Lecture 1

Topics:

some history
electrostatic forces
charge quantization and conservation
coulomb's law
superposition principle

History

600 BC Thales of Miletus - rupped amber which attracted straw

? observed naturally occurring magnetic attracts iron.

1706-1790 Ben Franklin identified change deposited on glass

1736 - 1806 Charles Coulomb - quantitative measurements of electrostatic force
1820 Hans Christian Oersted observed an electric current making a compass needle move.

1791-1867 Michael Faraday - important series of experiments on electromagnetic phenomena.

1831-1879 James Clerk Maxwell - mathematical formulation of the laws of electromagnetism.

The theory - called classical electromagnetism is responsible for most non-gravitational macroscopic physics.

**Context:**

It is believed that there are four fundamental forces of nature:

1. gravitational force - weak, attractive long range - responsible for orbits of planets around sun.
(2) Electromagnetic forces - intermediate strength, long range, attractive and repulsive

(3) Weak force - weak strength - short-range

responsible for radioactive decay

\[ n \rightarrow p^+ + e^- + \nu \]

(4) Strong force - strong, short range attractive

responsible for keeping atomic nucleus together.

* Electromagnetic force responsible for light, radio waves, chemical reactions, ...

Observations - Electrostatic forces

rub glass with silk
rub plastic with fur
2 glass rods repel
2 plastic rods repel
1 plastic + 1 glass rod attract.
Something is deposited on the rods that causes them to repel.

What is deposited on the glass and plastic rods are different.

What is deposited on each rod is called electric charge.

The exercise suggests that there are 2 kinds of charges. They are labeled + = positive charge, - = negative charge.

By convention, the charge on the glass is called positive charge, the charge on the plastic is called negative charge.

\[\begin{align*}
glass & \quad \rightarrow \quad \text{++++++} \\
rod & \quad | \quad \text{---} \\
plastic & \quad \uparrow \quad \text{rod}
\end{align*}\]
There are 2 types of electrical charge - positive and negative. Like charges repel and unlike charges attract.

What is going on?

Most visible matter is made of atoms. Atoms have a core consisting of protons and neutrons. Electrons orbit outside of the core.

It has been determined that each proton has a unit of positive charge, and each electron has a corresponding unit of negative charge.
A typical atom has the same number of electrons and protons. 
The total amount of positive charges = the total amount of negative charge. 
When the number of positive charges is equal to the number of negative charges, the object is electrically neutral.

If we bring a positive charge near a neutral atom, it will not move because the attractive forces due to the electrons cancel the repulsive forces due to the protons.

The nucleus of the atom looks like it should not be stable because the protons repel each other. The reason that the nucleus stays together...
Is because there is another stronger attractive force. It is called the strong force—it is much stronger than the electric force, but it has a very short range (about $10^{-15}$ meters).

What happened with the glass and plastic rods?

Both start out being electrically neutral—having approximately equal numbers of protons and electrons.

When the glass is rubbed with silk electrons in the glass get transferred to the silk, leaving the glass with an excess of positive charge.

Similarly—when the plastic rod is rubbed with fur, the fur deposits electrons on the plastic and it gets an excess of negative charge.
In most cases it is the electrons that move. The protons are more tightly bound in the atomic nucleus, in addition the mass of an electron is about $\frac{1}{2000}$ the mass of a proton.

$$
\begin{align*}
Me & \approx 9.1 \times 10^{-31} \text{ kg} \\
mp & \approx 1.67 \times 10^{-27} \text{ kg} \\
\frac{Me}{mp} & \approx 5.45 \times 10^{-4} = \frac{1}{1835}
\end{align*}
$$

Charge quantization - conservation

* Charge quantization electric charge has only been observed in integer multiples of the charge of an electron or proton.

* Protons and neutrons are believed to be composed of quarks that have charges

  \[ u_q = +\frac{2}{3} q_p \quad d_q = -\frac{1}{3} q_p \quad p = uud \]

  \[ n = ddu \]

(Up and down quarks), the quarks are believed to permanently confined in observable particles.
free quarks have never been seen.)

2) the net electric charge is always conserved.

\[ \text{N} \rightarrow \text{p} + \text{e} + \text{\nu} \]

a free neutron is unstable-it decays into a proton, electron and neutrino.

the net charge of p+e+\nu is 0 which is the same as the net charge of the neutron.

\[ \text{e}^+ + \bar{\text{e}} \rightarrow 2\gamma \]

an electron and positron (positively charged electron) decay into 2 photons each with energy \( E_{\gamma} = m_{e}c^{2} \). This mechanism is important in positron emission tomography (at UHMC).
unit of electric charge

1 coulomb = the amount of charge transported in 1 second by a 1 ampere current =
change of

624, 150, 907, 446, 076, 2607, 776 protons
≈ 6.24 × 10^{18} protons

conducus and insulatus

insulatus - these are substances where the electrons are tightly bound to the positive charges

conducus - these substances have some electrons that are weakly bound - electrons move more freely in conducus.

Copper, Silver, Aluminum are good conducus.

Plastic, wood, glass are good insulatus.
Coulomb's Law and the superposition principle.

What is the force on an electron due to a proton?

\[ \mathbf{F} = \text{constant} \times q_p q_e \frac{\mathbf{r}_e - \mathbf{r}_p}{|\mathbf{r}_e - \mathbf{r}_p|^3} \]

* The force is a vector quantity.
* Here \( \mathbf{F} \) points in the opposite direction to the vector \( \mathbf{r}_e - \mathbf{r}_p \) because \( q_e \) and \( q_p \) have different signs.
remarks

1. the force on the electron is proportional to both the charge of the electron and the charge of the proton.

2. the direction of the force is along the line between the charges (repulsive for like charges - attractive for unlike charges).

3. the magnitude of the force falls off like the inverse square of the distance between the charges.

4. the constant is

\[ k = \frac{1}{4\pi \varepsilon_0} = 8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2 \]

\[ \varepsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2 \]

C = coulombs
m = meters
N = Newtons
$k$ is called the Coulomb constant
$\varepsilon_0$ is called the permittivity of free space

The general form of Coulomb's Law

$$F_{12} = \frac{k q_1 q_2 (\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3}$$

= force on particle 1 due to particle 2

From this definition

$$\vec{F}_{12} = -\vec{F}_{21}$$

which is Newton's third law.

$$q_e = -1.602 \times 10^{-19} \text{ C}$$
$$q_p = 1.602 \times 10^{-19} \text{ C}$$

This means that the net force on any pair of charges is zero.
**Superposition Principle**

Consider charges \( q_1 \ldots q_n \) at positions \( \mathbf{r}_1 \ldots \mathbf{r}_n \).

The force on charge \( q \) at \( \mathbf{r} \) due to the charges \( q_1, q_2, \ldots, q_n \) at \( \mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n \) are the sum of the forces on \( q \) due to the other charges:

\[
\mathbf{F}_q = \sum_{n=1}^{N} \frac{q q_n}{|\mathbf{r} - \mathbf{r}_n|^2} \mathbf{r} - \mathbf{r}_n
\]

This is called the **superposition principle**.
Example: Superposition principle

\[ \begin{align*}
q_1 & \quad \Theta \quad 3 \\
1 & \quad \Theta \quad h \\
q_1 & \quad \Theta \quad l \\
\Theta & \quad l \\
q_2 & \quad \Theta
\end{align*} \]

What is the force on particle 3 due to the charges on particles 1 and 2?

Superposition implies:

\[ \vec{F}_{q_3} = \vec{F}_{q_3 q_1} + \vec{F}_{q_3 q_2} \]

Since the forces are vectors, we have to use vector addition.

Choose a coordinate system:

- above = origin

\[ \vec{r}_1 = (-l, 0) \]
\[ \vec{r}_2 = (+l, 0) \]
\[ \vec{r}_3 = (0, h) \]

\( (\vec{r}_3 - \vec{r}_1) = (l, h) \)
\( |\vec{r}_3 - \vec{r}_1| = \sqrt{l^2 + h^2} \)

\( (\vec{r}_3 - \vec{r}_2) = (-l, h) \)
\( |\vec{r}_3 - \vec{r}_2| = \sqrt{(-l)^2 + h^2} \)
using coulombs law

\[ \vec{F}_{q_3} = k \frac{q_3 q_1}{r_3 - r_1} + k \frac{q_3 q_2}{r_3 - r_2} \]

\[ = k \left( \frac{q_3 q_1}{(l^2 + h^2)^{3/2}} + \frac{q_3 q_2}{(l^2 + h^2)^{3/2}} \right) \]

\[ = \frac{k q_3}{(l^2 + h^2)^{3/2}} \left( (q_1 - q_2) l \right) \left( q_1 + q_2 \right) h \]

consider the case where \( q_1 = q_2 \)

then

\[ \vec{F}_{q_3} = \frac{2 k q_3 q_1}{(l^2 + h^2)^{3/2}} \left( 0, h \right) \]

if \( q_1 = -q_2 \)

\[ \vec{F}_{q_3} = \frac{2 k q_3 q_1}{(l^2 + h^2)^{3/2}} \left( l, h \right) \]

numerically

\[ q_1 = q_2 = q_3 = 1 \text{ coulomb} \]

\[ l = h = 1 \text{ meter} \]

\[ \vec{F}_{q_3} = \frac{2 k}{(2^{3/2})} \left( 0, 1 \right) \approx \frac{2 \times 9 \times 10^9}{2\sqrt{2}} \left( 0, 1 \right) \]

\[ = \left( 0, \frac{9}{\sqrt{2}} \times 10^9 \text{ N} \right) \]
In this case the force is a vector in the "y" direction with magnitude \( \frac{9}{52} \times 10^9 \text{N} \approx 6.36 \times 10^9 \text{N} \)

Comparison of the strength of the strong, coulomb and gravitational forces between 2 protons separated by \(10^{-15}\) meters

**Gravity**

\[
|F| = \frac{G m_p^2}{r^2} \approx \frac{(6.67 \times 10^{-11}) (1.7 \times 10^{-27})^2}{(10^{-15})^2} \approx 19.3 \times 10^{-35} \text{N}
\]

**Coulomb**

\[
|F| = \frac{k q_1 q_2}{r^2} \approx \frac{(9 \times 10^9) (1.6 \times 10^{-19} \text{C})^2}{(10^{-15})^2} \approx 2.3 \times 10^2 \text{N}
\]

**Strong**

\[
|F| = \frac{\lambda}{r^2} \approx \frac{3.4 \times 10^{-20} \text{e}^4}{(10^{-15})^2} \approx 1.7 \times 10^4 \text{N}
\]

\( F \) at \(10^{-15}\) m \( \approx 60.1 \) \( F \) at \(10^{-15}\) m \( \approx 8.4 \times 10^{-37} \text{N} \)