1.) Consider the function \( f(x) = (1 - x^2) \) on the interval \([-1, 1]\). Calculate a 4-th degree Weierstrass polynomial approximation to this function. Compare the exact and approximate functions.

2.) Assume that \( |f(x) - p_n(x)| < \epsilon \) for all \( x \in [a, b] \), where \( p_n(x) \) is a polynomial. Let \( M \) be a \( 2 \times 2 \) Hermitian matrix with real eigenvalues \( \lambda \) satisfying \( a < \lambda < b \). Show that

\[
\| (f(M) - p_n(M)) \| < \epsilon
\]

where \( \|O\| \) is the matrix or operator norm of \( O \).

3.) Let \( f(x) \) and \( g(x) \) have Fourier Transforms.

\[
\hat{f}(k) = \frac{1}{\sqrt{2\pi}} \int dy e^{iky} f(y) \quad \hat{g}(k) = \frac{1}{\sqrt{2\pi}} \int dy e^{iky} g(y)
\]

Find an expression for the Fourier transform of the product \( f(x)g(x) \) in terms of their individual Fourier transforms, \( \hat{f}(k) \) and \( \hat{g}(k) \),

4.) Show

\[
\lim_{\lambda \to \infty} \int_{-\infty}^{\infty} e^{i\lambda x} f(x) dx = 0
\]

if \( f \) is absolutely integrable and differentiable for every \( x \).

5.) Show that if \( f(x) \) is a continuous function that is identically zero for \( |x| > L \), then its Fourier transform is an entire function.

6.) Calculate the Fourier transform of \( e^{-x^2/2} \).