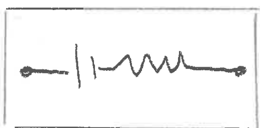
Lecture 26

exam review - since last exam

EMF devices, single loop circuits

$\mathcal{E} = \frac{dW}{dq}$ work / charge

Real batteries - have internal resistance



measured voltage = $\mathcal{E} - IR_{int}$

Power = $IV = I^2R = V^2/R$

(relations use Ohms law)

Key relations for loops

① conservation of charge

currents into a point =
currents out of a point

② conservation of energy

net change in potential around
any closed loop = 0

ure

① draw arrows representing direction of currents (these are guesses)

② EMF

 $\uparrow + \varepsilon \quad \downarrow - \varepsilon$

Resistors

$\longrightarrow \quad -IR = V$

$\longleftarrow \quad +IR = V$

Capacitors

$\longrightarrow \quad -\frac{Q}{C} = V$

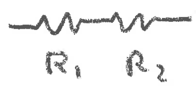
$\longleftarrow \quad \frac{Q}{C} = V$

③ use energy and charge conservation to get a set of linear equations for the currents

④ If $I_n > 0$ the current is in the direction of your arrows, If $I_n < 0$ the current is in the opposite direction to your arrows

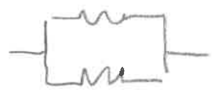
(5) simplify before solving

Resistors in series



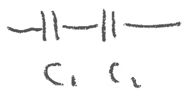
$R = R_1 + R_2$

Resistors in parallel



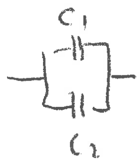
$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

Capacitors in series



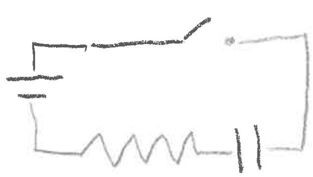
$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

Capacitors in parallel



$C = C_1 + C_2$

RC circuits



charging

$Q(t) = \mathcal{E}C\left(1 - e^{-t/RC}\right)$

$RC = \tau$ = time constant

discharging

$Q(t) = Q(0)e^{-t/RC}$

Thes solutions come from the loop equation:

$$\mathcal{E} - IR - Q/C = 0 \quad \text{charging}$$
$$- IR - Q/C = 0 \quad \text{discharging}$$

using $I = \frac{dQ}{dt}$

Note $\ln e^A = A$ $\quad \ln e = 1$ $\quad \ln A + \ln B = \ln A \cdot B$

complex circuits

Magnetic Field + Forces

$$\boxed{\bar{F} = q \bar{V} \times \bar{B}}$$

magnetic force on moving charge

$\bar{F} \perp \bar{v} \Rightarrow$ magnetic forces do no work

* $\bar{v}$ parallel to $\bar{B}$ - magnetic field has no effect

v perpendicular to $\bar{B}$

$$\boxed{\frac{m v_\perp^2}{r} = q v_\perp B}$$ Newton's 2^n law

motion is circular

remarks

$r = \frac{m v_\perp}{qB}$ radius proportional to perpendicular velocity

$\omega = \frac{qB}{m}$ $v_\perp = r\omega$ ω angular frequency (does not depend on r or $v_\perp$)

general motion

circular motion around field line, $\parallel$ motion follows field line

magnetic force on wire

$I = \int \bar{J} \cdot n \, dA$ J current density

$\bar{J} = q \bar{v}_D n$ v_D drift velocity

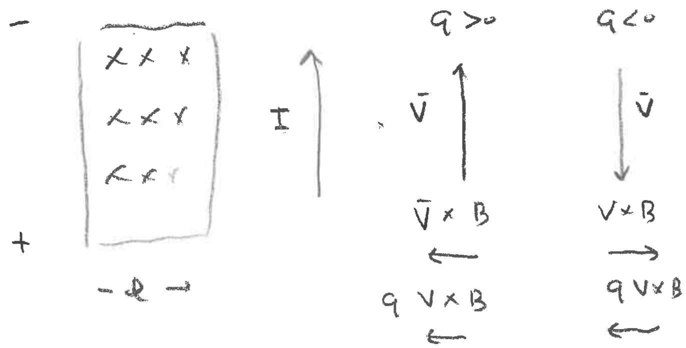
$d\bar{F} = I \, d\bar{l} \times \bar{B}$ $d\bar{l}$ is in direction of current

also misc

$R = \rho \frac{L}{A}$ $\rho = \rho(20)(1 + \alpha(T-20))$

$C = \frac{1}{\epsilon_0} \frac{A}{d}$

Hall effect – determines charge carriers



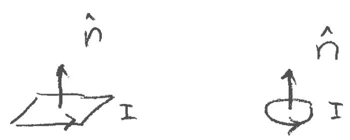
$$\boxed{\begin{aligned} qvB &= qE \\ &= q\frac{V}{l} \end{aligned}}$$

charge carriers accumulate on left

$V_{left} > V_{right}$ + carriers
$V_{left} < V_{right}$ – carriers

induced electric field cancels out magnetic force – leads to potential difference

magnetic dipoles



$\bar{\mu} \equiv IA\hat{n}$

we showed

① $U = -\bar{\mu}\cdot\bar{B}$ potential energy

② $\bar{\tau} = \bar{\mu}\times\bar{B}$ torque

similar to

$U_E = -\bar{p}\cdot\bar{E}$

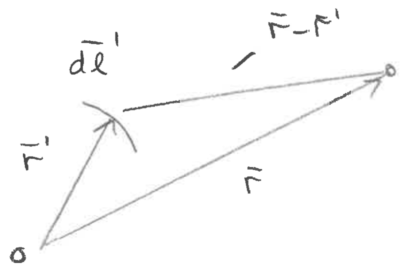
$\tau_E = \bar{p}\times\bar{E}$

sources of magnetic fields

$\Rightarrow$ currents or moving charges

* Biot Savart Law - magnetic field due to current

$$d\bar{B} = \frac{\mu_0 I}{4\pi} \frac{d\bar{l}' \times (\bar{r}-\bar{r}')}{|\bar{r}-\bar{r}'|^3}$$



$$\bar{B}(r) = \int \frac{\mu_0 I}{4\pi} \frac{d\bar{l}' \times (\bar{r}-\bar{r}')}{|\bar{r}-\bar{r}'|^3}$$

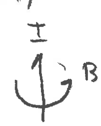
* Field at center of an arc

$$\bar{B}(r) = \frac{\mu_0 I}{4\pi} \frac{\theta}{r}$$

direction right hand rule
out of plane of $\bar{r}'$ and $d\bar{r}'$

* Field due to infinitely long wire

$$\bar{B} = \frac{\mu_0 I}{2\pi r} \hat{\theta}$$



superposition principle

* add (vector) fields due to multiple parallel wires

* add fields of central concentric arcs

magnetic field due to a magnetic dipole - used Biot Savart $r >> R$

$$\bar{B} = \frac{\mu_0}{4\pi} \frac{3(\bar{\mu}\cdot\bar{r})\bar{r} - r^2\bar{\mu}}{r^5} \qquad \bar{\mu} = IA\hat{n}$$

$r >> d$

similar to

$$\bar{E} = \frac{1}{4\pi\epsilon_0} \frac{3(\bar{p}\cdot\bar{r})\bar{r} - r^2\bar{p}}{r^5}$$

(field due to electric dipole)

Amperes law

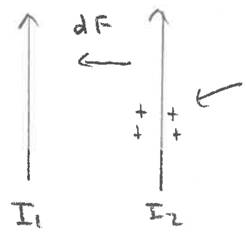
$$\oint \bar{B}\cdot d\bar{l} = \mu_0 I$$

Applications

① long straight wire
② solenoid
③ toroid

$$\boxed{\begin{array}{l} \oint \bar{B}\cdot d\bar{\ell} = \mu_0 \int \bar{J}\cdot\hat{n}\, dA \\ \bar{J} = \text{current density} \end{array}}$$

* force on wires



$$B = \frac{\mu_0 I_1}{2\pi r}$$

$$dF_2 = I_2\, d\bar{z} \times \bar{B}$$

remark

$$\oint \bar{B}\cdot d\bar{\ell} = \int (\nabla\times B)\cdot\hat{n}\, dA = \int \mu_0 \bar{J}\cdot\hat{n}\, dA \Rightarrow$$

$$\boxed{\nabla\times\bar{B} = \mu_0 \bar{J}}$$

differential form of amperes law - can be used for moving charges