# Summary - Electromagnetic Radiation

① $\bar{E} = \bar{E}_0 \cos(\bar{k}\cdot\bar{r} - \omega t)$

$k\lambda = 2\pi$ gives wavelength $\lambda$

$\lambda = \frac{2\pi}{k}$

$k$ = wave number

$kc = \omega = 2\pi f = \frac{2\pi}{T}$

$f$ = frequency

$\omega$ = angular frequency

$\hat{k}$ = direction of wave

$\bar{E}_0 \cdot \hat{k} = 0$

$\bar{B}_0 = \frac{1}{c}\hat{k}\times\bar{E}_0$

$\bar{B}_0 \cdot \hat{k} = 0$

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$\bar{B} = \frac{1}{c}\hat{k}\times\bar{E}_0 \cos(\bar{k}\cdot\bar{r} - \omega t)$

② more complex waves can be constructed using superposition principle

③ energy transport

$$\bar{S} = \frac{1}{\mu_0}\bar{E}\times\bar{B} = c\left(\tfrac{1}{2}\epsilon_0 \bar{E}\cdot\bar{E} + \tfrac{1}{2\mu_0}\bar{B}\cdot\bar{B}\right)\hat{k}$$

$\langle \bar{S} \rangle = I$ = intensity

$\bar{E}_{rms} = \frac{1}{\sqrt{2}}\bar{E}_{max}$ $\quad \bar{B}_{rms} = \frac{1}{\sqrt{2}} B_{max}$

$I = \frac{1}{\mu_0}\bar{E}_{rms}\times\bar{B}_{rms} = \frac{1}{2}\cdot\frac{1}{\mu_0}(\bar{E}_{max}\times\bar{B}_{max})$

$IA$ = Average power deposited in area $A$

④ radiation pressure

$$P = \frac{I}{c}$$

if radiation absorbed $P = \frac{I}{c}$

if radiation reflected $P = 2\frac{I}{c}$

case 1 field momentum absorbed

case 2 field momentum changes direction - absorbed momentum is $2\Delta p$

⑤ polarization


only let polarization in an direction

$$I = c\,\epsilon_0\,E_{rms}^2\,\underline{\cos^2\theta}$$

$$(\cos 90^\circ)^2 = 0$$

$$(\cos 45)^2 (\cos 45)^2 = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$$

possible to rotate polarization without much loss role of frequency

$$\frac{\text{Energy}}{\text{photon}} = hf = h\frac{\omega}{2\pi} = \hbar\omega$$

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Magnetic Properties of matter

① magnetic dipoles

$\bar{\mu} = IA$ current loops

direction right hand rule

magnetic dipole moment

① $U = -\bar{\mu}\cdot\bar{B}$

② $\bar{B}_d = \frac{\mu_0}{4\pi}\frac{3(\bar{\mu}\cdot\bar{r})\bar{r} - \bar{\mu}(\bar{r}\cdot\bar{r})}{r^5}$

analogous to electric case

$U = -\bar{p}\cdot\bar{E}$

$\bar{E}_d = \frac{1}{4\pi\epsilon_0}\left(\frac{3(\bar{p}\cdot\bar{r})\bar{r} - \bar{p}(\bar{r}\cdot\bar{r})}{r^5}\right)$

② magnetic moments and angular momentum

$\frac{mv^2}{r} = \frac{q^2}{4\pi\epsilon_0}\frac{1}{r^2}$ orbiting electron

$I = \frac{q}{T} = qf = q\frac{\omega}{2\pi}$

$IA = \pi r^2\left(q\frac{\omega}{2\pi}\right) = \frac{q}{2}r^2\omega = \frac{q}{2}rv = \frac{q}{2m}mrv$

$$L = \bar{r} \times \bar{p} = \bar{r} \times m\bar{v}$$

$$|\bar{L}| = rmv \quad \text{for } \bar{v} \perp \bar{r}$$

$$\boxed{\bar{u} = \frac{q_e}{2m}\bar{L}}$$

In quantum mechanics orbital angular momentum is quantized $\vec{L}$ can have integer multiples of $\hbar$

electrons also have a spin angular momentum

$$\boxed{\bar{u}_{spin} = \frac{q}{m}\vec{S}}$$

$\bar{s}$ has values $\pm\frac{\hbar}{2}$.

The absence of $\frac{1}{2}$ for $\bar{u}_{spin}$ is an experimentally determined number — the big success of the Dirac equation for an electron is that it explains the factor of 2.

In a general atom or molecule the different angular moments are added, so the total magnetic dipole moment can be more complicated.

In general if there are a lot of magnetic dipoles in a substance the moments will be randomly oriented

If a magnetic field is applied they tend to want to rotate until they are parallel to the magnetic field

since each dipole behaves like a small bar magnet, the oriented magnetic dipoles will tend to increase the strength of the field.

consider

$$\bar{B}_{dipole} = \frac{\mu_0}{4\pi} \frac{3(\bar{\mu}\cdot\hat{r})r - \bar{\mu}(\bar{r}\cdot\bar{r})}{r^5}$$

$$\sim \mu_0 \cdot \frac{\mu}{\text{volume}}$$

$\frac{\bar{B}}{\mu_0}$ has dimension magnetic moment / volume

for a system of magnetic dipoles we define the magnetization $\bar{M}$ as the dipole moment / volume

The total magnetic field is

$$\bar{B}_{TOT} = \bar{B}_{applied} + \mu_0 \bar{M}$$

it is useful to define a new field

$$\boxed{\bar{B}_{applied} = \mu_0 H}$$

then

$$\boxed{\bar{B}_{TOT} = \mu_0 (\bar{H} + \bar{M})}$$

In many case $\bar{M}$ is proportional to $\bar{H}$

$$\bar{M} = \chi \bar{H}$$

$\chi$ is called the magnetic susceptibility

$$\boxed{\bar{B}_{TOT} = \mu_0 (1+\chi) \bar{H} = \mu \bar{H}}$$

where $\bar{B}$ is the actual field - including magnetization, that includes the dipole field

$\mu$ is called the magnetic permeability when $\bar{M} = \chi \vec{H}$

there are 2 cases of interest

① $\chi > 0$

② $\chi < 0$

when $\chi > 0$ the material is called paramagnetic material. This happens when the material has dipoles that align with the field

when $\chi < 0$ the material is called diamagnetic. Usually this is a very small effect. It was discovered by Faraday. It tends to weaken the field The only exception to being small is in superconductors, where it completely eliminates the field in the superconductor.

For paramagnetic substances the magnetization also has a temperature dependence given by Curie's law

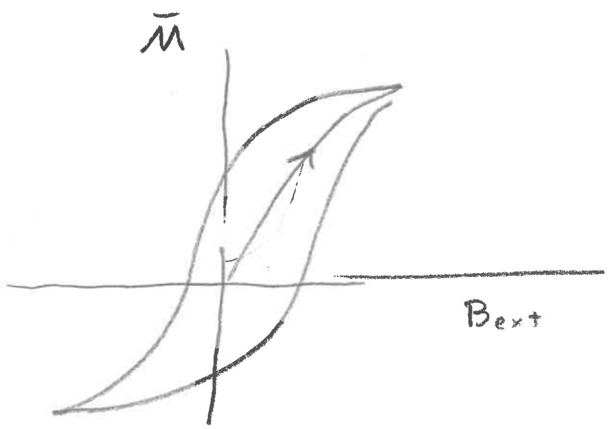
$$M = C \frac{B_{ext}}{T} \qquad \bar{B}_{ext} = \mu_0 \bar{H}$$

C is called the Curie constant

T is the absolute temperature

There is a third type of magnetic material called Ferromagnetic material

In these materials the magnetic field will persist, even after the external field is removed. In this case since $U = -\bar{\mu} \cdot \bar{B}$ once the dipoles reach a low energy state the fields due to the dipoles keep them aligned.



In this case the magnetization is a multivalued function of the applied field.

The alignment can be destroyed by heating. The Curie temperature is a temperature above which they become paramagneti

# Special Relativity

If you throw a ball in a car moving with constant velocity it can be treated using the same laws of physics that apply to the car at rest.

This is not true if the car is accelerating

This suggest that there are preffered coordinate systems where the laws of physics have the same form. These are called inertial coordinate systems

They are coordinate systems where non-interacting particles move with constant velocity

In these systems Newton's second law for free particles must be preserved

$$m \frac{d^2\bar{x}}{dt^2} = 0 \qquad x \to \bar{y}(\bar{x}) \qquad m \frac{d^2\bar{y}}{dt^2} = 0$$

Transformations that do this are

$$\bar{y}_i(t) = \bar{x}_i(t) + \bar{a} \qquad \bar{a} = \text{constant}$$

$$\bar{y}_i(t) = \bar{x}_i(t-t_0) \qquad t_0 = \text{constant}$$

$$\bar{y}_i(t) = \bar{x}_i(t) + \bar{b}t \qquad \bar{b} = \text{constant}$$

$$y_1(t) = \cos\theta\, x_1(t) + \sin\theta\, x_2(t)$$
$$y_2(t) = \cos\theta\, x_2(t) - \sin\theta\, x_1(t)$$
$$y_3(t) = x_3(t)$$

These correspond to space translations, time translations, velocity shifts, and rotations. These all preserve

$$m\ddot{\tilde{y}} = 0$$

These elementary transformations generate the most general transformations relating inertial coordinate systems in classical mechanics

Maxwells equation → wave equation

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\begin{Bmatrix}\bar{E}(r,t)\\ \bar{B}(r,t)\end{Bmatrix}=0$$

leads to waves traveling at the speed of light

The speed of light can't be the same in 2 coordinate systems related by a velocity shift unless the identification of inertial coordinate systems is wrong.

It was know before Einstein that transformations that preserve $\bar{r}_1-\bar{r}_2$

$$(x_1-x_2)^2+(y_1-y_1)^2+(z_1-z_2)^2-c^2(t_1-t_2)^2=$$

$$(x_1'-x_2')^2+(y_1'-y_2')^2+(z_1'-z_2')^2-c^2(t_1'-t_2')^2$$

preserves the speed of light, the wave equation and Maxwells equations