Lecture 18

Magnetic Forces and Fields

* There is a second force experienced by electrically change particles called the Lorentz force

(1) it is proportional to the electric charge

(2) it vanishes for electrically charged particles at rest

(3) for moving particles the force is perpendicular to the particles velocity

(4) it is proportional to the velocity

(5) since nothing is touching the particle when it feels this force, we expect that it is due to a field.

The relation consistent with the above properties is

$$\bar{F} = q\,\bar{V} \times \bar{B}$$

where

q = charge of particle in coulombs

$\bar{v}$ = velocity of particle

$\bar{B}(\bar{r},t)$ = magnetic field at $\bar{r}, t$

units of $B = \frac{\text{Newtons}}{\text{Coulomb}} \frac{\text{sec}}{\text{meters}} = 1$ Tesla

In terms of components

$$F_x = q(V_y B_z - V_z B_y)$$

$$F_y = q(V_z B_x - V_x B_z)$$

$$F_z = q(V_x B_y - V_y B_x)$$

the direction of the force can be determined by the right hand rule.

the magnitude of $|\vec{F}|$ is

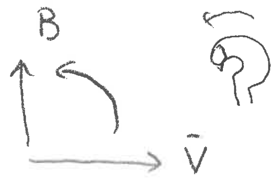
$$|\vec{F}| = |q|\,|V|\,|\bar{B}| \sin\theta$$

θ = angle between $\bar{v}$ and $\bar{B}$

Source of magnetic fields

* permanent magnets
* electromagnets - moving charges
* earth - permanent field (compass)

Right Hand Rule



curl hands
$\bar{F}$ direction
thumb points

* Because the magnetic force is $\perp$ to the velocity of a particle

$$\bar{F} \cdot d\bar{r} = \bar{F} \cdot \frac{d\bar{r}}{dt} dt = \bar{F} \cdot \bar{V} dt = 0$$

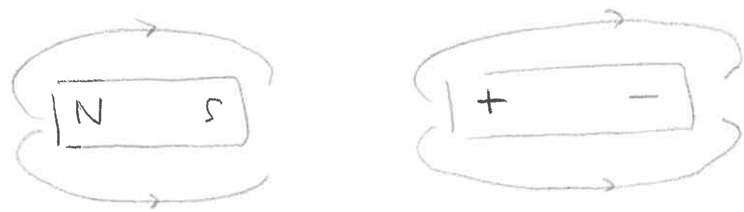
this means magnetic forces do no work on a moving charged particle

Like the electric field, there are field lines with the property that they are in the direction of the tangent to the field line

While some theories have predicted magnetic charges called magnetic monopoles, none have been observed

The next simplest source of fields is a magnetic dipole; an example is a bar magnet. Like an electric dipole it has 2 sides, one is called the north pole and the other is called the south pole

by convention field lines run from the north pole to the south pole



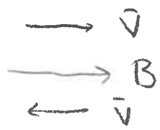
magnetic dipole   electric dipole

A permanent magnet is constructed out of a large number of parallel microscopic dipoles. The microscopic dipoles are related to moving or spinning charges

protons, neutrons, and electrons have permanent dipole moments.

motion of a charged particle in a magnetic field

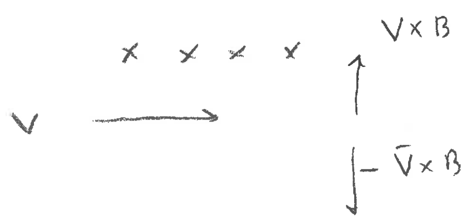
$\bar{V}$ parallel to $\bar{B}$ $\quad \bar{V} \times \bar{B} = 0$



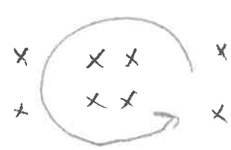
in this case $\bar{F} = q\,\bar{V} \times \bar{B} = 0$ and the particle continues to move with constant velocity

case 2

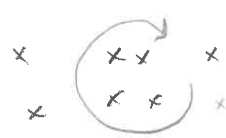
$\bar{V}$ perpendicula to $\bar{B}$ $\quad$ × × × = into plane of pape



for a positively charged particle



for a negatively charge particle



For circular motion

$$ma = m\frac{v^2}{r} = qvB$$

canceling one factor of v

$$\boxed{\frac{q}{m} = \frac{v}{rB} = \frac{r\omega}{rB} = \frac{\omega}{B}}$$

ω = angular frequency.

measuring the mass of an electron

choose an electric field that eliminates the Lorentz force

$(E \perp B)$

$$qvB = qE$$

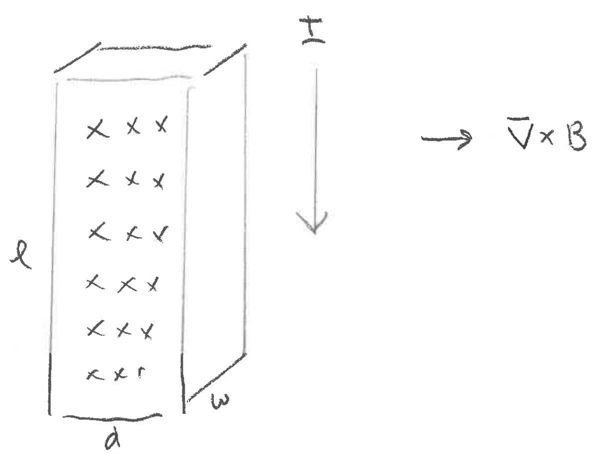
this gives $v = \frac{E}{B}$. Using this in the above equation with the electric field turned off gives

$$\boxed{\frac{q}{m} = \frac{v}{rB} = \frac{E}{rB^2}}$$

In this case if we know E, B and the radius of the circle we can find the charge to mass ratio of an electron

Hall effect

determining charge carriers



If the carriers are positive, positive charge builds up on the right until the magnetic force is canceled

If the carriers are negative $\bar{V}$ ↑ and q is negative, so in this case negative charge builds up on the right until the Lorentz force is canceled.

If the potential increases - left to right the charge carriers are positive

If the potential decreases - left to right the charge carriers are electrons in this case

(1) $V = Ed$

(2) $qvB = qE \qquad v = E/B = \bar{V}/dB$

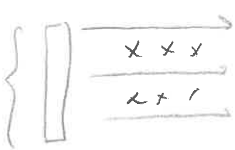
(3) $I = nev_D \cdot A = nev(dw)$

$$\boxed{n = \frac{I}{e v_D A} = \frac{I}{e(V/dB)\,dw} = \frac{IB}{e\bar{V}w}}$$

which can be used to measure the number of charge carriers / unit volume

$$\boxed{v_D = \frac{V}{dB}}$$

moving conductors



$\bar{v} \times \bar{B} \uparrow$

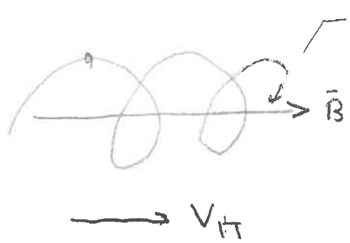
develops a potential difference when it move $\perp$ to magnetic field.

sets up a potential $\quad E = \bar{V}\ell \qquad E = vB$

$$\boxed{\bar{V} = E\ell = vB\ell}$$

motion of a charged particle in a magnetic field $\bar{B}$

① write $\bar{V} = \bar{V}_{\parallel} + \bar{V}_{\perp}$



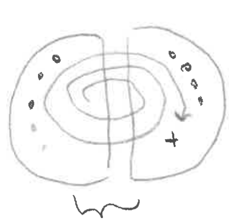
$$\frac{mV_\perp^2}{r} = qV_\perp B \qquad m\omega = qB$$

$$r = \frac{mV_\perp}{qB} \qquad \omega = \frac{qB}{m}$$

the helix gets tighter as B increases, larger as $V_\perp$ increases

ccw for positive charge
cw for negative charge

Operation of a Cyclotron



$$\frac{mv^2}{r} = qvB$$

$$r = \frac{mv}{qB} \; ; \quad \omega = \frac{qB}{m}$$

We note that the angular frequency ω $\frac{qB}{m}$ is independent of the particles' velocity

$$\omega T = 2\pi \Rightarrow T = \frac{2\pi}{\omega} = \frac{1}{f} \Rightarrow \omega = 2\pi f$$

if the electric field oscillates

$$E = E_0 \cos(\omega t) = E \cos\left(\frac{qB}{m} t\right)$$

$$E_0 \cos(2\pi f t)$$

In this way whenever the proton reaches the gap it will be accelerated the speed it acquires when it leaves the cyclotron

$$V = r\omega = \frac{qBr}{m}$$

so it only depends on the radius of the cyclotron

It will also increase with B.

This simple idea breaks down when the speed approach the speed of light. Then Newton's second law must be modified due to special relativity

This is fixed with a synchotron where the magnetic field is adjusted to keep in step with the oscillating electric fied.

Magnetic force on a current carrying wire

$$\bar{J} = nq\bar{V}$$

$$I = \int \bar{J}\cdot\hat{n}\, dA$$

actually the current is directed along the wire, which the direction of the drift velocity

consider

$$I\, d\bar{l} \times \bar{B} = J A\, d\bar{l} \times \bar{B}$$

$$= nqv\underbrace{A\, d\bar{l}}_{\text{volume}} \times B$$

$n A\, dl$ = Total charges in dl

$$I\, dl \times \bar{B} = (\#\text{ charges}) \times (q\bar{V}\times\bar{B}) = d\bar{F}$$

This means that the net force on a wire is

$$\bar{F} = \int dF = \int I\, d\bar{l}\times\bar{B} \simeq \lim I \sum d\bar{l}_i \times B(r_i)$$

integral along the path of the wire