1.) Use the power series representation of \( J_{1/2}(x) \) to calculate the power series representation of the function

\[
j_0(x) = \sqrt{\frac{\pi}{2x}} J_{1/2}(x).
\]

Can you relate this series to the series expansion of an elementary function?

2.) Consider the differential operator

\[
L = \frac{1}{x} \frac{d^2}{dx^2} + 1
\]

a. Find independent solutions to the homogeneous equations on the interval \([0, a]\).

b. Find the Green function satisfying Dirichlet boundary on \([0, a]\)

3.) Use the generating function

\[
e^x (t^2 - 1) = \sum_{n=-\infty}^{\infty} t^n J_n(x)
\]

to derive the recurrence relation

\[
J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)
\]

4.) Consider the differential equation

\[
\frac{d^2}{dz^2} f(z) + \frac{2}{z} \frac{d}{dz} f(z) - zf(z) = 0
\]

Find the indicial equation for a solution about \( z = 0 \). Find the roots.

5.) Verify that

\[
z^{1-c} \Phi(a - c + 1, 2 - c, z)
\]

satisfies the same differential equation as the confluent Hypergeometric function (c not integer) \( \Phi(a, c, z) \).

6.) Find the general solution to

\[
u'(x) + xu(x) = ce^{-x^2/2}
\]