Three-body scattering in Poincaré invariant quantum mechanics

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Outline

- Motivation
- Poincaré invariant quantum mechanics
- Construction of the three-body dynamics
- Solving the Faddeev equations
- Results
- Observations

(T. Lin et. al. Phys. Rev. C76,014010(2007))

Motivation

- Extend NR QM so it is applicable at energy scales where sub-nucleon degrees of freedom may be relevant:

$$\Delta pc \geq \frac{\hbar c}{(\Delta x)} \gtrsim m_n c^2$$

- Poincaré invariance is required for a consistent treatment of dynamics.
- What are the observable consequences of Poincaré symmetry in the three-nucleon system?

Poincaré invariant quantum mechanics

(Wigner - 1939)

$$P = |\langle \Psi_f | \Psi_i \rangle|^2 = |\langle \Psi'_f | \Psi'_i \rangle|^2 = P'$$

$$\Updownarrow$$

$$|\Psi'\rangle = U(\Lambda, a)|\Psi\rangle$$

$$\Updownarrow$$

$$P^\mu, J^{\mu\nu}$$

Single-nucleon Hilbert space, $U(\Lambda, a) \to U_n(\Lambda, a)$

$$||$$

$(m = m_n, j = \frac{1}{2})$ irreducible representation space

$$P^\mu_n, J^{\mu\nu}_n \quad \Rightarrow \quad m, j, h_i, \Delta h_i \qquad 1 \le i \le 4$$

$$\mathcal{H}_n = L^2(\sigma(h)) \qquad |(m,j)\,\mathbf{h}\rangle$$

$$U_n(\Lambda, a)|(m,j)\,\mathbf{h},\rangle = \sum\!\!\!\!\!\!\int |(m,j)\,\mathbf{h}'\,\rangle d\mathbf{h}' D^{mj}_{\mathbf{h}'\mathbf{h}}(\Lambda, a)$$

eg. $\qquad \mathbf{h} = (\mathbf{p}, \hat{\mathbf{z}} \cdot \mathbf{j}_c)$

N-nucleon Hilbert space

$$\mathcal{H} = \bigotimes_{i=1}^{N} \mathcal{H}_n$$

Tensor product basis

$$|(m_n, j_n), \mathbf{h}_1, \cdots (m_n, j_n), \mathbf{h}_N\rangle$$

Kinematic representation of the Poincaré group

$$U_0(\Lambda, a) := \bigotimes_{i=1}^{N} U_n(\Lambda, a)$$

$$U_0(\Lambda, a)|(m_n, j_n), \mathbf{h}_1, \cdots, (m_n, j_n), \mathbf{h}_N\rangle =$$

$$\sum\!\!\!\!\!\!\int |(m_n, j_n), \mathbf{h}'_1, \cdots, (m_n, j_n), \mathbf{h}'_N\rangle \prod_{i=1}^{N} d\mathbf{h}'_i D^{m_n j_n}_{\mathbf{h}'_i \mathbf{h}_i}(\Lambda, a)$$

This is reducible!

N free particle irreducible representations

Poincaré Clebsch-Gordan coefficients

$$|(m,j),\mathbf{h};\mathbf{d}\rangle = \sum\!\!\!\!\!\!\!\int |(m_n,j_n),\mathbf{h}_1,\cdots(m_n,j_n),\mathbf{h}_N\rangle < CG >$$

$$\bigotimes_{i=1}^{N} D^{m_n j_n}_{\mathbf{h}'_i \mathbf{h}_i}(\Lambda,a) < CG > = \int_{\oplus m,j,d} < CG > D^{mj}_{\mathbf{h}'\mathbf{h}}(\Lambda,a)$$

$$\Downarrow$$

$$U_0(\Lambda,a)|(m,j),\mathbf{h};\mathbf{d}\,\rangle = \sum\!\!\!\!\!\!\!\int |(m,j),\mathbf{h}';\mathbf{d}\,\rangle d\mathbf{h}' D^{mj}_{\mathbf{h}'\mathbf{h}}(\Lambda,a)$$

Dynamics

$$M = M_0 + V \geq 0$$

$$[V, j^2] = [V, \mathbf{h}] = [V, \Delta\mathbf{h}] = 0$$

Diagonalize M in free particle irreducible basis, $|(m, j), \mathbf{h}, \mathbf{d}\rangle$

$$\Downarrow$$

Simultaneous eigenstates of $M, j^2, \mathbf{h}$

$$|(\lambda, j), \mathbf{h}, \mathbf{d}_I\rangle$$

(Bakamjian & Thomas - 1953)

Dynamical representation of the Poincaré group

$$\{|(\lambda, j), \mathbf{h}, \mathbf{d}_I\rangle\}$$

complete and

$$U(\Lambda, a)|(\lambda, j), \mathbf{h}, \mathbf{d}_I\rangle = \sum\!\!\!\!\!\!\int |(\lambda, j), \mathbf{h}'; \mathbf{d}_I\rangle d\mathbf{h}' D^{\lambda j}_{\mathbf{h}'\mathbf{h}}(\Lambda, a)$$

$U(\Lambda, a) =$ solution of dynamical problem

Scattering Theory

$$S = \Omega_+^\dagger(H, H_0)\Omega_-(H, H_0)$$

$$\Omega_\pm(H, H_0) = \Omega_\pm(M, M_0) = \Omega_\pm(M^2, M_0^2)$$

(Kato - Birman - 1966)

$$\bar{M} := WMW^\dagger \qquad \bar{M}_0 := M_0 \qquad WW^\dagger = I$$

$$S(M, M_0) = S(\bar{M}, M_0)$$

$$\Updownarrow$$

$$\boxed{\lim_{t\to\pm\infty} \|(W - I)e^{-iM_0 t}|\psi\rangle\| = 0}$$

(Ekstein - 1960)

$$N = 2$$

$$V = 4 m_n V_{nn} \qquad M_0^2 = 4(\mathbf{k}^2 + m_n^2) \qquad M^2 = M_0^2 + V = 4 m_n (H_{nr} + m_n)$$

$$\frac{d\sigma}{d\Omega}(k) = \frac{(2\pi)^4}{v_{nr}} |\langle f | T_{H_{nr}} | i \rangle|^2 k^2 \frac{dk}{dE_{nr}} = \frac{(2\pi)^4}{v_r} |\langle f | T_M | i \rangle|^2 k^2 \frac{dk}{dE_r}$$

No relativistic corrections for $N = 2$! $\qquad (k_{nr} = k_r)$

No loss of generality in choosing $V = 4 m_n V_{nn}$!

(Coester, Pieper & Serduke - 1975)

$$N = 3$$

Structure of M dictated by two-body M and cluster properties

$$M = W(V)\bar{M}(V)W^{\dagger}(V) \qquad \bar{M}(V) = M_0 + V_{12} + V_{23} + V_{31}$$

$$V_{12} = \sqrt{M_{12}^2 + q_3^2} - \sqrt{M_{012}^2 + q_3^2} \quad \cdots$$

$$M_{12}^2 = 4(\mathbf{k}^2 + m_n^2) + 4m_n V_{NN} \qquad M_{012}^2 = 4(\mathbf{k}^2 + m_n^2)$$

$W(V)$ not required to calculate S!

(Coester 1965, Sokolov 1977, F.C & W.P. 1982)

Predictions

Binding energies

$$M(V)|\Psi\rangle = \lambda|\Psi\rangle$$

$$|\Psi\rangle = W(V)|\bar{\Psi}\rangle \qquad \bar{M}(V)|\bar{\Psi}\rangle = \lambda|\bar{\Psi}\rangle$$

Cross sections

$$S = \langle\Psi_f^+|\Psi_i^i\rangle = \langle\bar{\Psi}_f^+|\bar{\Psi}_i^i\rangle$$

Current matrix elements

$$\langle\Psi_f|J^\nu(0)|\Psi_i\rangle = \langle\bar{\Psi}_f|W^\dagger(V)J^\nu(0)W(V)|\bar{\Psi}_i\rangle$$

Faddeev Equations

$$T^{ab}(z) = \bar{\delta}^{ab}(z - M_0) + \sum_{c \neq a} V_c(z - M_0 - V_c)^{-1} T^{cb}(z)$$

$$a, b, c \in \{(12)(3), (23)(1), (31)(2)\}$$

Kernel

$$\int' \langle a|c'\rangle dc' \langle c'| V_c(z - M_0 - V_c)^{-1} |c\rangle$$

Solution by direct integration - no partial waves !

(Liu, Elster, Glöckle 2005)

Technical considerations

$$\langle a|c\rangle = \quad \text{Poincaré Group Racah coefficient}$$

(Changes order of coupling irreducible representations.)

$$\langle a|c\rangle = \delta[\mathbf{h};\mathbf{h}']\delta(M-M')\delta_{JJ'}R^{MJ}(\mathbf{d}_a,\mathbf{d}_c)$$

Become NR permutation operators in static limit

Technical considerations

$$z = z_c$$

$$\langle c' | V_c (z_c - M_0 - V_c)^{-1} | c \rangle =$$

$$\frac{4 m_n}{m'_c + m_c} \langle c' | t_{NN}(z_c) (z_c - M_0)^{-1} | c \rangle$$

NR half-shell transition matrix elements can be used to directly construct half-shell Faddeev kernel

Technical considerations

$$z \neq z_c$$

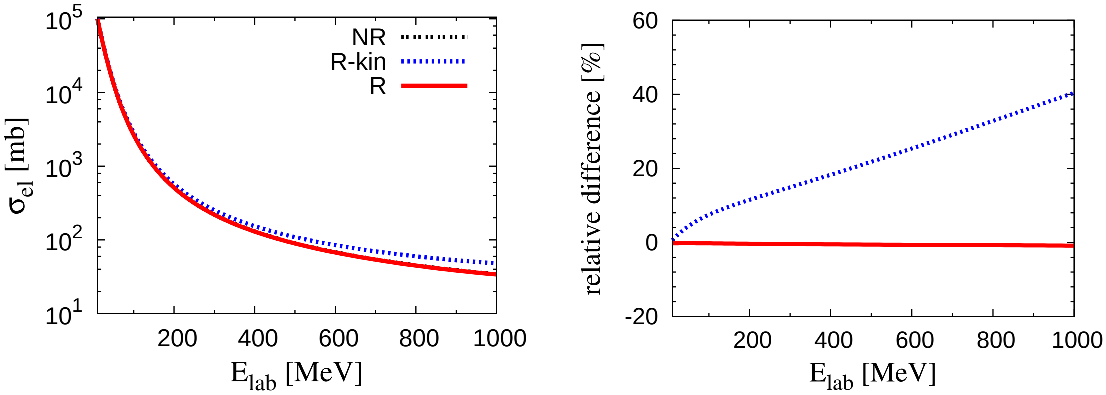
First resolvent equation gives fully off-shell Faddeev kernel

$$T_c(z) = T_c(z_c) + T_c(z) \frac{(z - z_c)}{(z - M_0)(z_c - M_0)} T_c(z_c)$$

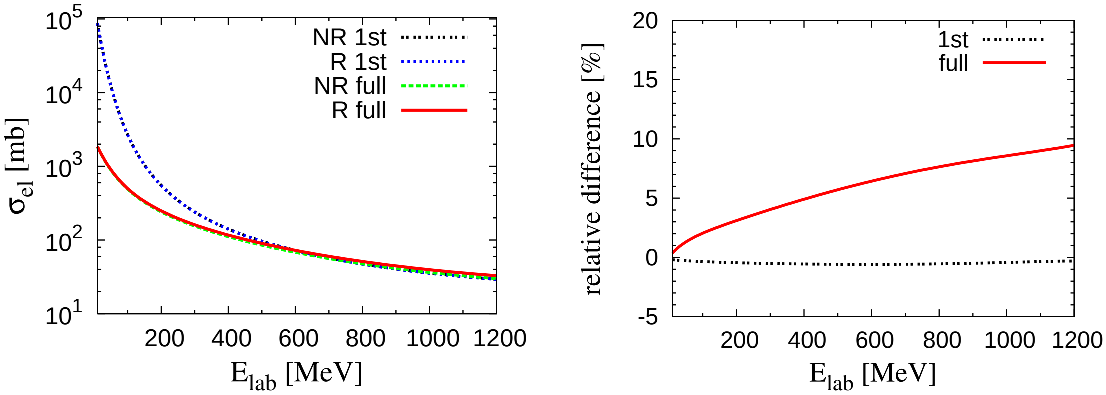
$$\langle c' | T_c(z_c) | c \rangle = \frac{4 m_n}{m_c + m_c'} \langle c' | t_{NN}(z_c) | c \rangle$$

Input to Faddeev equations

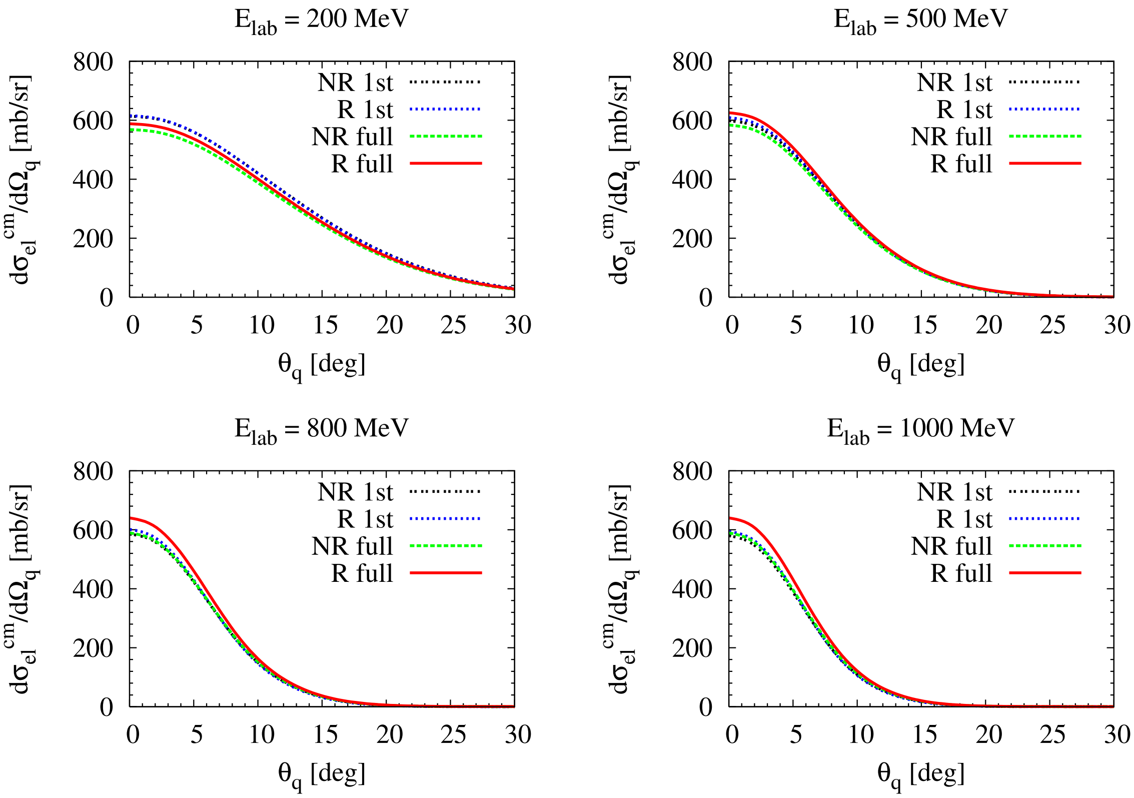
(B. Keister and W.P. 2006)

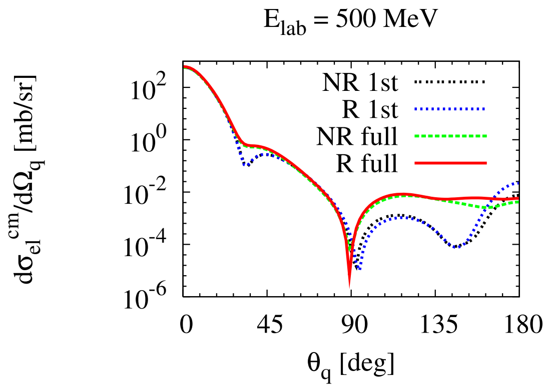


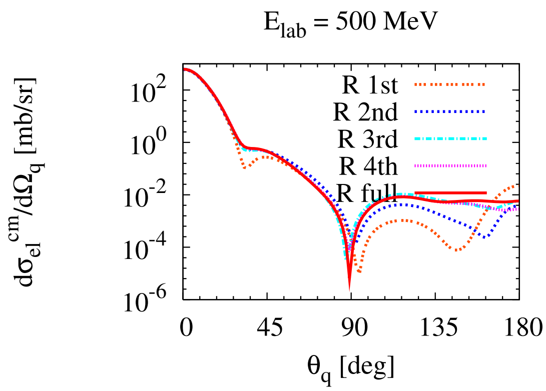
σ_{el} [mb]
NR
R-kin
R
E_{lab} [MeV]
relative difference [%]
E_{lab} [MeV]

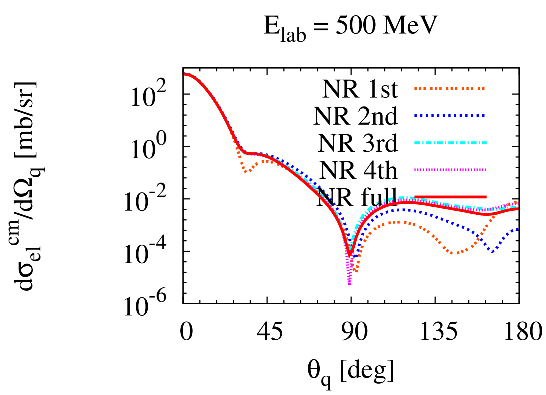


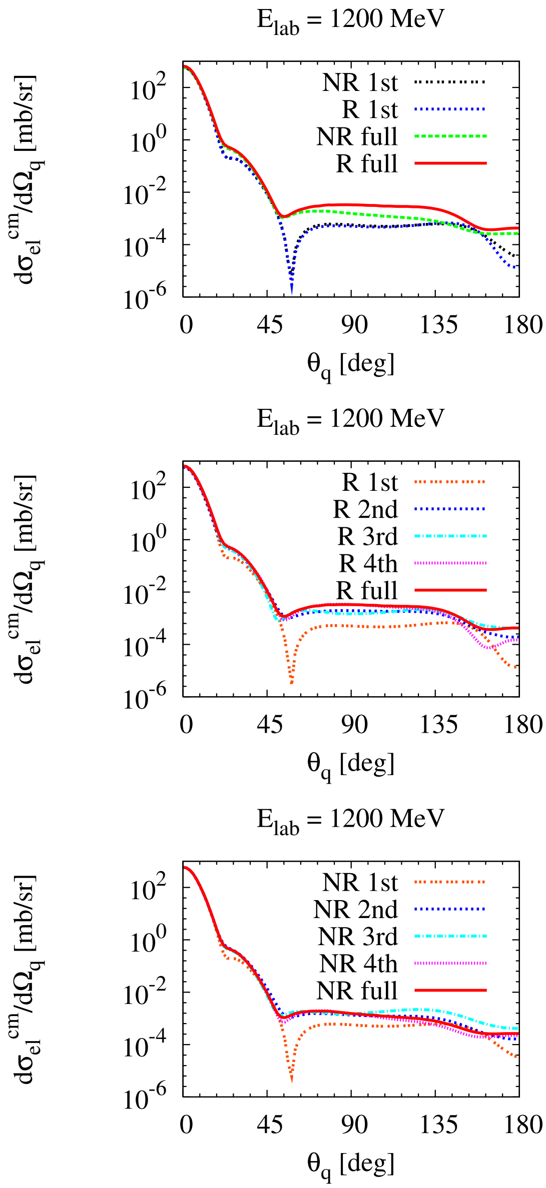
σ_{el} [mb]
NR 1st
R 1st
NR full
R full
E_{lab} [MeV]
relative difference [%]
1st
full
E_{lab} [MeV]


E_lab = 200 MeV
E_lab = 500 MeV
E_lab = 800 MeV
E_lab = 1000 MeV
dσ_el^cm/dΩ_q [mb/sr]
θ_q [deg]
NR 1st
R 1st
NR full
R full
0
200
400
600
800
0
5
10
15
20
25
30


E_lab = 500 MeV
NR 1st
R 1st
NR full
R full
dσ_el^cm/dΩ_q [mb/sr]
θ_q [deg]
0
45
90
135
180
10^2
10^0
10^-2
10^-4
10^-6


E_lab = 500 MeV
R 1st
R 2nd
R 3rd
R 4th
R full
dσ_el^cm/dΩ_q [mb/sr]
θ_q [deg]
0
45
90
135
180
10^2
10^0
10^-2
10^-4
10^-6


E_lab = 500 MeV
NR 1st
NR 2nd
NR 3rd
NR 4th
NR full
dσ_el^cm/dΩ_q [mb/sr]
θ_q [deg]
0
45
90
135
180
10^2
10^0
10^-2
10^-4
10^-6


E_lab = 1200 MeV
NR 1st
R 1st
NR full
R full
dσ_el^cm/dΩ_q [mb/sr]
θ_q [deg]
E_lab = 1200 MeV
R 1st
R 2nd
R 3rd
R 4th
R full
dσ_el^cm/dΩ_q [mb/sr]
θ_q [deg]
E_lab = 1200 MeV
NR 1st
NR 2nd
NR 3rd
NR 4th
NR full
dσ_el^cm/dΩ_q [mb/sr]
θ_q [deg]

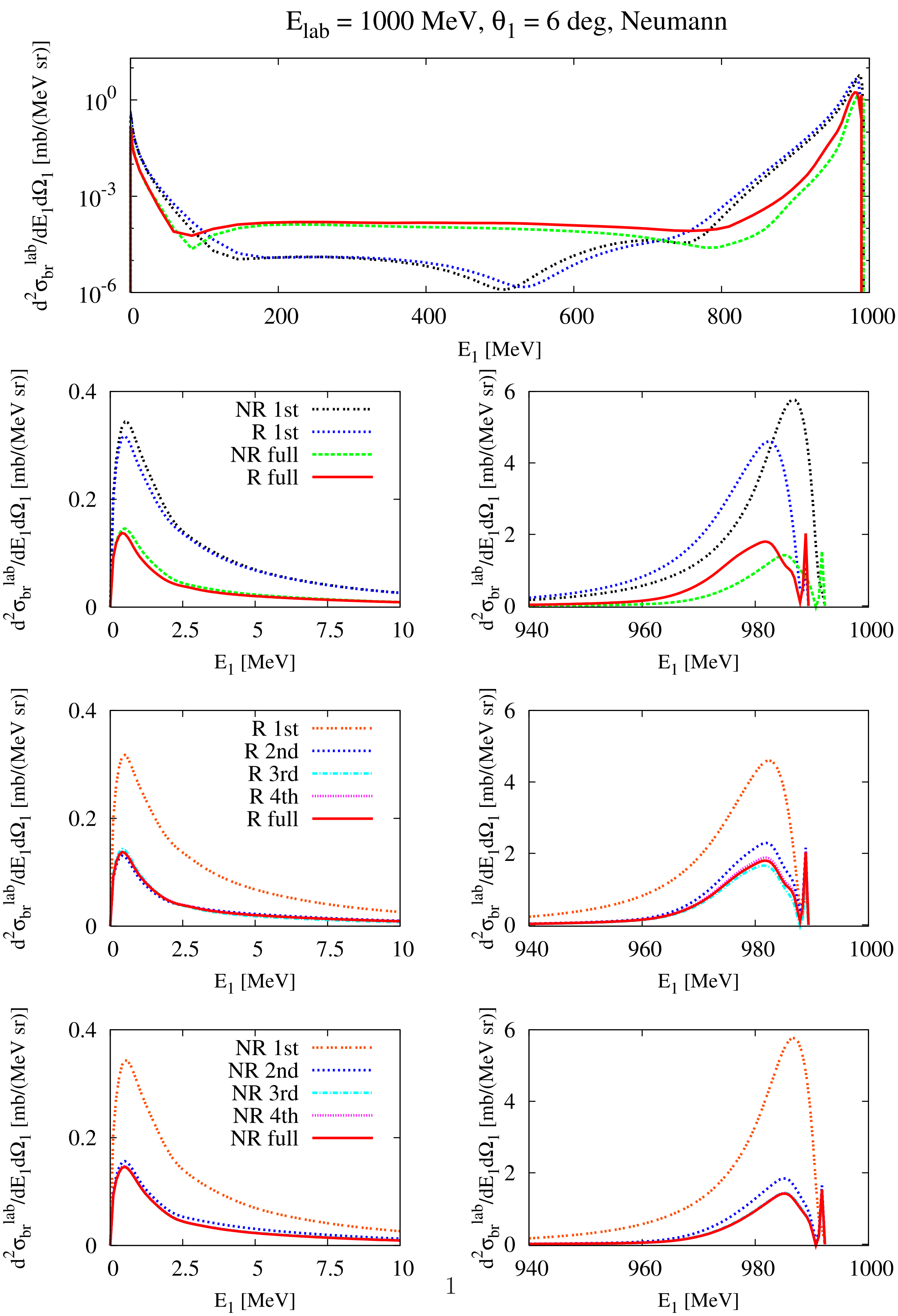


Figure 1: The incls differential cross section at projectile energy $E_{lab} = 1000\mathrm{MeV}$ and $\theta_1 = 6^o$. Neumann

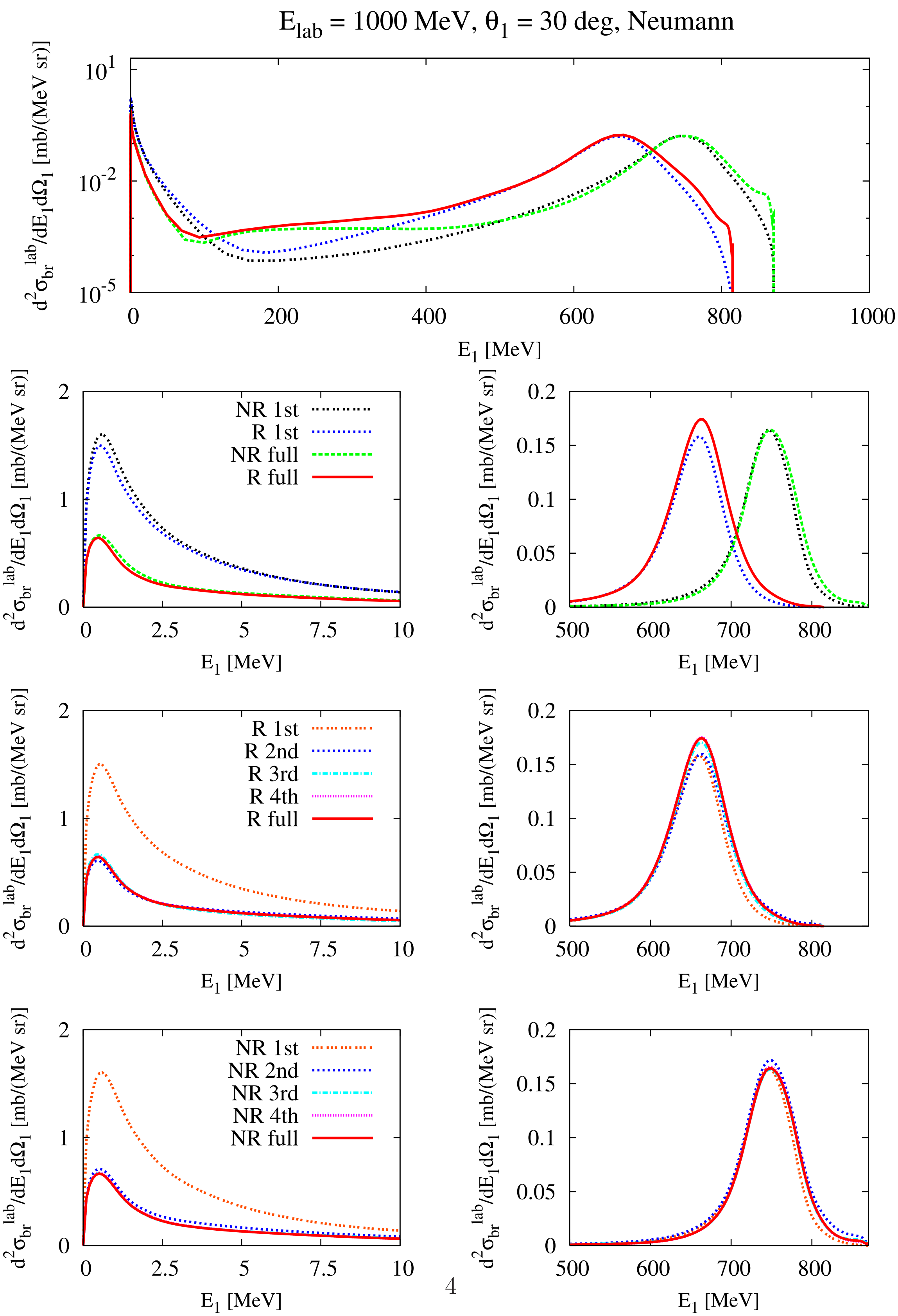


Figure 4: The incls differential cross section at projectile energy E_{lab} = 1000MeV and $\theta_1 = 30^o$. Neumann

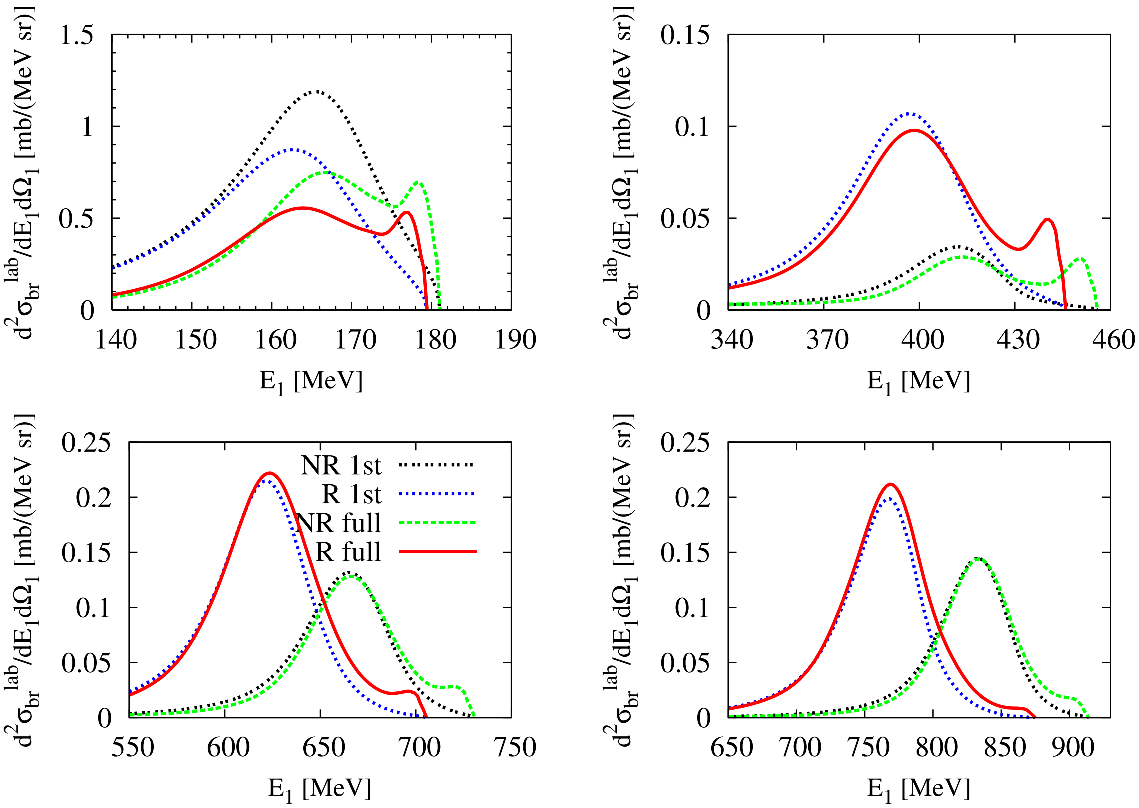
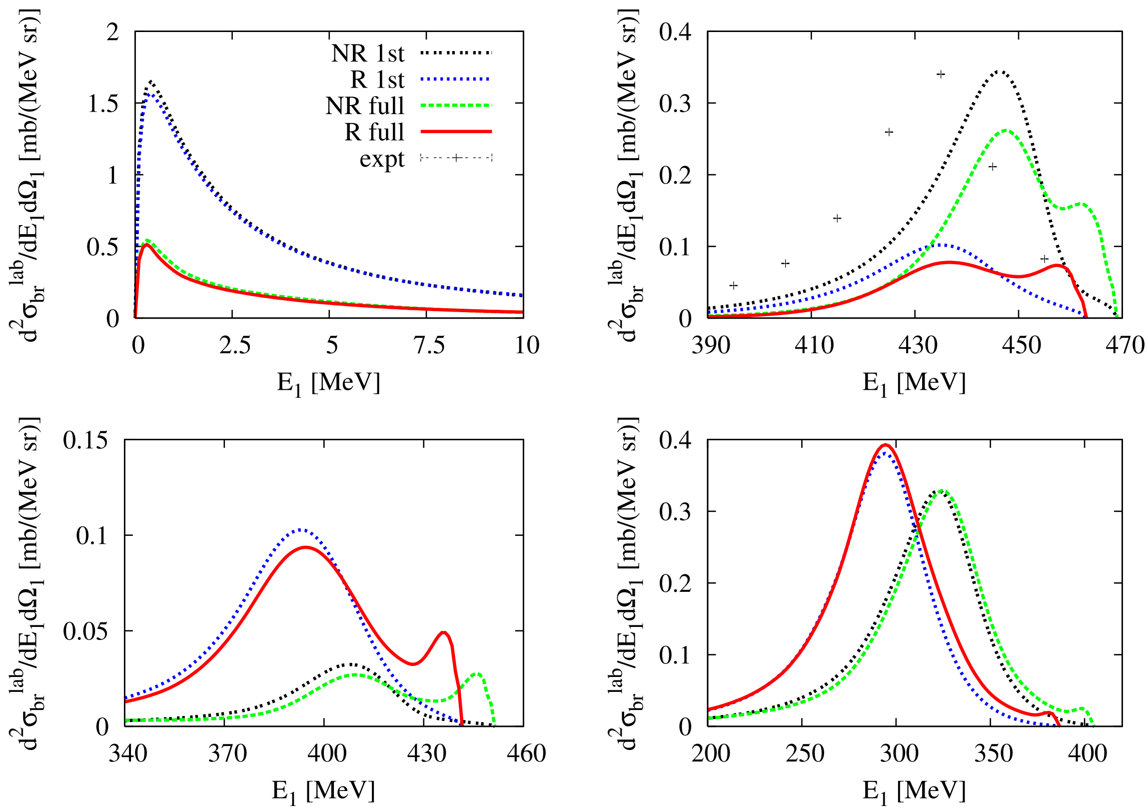


Figure 1: The quasifree peak of inclusive breakup scattering at $\theta_1 = 24^o$. From left ro right, the corresponding projectile energies are: upper panel 200 and 500 MeV; lower panel 800 and 1000 MeV.

$E_{lab} = 495$ MeV


NR 1st
R 1st
NR full
R full
expt
d²σ_br^lab/dE₁dΩ₁ [mb/(MeV sr)]
E₁ [MeV]
0
0.5
1
1.5
2
2.5
5
7.5
10
0.1
0.2
0.3
0.4
390
410
430
450
470
0.05
0.15
340
370
400
460
200
250
300
350

Observations

- Differences in the relativistic and non-relativistic calculations with same two-body input appear (1) in the Poincaré Racah coefficients and (2) in the embedding of the two-body interaction in the three-body mass operator.
- The computational method is stable beyond 1 GeV.
- High precision NN interactions can be used as input without modification. Calculations shown are with a Malfliet-Tjon potential.
- With the same two-body input there are measurable differences in the NR and relativistic three-body cross sections. In some parts of phase space there are differences even at 200 MeV.
- Watson series requires 3-4 iterations for convergence near quasi-elastic peak at 1 GeV for forward angles.
- Kinematic effects of relativity are largely canceled by dynamical effects.

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