Q_1 + \frac{C_2}{C_1} Q_1 = \left(1 + \frac{C_2}{C_1}\right) Q_1$$

$$= \left(\frac{C_1 + C_2}{C_1}\right) Q_1$$

$$Q_1 = \frac{C_1}{C_1 + C_2} \cdot Q =$$

$$= \frac{C_1}{C_1 + C_2} \mathcal{E} C' \left(1 - e^{-t/RC'}\right)$$

$$= \frac{C_1}{C_1 + C_2} \mathcal{E} (C_1 + C_2) \left(1 - e^{-t/RC'}\right)$$

$$= \mathcal{E} C_1 \left(1 - e^{-t/RC'}\right)$$

similarly

$$Q_2 = \mathcal{E} C_2 \left(1 - e^{-t/RC'}\right)$$

we can also find the current in each capacitor

$$I_1 = \frac{dQ_1}{dt} = \frac{\mathcal{E}}{R} \frac{C_2}{C'} e^{-t/RC'} = \frac{\mathcal{E}}{R} \frac{C_2}{C_1 + C_2} e^{-t/RC'}$$

$$I_2 = \frac{dQ_2}{dt} = \frac{\mathcal{E}}{R} \frac{C_1}{C'} e^{-t/RC'} = \frac{\mathcal{E}}{R} \frac{C_1}{C_1 + C_2} e^{-t/RC'}$$

Leaky capacitor

$C = 2\mu F$ $\quad Q(2) = \frac{1}{4} Q(0)$ $\quad$ Find R $\quad t = 1$

$$Q(2) = \frac{1}{4} \cancel{Q(0)} = \cancel{Q(0)}\, e^{-2/RC}$$

$$-\ln 4 = -\frac{2}{RC}$$

$$\boxed{R = \frac{2}{C \ln 4}}$$

There is another kind of force on charge particles that occurs when moving electric charges pass by a magnet.

so far no one has observed magnetic charges, so for this reason it is useful to start with the definition of the magnetic field

recall $\bar{E} = \lim_{q \to 0} \frac{\bar{F}}{q}$

for magnetic forces on a charged particle

(1) the force is proportional to the speed of the particle

(2) the force is perpendicular to the velocity

(3) the force is proportional to the charge

It turns out observations are consistent with

$$\boxed{\bar{F}_m = q \, \vec{V} \times \bar{B}}$$

This force is called the Lorentz force - the direction of $\bar{F}$ and $\bar{v}$ is consistent with a magnetic field vector $\bar{B}(x,t)$

$$\bar{F} = q\,\bar{v}\times\bar{B}$$

$$\bar{v}\times\bar{F} = q\,\bar{v}\times(\bar{v}\times\bar{B})$$

$$= q\left(\bar{v}(\bar{v}\cdot\bar{B}) - \bar{B}(v^2)\right)$$

$$\bar{B} = q\bar{v}(\bar{v}\cdot\bar{B}) - \frac{1}{q}(\bar{v}\times\bar{F})$$

This is maximized when $\bar{v}\perp\bar{B}$

In terms of magnitude

$$|\bar{F}| = |q|\,|\bar{v}\times\bar{B}| = |q|\,vB\sin\theta$$

where θ is the angle between $\bar{B}$ and $\bar{v}$ and the direction is given by the right hand rule

remarks

1. The Lorentz force does no work because $\bar{F}$ is $\perp$ to $\bar{v} = d\bar{x}/dt$

2. The Lorentz force vanishes when the particle is at rest

The unit of magnetic field is a

$$1 \text{ Tesla} = \frac{\text{Newton sec}}{\text{Coulomb} \cdot \text{meter}}$$

Like the electric field, there is a magnetic field vector at every point is space.

Where do magnetic fields come from

(1) electric currents can cause magnetic fields

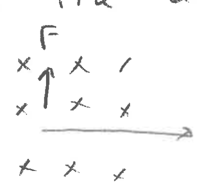
(2) there are also permanent magnets that create magnetic fields

(3) the earth has a magnetic field

Motion of a charged particle moving with constant velocity $\bar{v}$

$$\bar{F} = q\,\bar{v} \times \bar{B}$$

assume $\bar{B}$ has a component $\perp$ to $\bar{v}$, then $|F| = q|\bar{v}||B| \sin\theta$ which is the angle between $\bar{B}$ and $\bar{v}$



($\times$ = into plane of paper)

consider first $\bar{V} \perp \bar{B}$

then there is a force $\perp$ to $\bar{v}$. This does no work. It causes the particle to move in a circle

$$\frac{mv^2}{r} = qvB$$

$$\boxed{r = \frac{mv^2}{qvB} = \frac{m}{q}\left(\frac{v}{B}\right)}$$

If the velocity also has a component parallel to $\bar{B}$, there is no magnetic force in this direction.

When this is combined with the circular motion, the motion is helical

If we know the component of $\bar{v} \perp \bar{B}$ we can measure the charge to mass ratio by measuring $r, \bar{v}_\perp, \bar{B}$

This is a standard way to measure the mass of an electron.

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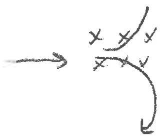
measuring mass of electron

L

$|\leftarrow V - |$

$\overleftarrow{qE}$



$\downarrow E$

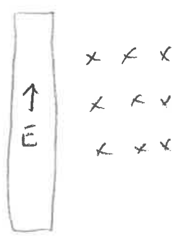
$qE = qvB$

$v = \frac{E}{B}$

$\frac{1}{2}mv^2 = qE'L = qV$

$\frac{q}{m} = \frac{v^2}{2V} = \frac{1}{2V}\left(\frac{E^2}{B^2}\right)$

Hall effect



positive charge $\leftarrow$

negative charge $\rightarrow$

w

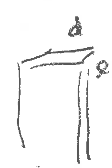
$\hookrightarrow$ gives a potential on either side of conductor. Depending on the sign of the potential — we can determine the sign of the charge carrier.

We can use an electric field to cancel the $\perp$ component of the motion

$E = \frac{V}{d}$ $\quad qE = q\frac{V}{d} = qvB$

where v is the drift velocity on electron

$I = JA = nev_D A$



$n = \frac{I}{ev_D A} = \frac{I}{eA}\cdot\frac{B}{E} = \frac{IB}{e(\ell d)E} = \frac{IB}{eV\ell}$