29:4761 - Homework Assignment #8

1. By integrating
\[ \int \frac{zdz}{a - e^{-iz}} \]
over a rectangular curve with corners at \(-\pi, \pi, \pi + i n\) and \(-\pi + i n\) and letting \(n \to \infty\) show
\[ \int_{0}^{\pi} \frac{x \sin(x)dx}{1 + a^2 - 2a \cos(x)} = \frac{\pi}{a} \log(1 + a) \quad (0 < a < 1) \]

2. Evaluate
\[ \int_{0}^{\infty} \frac{\ln^2(z)dz}{z^2 + 1} \]

3. Express the integral
\[ \int_{0}^{\infty} e^{-\alpha x^2} x^\beta dx \]
where \(\alpha\) and \(\beta\) are real and positive in terms of the Gamma function.

4. Prove that if \(a > 0\), \(-\frac{1}{2} \pi < a \lambda < \frac{1}{2} \pi\)
\[ \int_{0}^{\infty} e^{-r^n \cos(a \lambda)} \cos(r^n \sin(a \lambda))dr = \cos(\lambda) \frac{1}{a} \Gamma\left(\frac{1}{a}\right) \]

5. Calculate
\[ \int_{0}^{\pi/2} \sin^\alpha(\theta) \cos^\beta(\theta)d\theta \]
for \(\alpha, \beta > 0\).

6. Evaluate \(\beta(m, n)\) and relate it to the binomial coefficients.