1. Consider a plane electromagnetic wave interacting with an electron bound to a symmetric three-dimensional harmonic oscillator well with spring constant $k$. Assume that the vector potential of the wave has the form

$$A = -i \frac{E}{\omega} e^{i \phi/c - i \omega t}$$

Find an expression, using first-order time-dependent perturbation theory for this wave to induce a transition from the ground state to first excited state of the oscillator.

2. Consider a two state system with unperturbed Hamiltonian $H_0 := E_1 |1\rangle \langle 1| + E_2 |2\rangle \langle 2|$ with $E_2 > E_1$ and perturbing interaction for $t > 0$, $V(t) = \gamma e^{i \omega t} |1\rangle \langle 2| + \gamma e^{-i \omega t} |2\rangle \langle 1|$ where $\omega$ and $\gamma$ are real positive constants.

   a. Do the same calculation using first order perturbation theory.

   b. Both probabilities exhibit oscillations. Find the frequency that maximizes the amplitude of the oscillations of the probability to find the system in the second state.

3. A one-dimensional harmonic oscillator is in its ground state for $t < 0$. For $t \geq 0$ it is subject to a time-dependent but spatially uniform force (not potential) in the $x$-direction

$$F(t) = F_0 e^{-i \omega t}$$

Use first order time-dependent perturbation theory to find the probability of finding the oscillator in its first excited state as a function of time.

4. Let

$$H_0 = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \quad V(t) = \theta(t) \begin{pmatrix} 0 & \eta e^{-i \omega t} \\ \eta e^{i \omega t} & 0 \end{pmatrix}$$

Find the probability as a function of time that the system makes a transition from the 1 state to the 2 state.
\[ \langle 1 \left| 0 \right| 000 \rangle \rightarrow 1001 \rangle \]

\[ |C_{001}|^2 = \frac{i \omega z/c}{\sin \left( \frac{(\sqrt{\frac{k}{m}} - \omega)^2}{2} \right)} \]

The non-zero contribution comes from

\[ i \omega z/c \]

\[ \langle 111e \left| 0 \right| 10 \rangle \]

\[ \int_{-\infty}^{\infty} e^{\frac{1}{\sqrt{2\pi}} 2(mw/n) \frac{i \omega z/c - mw z^2}{2}} e^{\frac{-\omega}{\omega_0}} \frac{m \omega (Z - 1)}{c^4 m^2} + \frac{m \omega \omega_1^2}{c^2 4 m^2} \]

\[ e^{\frac{i 2}{\sqrt{2\pi}} m \omega \left( -Z c^2 4 m^2 \right) \frac{E}{E_1}} \left( -i \frac{\Delta}{\omega_0} \right) \left( \frac{E}{E_1} \right) \]

\[ \frac{\hbar \omega_1^2}{c^4 m^4 \omega} \]

\[ -i 2 \frac{E}{\omega} \left( -\frac{2 \omega_1^2}{c^4 m^2} \right) \frac{E}{E_1} \left( \frac{E}{E_1} \right) \frac{\hbar \omega_1^2}{c^4 m^2} \]

\[ \frac{i E \hbar}{c m \omega} \]

\[ \frac{\hbar \omega_1^2}{4 c^2 m^2} \]

\[ |C_{001}|^2 = \frac{E \hbar}{c^2 m^2 \omega_1^2} e^{\frac{\hbar \omega_1^2}{2 c^2 m^2 \omega_1^2}} \frac{\sin^2 \left( \frac{\omega - \omega_1}{2} \right)}{\left( \frac{\hbar \omega - \hbar \omega_1}{2} \right)^2} \]

\[ \omega = \sqrt{\frac{k}{m}} \]
\[ H_0 = \begin{pmatrix} E_1 & 0 \\ 0 & E_1 \end{pmatrix} \]

we did the exact calculation last week.

\[ |C_2|^2 = K^2 |\langle n | \{ x \} | n \rangle|^2 \frac{\sin^2 \left( \frac{E_n - E_1 - \omega_n}{2} \right)}{\left( \frac{E_n - E_1 - \omega_n}{2} \right)^2} \]

\[ \omega_m = \frac{E_2 - E_1}{\hbar \omega} \]

\[ F(t) = F_0 e^{-i\omega_0 t}, \quad V = -F_0 x e^{-i\omega t} \]

\[ |C_3|^2 = \langle 11 (-F_0 \chi) 10 \rangle \frac{\sin^2 \left( \frac{\sqrt{\hbar m} - \omega}{2} \right)}{\left( \frac{\hbar (\sqrt{\hbar m} - \omega)}{2} \right)^2} \]

\[ X = \sqrt{\frac{\hbar}{2m\omega_0}} (a + a^\dagger) \]

\[ \langle 11 (-F_0 \sqrt{\frac{\hbar}{2m\omega_0}}) (a + a^\dagger) 10 \rangle = \]

\[ -F_0 \sqrt{\frac{\hbar}{2m\omega_0}} = F_0 \sqrt{\frac{\hbar}{2\sqrt{\hbar m}}} \]

\[ |C_1|^2 = \frac{F_0^2 \sqrt{\frac{\hbar}{2\sqrt{\hbar m}}}}{2} \frac{\sin^2 \left( \frac{\sqrt{\hbar m} - \omega}{2} \right)}{\left( \frac{\hbar (\sqrt{\hbar m} - \omega)}{2} \right)^2} \]
In this case

\[ c_2 = -\frac{i}{\hbar} \int_0^t e^{i(E_2 - E_1)\frac{t'}{\hbar}} e^{-i\omega t'} \text{d}t' \]

\[ = -i \frac{m}{\hbar} \frac{i}{i(E_2 - E_1) - \omega} \left( e^{\frac{i(E_2 - E_1)t}{\hbar}} - 1 \right) \]

\[ = -\frac{m}{\hbar} e^{-\omega t} e^{\frac{i(E_2 - E_1)t}{2\hbar}} \frac{2i}{i(E_2 - E_1) - \omega} \sin \left( \frac{E_2 - E_1}{2\hbar} t \right) \]

\[ |c_2|^2 = \frac{m^2}{\hbar^2} e^{-2\omega t} \frac{4}{(E_2 - E_1)^2 + \omega^2} \sin^2 \left( \frac{E_2 - E_1}{2\hbar} t \right) \]

\[ \eta^2 e^{-2\omega t} \frac{\sin^2 \left( \frac{E_2 - E_1}{2\hbar} t \right)}{(\frac{E_2 - E_1}{2})^2 + \omega^2 \frac{\hbar^2}{4}} \]