Few-body nuclear physics: working towards the transition region

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Outline

- Problem
- Discussion of scales
- Essential quantum mechanics
- Symmetries - Galilean and Poincaré
- Model description
- Some results
- Future

Problem

Nucleus = quarks + gluons

Nucleus = nucleons + mesons

?

Scales

- **Mass of nucleon** $mc^2 = 940$ **MeV**
- **Size of nucleon** $r \approx 1 \times 10^{-15}$ **meters** $= 1$ **fm.**
- **Binding energy per nucleon** 8 **MeV.**
- **Nuclear excitation energies - a few MeV.**
- **Range of nuclear force** $r \approx 1 \times 10^{-15}$ **meters**
- **Nucleon excitation energies** 500 **MeV**

Elementary analysis

$$\Delta x \Delta p \geq \hbar$$

$$\Delta p \geq \frac{\hbar}{\frac{1}{2}\textbf{nucleon radius}}$$

$$\Delta pc \gtrsim \frac{1}{2} GeV \approx \frac{1}{2} \times \textbf{nucleon rest energy}$$

Implications

- **Maximal cross section $\Rightarrow$ minimal beam momentum**
- **Minimal beam momentum $\Rightarrow$ $\Delta p \Delta x \approx \hbar$**
- $\Delta p \Delta x \approx \hbar \Rightarrow$ **quantum mechanical treatment**
- **Minimal beam momentum $+$ $\Delta x \approx \frac{1}{2}$nucleon radius $\Rightarrow$ minimal $\Delta p \approx \frac{1}{2}$nucleon mass**
- $\Delta p \approx \frac{1}{2}$ **nucleon mass $\Rightarrow$ relativistic treatment necessary**

Goals

- Formulate a mathematical model of nuclei that provides a quantitative description of nuclear structure and reaction observables for energies up to a few GeV.
- The model should be quantum mechanical.
- The model should be relativistically invariant.
- The model should be as simple as is necessary.
- The model should be able to describe complete sets of observables for a large class of nuclei.

Poincaré invariant quantum mechanics of

few-nucleon or

few-quark systems

Minimal quantum theory

- **Complex vector space**

- **Scalar product**

$$\boxed{(\vec{a})^* \cdot \vec{b} := \langle a|b\rangle}$$

Example: nucleon at rest

- **Basis vectors**

$$|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- **Scalar product**

$$\langle\uparrow|\uparrow\rangle = \langle\downarrow|\downarrow\rangle = 1 \qquad \langle\downarrow|\uparrow\rangle = \langle\uparrow|\downarrow\rangle = 0$$

Measurements

The state of a nucleon is represented by a unit vector

$$|e\rangle = \cos(\theta)\,|\uparrow\rangle + \sin(\theta)\,|\downarrow\rangle$$

$$P_\uparrow := |\langle e|\uparrow\rangle|^2 = |\cos(\theta)|^2 = \cos^2(\theta)$$

$$P_\downarrow := |\langle e|\downarrow\rangle|^2 = |\sin(\theta)|^2 = \sin^2(\theta)$$

$$P_\downarrow + P_\uparrow = \sin^2(\theta) + \cos^2(\theta) = 1$$

Quantum probabilities

and inner products

$$\boxed{P_{a,b} = |\langle a|b\rangle|^2}$$

Key observations

- **The Schrödinger equation plays no role in quantum measurements.**
- **The result of any measurement is a probability.**

Symmetries of quantum theories

A vector correspondence

$$|a\rangle \longrightarrow |a'\rangle$$

is a symmetry of a quantum theory if

$$P_{ab} = |\langle a|b\rangle|^2 = |\langle a'|b'\rangle|^2 = P_{a'b'}$$

If $|a\rangle \to |a'\rangle$ is a symmetry

$$\Downarrow$$

physics in the unprimed world is indistinguishable from physics in the primed world.

Relativity = existence of inertial reference frames

Physics in different inertial reference frames is indistinguishable

$$|a\rangle \to |a'\rangle$$

$$\Downarrow$$

$$P_{ab} = P_{a'b'}$$

Differs from the historical development.

How are inertial reference frames related?

Use classical physics

- By coordinate transforms that preserve the form of Newton's second law ? (Galilean relativity)

- By coordinate transforms that preserve the form of Maxwell's equations of electricity and magnetism ? (special relativity)

Galilean relativity preserves

$$|\vec{x} - \vec{y}| = |\vec{x}' - \vec{y}'| \qquad |t_x - t_y| = |t'_x - t'_y|$$

$$\vec{x}\,' = R\vec{x} + \vec{v}t + \vec{a} \qquad t' = t + c$$

Special relativity preserves

$$|\vec{x} - \vec{y}|^2 - c^2|t_x - t_y|^2 = |\vec{x}\,' - \vec{y}\,'|^2 - c^2|t'_x - t'_y|^2$$

$c =$ **speed of light**

Which is the correct symmetry of nature?

Michelson-Morley experiment

$$\Downarrow$$

Special relativity

Poincaré group

$$|\vec{x}-\vec{y}|^2 - c^2|t_x - t_y|^2 = |\vec{x}\,' - \vec{y}\,'|^2 - c^2|t'_x - t'_y|^2$$

$$x^\mu := (ct, x^1, x^2, x^3)$$

$$\Downarrow$$

$$x'^\mu = \sum_{\nu=0}^{3} \Lambda^\mu{}_\nu x^\nu + a^\mu \qquad \sum_{\mu,\nu} \Lambda_\mu{}^\alpha \eta^{\mu\nu} \Lambda_\nu{}^\beta = \eta^{\alpha\beta}$$

$$\eta^{\mu\nu} := \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Elementary Poincaré transformations

Rotations (3):

$$\vec{x}\,' = O\vec{x} \qquad O^t O = I$$

Translations (4):

$$\vec{x}\,' = \vec{x} + \vec{a}, \qquad ct' = ct + a^0$$

Rotationless Lorentz transformations (3):

$$x' = \gamma(x + vt) \qquad t' = \gamma(t + vx/c^2) \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

- $\Lambda^{\mu}{}_{\nu}$ **and** a^{μ} **are labels for distinct inertial reference frames.**

- **All Poincaré transformations can be generated from 10 elementary transformations.**

- **Compositions of Poincaré transformations are Poincaré transforms.**

- **Poincaré transformations are elements of a group (closed under composition, inverse, identity, associative).**

Special relativity and quantum mechanics

$$\Updownarrow$$

The group of Poincaré transformations is a symmetry of quantum mechanics

$$|v\rangle \to |v_{\Lambda,a}\rangle$$

$$\Downarrow$$

$$P_{uv} = P_{u_{\Lambda,a},v_{\Lambda,a}}$$

Eugene P. Wigner

$$P_{uv} = P_{u_{\Lambda,a} v_{\Lambda,a}}$$

$$\Downarrow$$

$$\langle u_{\Lambda,a} | v_{\Lambda,a} \rangle = \langle u | v \rangle$$

$$\Downarrow$$

$$U(\Lambda_2, a_2) U(\Lambda_1, a_1) = U(\Lambda_2 \Lambda_1, \Lambda_2 a_1 + a_2)$$

Note that $U(\Lambda, a)$ has the multiplication properties of the Poincaré group but it acts on a vector space with different dimension than four.

$U(\Lambda, a)$ is a unitary representation of the Poincaré group.

How do we construct $U(\Lambda, a)$?

Rotating a nucleon at rest

$$U(R)|\uparrow\rangle = |\uparrow\rangle u_{\uparrow\uparrow} + |\downarrow\rangle u_{\downarrow\uparrow}$$

$$U(R)|\downarrow\rangle = |\uparrow\rangle u_{\uparrow\downarrow} + |\downarrow\rangle u_{\downarrow\downarrow}$$

$$\Downarrow$$

$$U(R)|\mu\rangle = \sum_{\nu=\uparrow,\downarrow} |\nu\rangle D^{1/2}_{\nu\mu}(R)$$

$D^{1/2}_{\nu\mu}(R)$ **is a** 2×2 **matrix representation of the rotation group.**

Translating a nucleon at rest

$$U(a^0, \vec{a})|\mu\rangle = |\mu\rangle e^{-imca^0}$$

Nucleon moving with velocity $\vec{v}$, $\quad (\vec{p} = m_n \gamma \vec{v})$

$$|\vec{p}, \mu\rangle := U(\vec{v})|\mu\rangle$$

Every Poincaré transformation can be expressed as

- Rotationless Lorentz transform to rest frame
- Rotation
- Translation of a rest state
- Rotationless Lorentz transform to final frame

Transformation law for a free particle

$$\boxed{U(\Lambda, a)|\vec{p}, \mu\rangle = \sum_{\mu'} |\vec{p}\,', \mu'\rangle D^{1/2}_{\mu'\mu}(R(\Lambda, p))e^{-ip'\cdot a}}$$

This transformation law is a mass m_n spin $1/2$ irreducible representation for the Poincaré group.

This can be done for particles with any mass and spin.

Two free nucleons in zero total momentum frame

$$|\vec{k}, \mu_1; -\vec{k}, \mu_2\rangle := |\vec{0}, \vec{k}, \mu_1, \mu_2\rangle =$$

- **Decompose into linear combinations that have definite angular momenta, J^2, $\vec{J} \cdot \vec{z}$**

- **Resulting states look just like free particle rest eigenstates with mass and spin**

$$M = 2\sqrt{k^2 + m_n^2} \qquad J$$

- **Simultaneous eigenstates of M, $\vec{P}$, J^2 and J_z transform like free particles of mass M and spin J:**

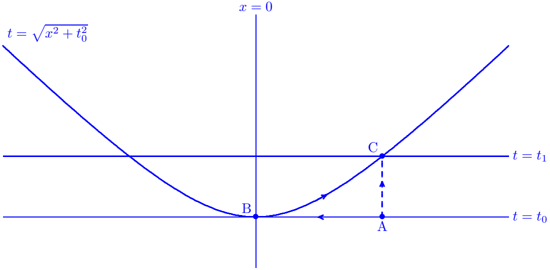
$$|\vec{p}, \mu(M(k), J, d)\rangle$$

$$U(\Lambda, a)|\vec{p}, \mu(M(k), J, d) =$$

$$\sum_{\mu'} |\vec{p}\,', \mu'(M(k), J, d)\rangle D^J_{\mu'\mu}[R(\Lambda, p)] e^{-ip'\cdot a}$$

Two interacting nucleons

Consistent initial value problem?



Solving the non-linear problem?

- **Add interactions to mass**

$$M = 2\sqrt{k^2 + m_1^2} + V$$

- **Require that V commutes with J^2**
- **R that V commutes with and is independent of $\vec{P}$ and J_z.**
- **Diagonalize the matrix M in the basis $|\vec{p}, \mu(M(k), J, d)\rangle$.**

$$(2\sqrt{k^2+m_n^2}+V)|\vec{p},\mu,(M',J)\rangle = M'|\vec{p},\mu,(M',J)\rangle$$

$$U_I(\Lambda,a)|\vec{p},\mu(M',J)\rangle =$$

$$\sum_{\mu'} |\vec{p}\,',\mu'(M',J)\rangle D^J_{\mu'\mu}(R(\Lambda,p))e^{-ip'\cdot a}$$

Simultaneous eigenstates of M, $\vec{P}$ J^2 and J_z complete.

This defines a relativistic interacting two-nucleon quantum theory

Relation to non-relativistic problem ?

$$M^2 = 4(m_n^2 + \vec{k}^2) + 4m_n v_{nn} = 4m_n(\frac{\vec{k}^2}{m} + v_{nn}) + 4m_n^2 =$$

$$4m_n h_{nr} + 4m_n^2$$

- **Has same eigenvectors as non-relativistic problem**
- **Has same scattering probabilities as non-relativistic problem.**

Nuclear potentials v_{nn}

$$v_{nn} = \sum_\alpha V_\alpha(r) O_\alpha$$

$$O_\alpha \in \{I, \mathbf{s}_1 \cdot \mathbf{s}_2, \mathbf{L} \cdot \mathbf{S}, (\hat{\mathbf{r}} \cdot \mathbf{s}_1)(\hat{\mathbf{r}} \cdot \mathbf{s}_2), (\mathbf{s}_1 \cdot \mathbf{L})(\mathbf{s}_2 \cdot \mathbf{L}), L^2, (\mathbf{s}_1 \cdot \mathbf{s}_2) L^2,$$

$$(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2), (\mathbf{s}_1 \cdot \mathbf{s}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2), (\mathbf{L} \cdot \mathbf{S})(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2), (\hat{\mathbf{r}} \cdot \mathbf{s}_1)(\hat{\mathbf{r}} \cdot \mathbf{s}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2), \cdots$$

- $\boldsymbol{\tau}_i$ = isospin of particle i
- $\mathbf{L}$ = orbital angular momentum of system
- $\mathbf{s}_i$ spin of particle i
- $\mathbf{S}$ = sum of nucleon spins
- $V_\alpha(r)$ = Yukawa-like interactions $\left(\frac{e^{-m_\pi c r/\hbar}}{r}\right)$

$$N > 2$$

Cluster properties - separate region A from region B

$$\Downarrow$$

$$U(\Lambda, a) - U(\Lambda, a)_B \otimes U(\Lambda, a)_B \to 0$$

$$\Downarrow$$

$$H = \sum_i H_i + \sum_{ij} V_{ij} + \sum_{ijk} V_{ijk} + \cdots$$

Specific case: $N = 3$

Structure of M dictated by two-body M and cluster properties

$$M = W(V)\bar{M}(V)W^\dagger(V) \qquad \bar{M}(V) = M_0 + V_{12} + V_{23} + V_{31}$$

$$V_{12} = \sqrt{M_{12}^2 + q_3^2} - \sqrt{M_{012}^2 + q_3^2} \quad \cdots$$

$$M_{12}^2 = 4(\mathbf{k}^2 + m_n^2) + 4m_n v_{nn} \qquad M_{012}^2 = 4(\mathbf{k}^2 + m_n^2)$$

$W(V)$ **not required to calculate** S**!**

(Coester 1965, Sokolov 1977, F.C & W.P. 1982)

Predictions

Binding energies

$$M(V)|\Psi\rangle = \lambda|\Psi\rangle$$

$$|\Psi\rangle = W(V)|\bar{\Psi}\rangle \qquad \bar{M}(V)|\bar{\Psi}\rangle = \lambda|\bar{\Psi}\rangle$$

Scattering probabilities

$$|S_{fi}|^2 = |\langle\Psi_f^+|\Psi_i^i\rangle|^2 = |\langle\bar{\Psi}_f^+|\bar{\Psi}_i^i\rangle|^2$$

Electromagnetic and weak current matrix elements

$$\langle\Psi_f|I^\nu(0)|\Psi_i\rangle = \langle\bar{\Psi}_f|W^\dagger(V)I^\nu(0)W(V)|\bar{\Psi}_i\rangle$$

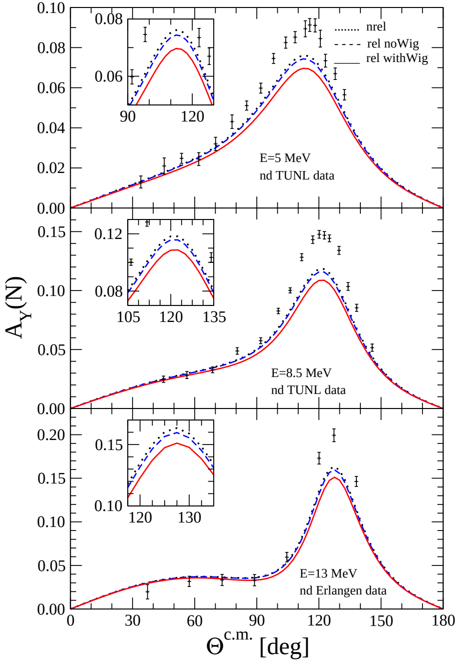
Three-body scattering

$$S_{ab} = \delta_{ab} - 2\pi i \delta(M_a - M_b) T^{ab}$$

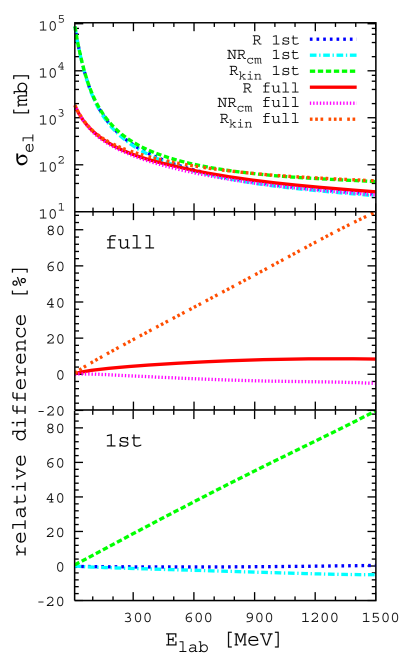
Faddeev equation

$$T^{ab}(z) = \bar{\delta}^{ab}(z - M_0) + \sum_{c \neq a} V_c (z - M_0 - V_c)^{-1} T^{cb}(z)$$

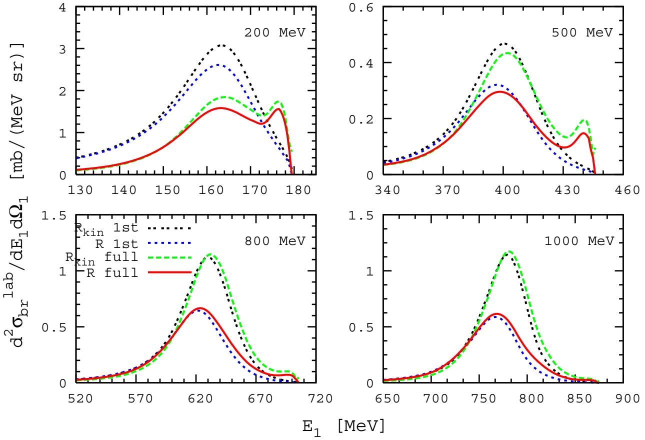
$$a, b, c \in \{(12)(3), (23)(1), (31)(2)\}$$



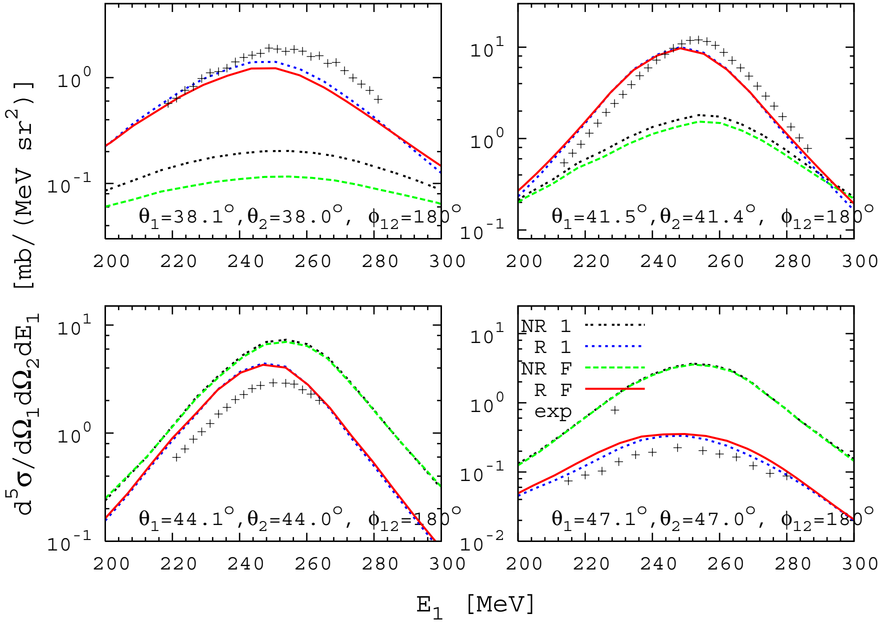
nrel
rel noWig
rel withWig
E=5 MeV
nd TUNL data
E=8.5 MeV
nd TUNL data
E=13 MeV
nd Erlangen data
$A_Y(N)$
$\Theta^{c.m.}$ [deg]



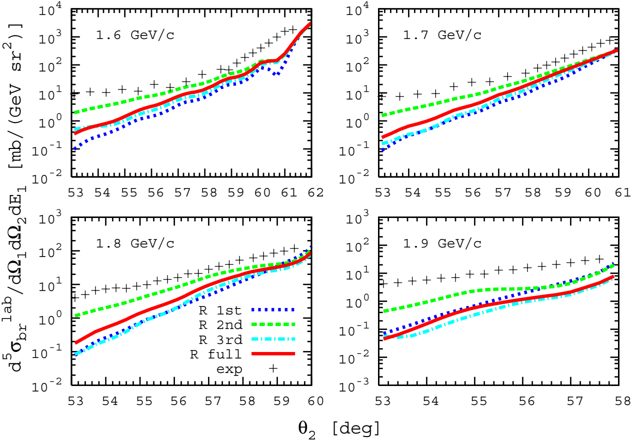
R 1st
NR_cm 1st
R_kin 1st
R full
NR_cm full
R_kin full
σ_el [mb]
full
1st
relative difference [%]
E_lab [MeV]



200 MeV
500 MeV
800 MeV
1000 MeV
R_{kin} 1st
R 1st
R_{kin} full
R full
$d^2\sigma_{br}^{lab}/dE_1 d\Omega_1$ [mb/(MeV sr)]
E_1 [MeV]



$d^5\sigma/d\Omega_1 d\Omega_2 dE_1$ [mb/(MeV sr^2)]
E_1 [MeV]
θ_1=38.1°,θ_2=38.0°, ϕ_{12}=180°
θ_1=41.5°,θ_2=41.4°, ϕ_{12}=180°
θ_1=44.1°,θ_2=44.0°, ϕ_{12}=180°
θ_1=47.1°,θ_2=47.0°, ϕ_{12}=180°
NR 1
R 1
NR F
R F
exp



1.6 GeV/c
1.7 GeV/c
1.8 GeV/c
1.9 GeV/c
R 1st
R 2nd
R 3rd
R full
exp
$d^5\sigma_{br}^{lab}/d\Omega_1 d\Omega_2 dE_1$ [mb/(GeV sr^2)]
θ_2 [deg]

Summary

- Poincaré invariant quantum mechanics provides a mathematical framework for modeling nuclei that can be used effectively for energies up to a few GeV.

- Calculations have been performed for three-nucleon bound-state and scattering observables.

- Spin has been treated up to 250 MeV and spin-independent observable have been computed up to 2 GeV.

- Two-body electromagnetic observables have been computed for bound systems of quarks and nucleons.

Conclusions

- A Poincaré invariant model is needed to compute scattering observables above 250 MeV.
- Kinematic corrections alone lead to misleading results.
- Many standard approximations are not accurate in all kinematic regions.
- There are some surprising low-energy effects in spin observables.

Future

- Include explicit particle production.
- Include baryon resonances.
- More theory - cluster properties + particle production.
- Effects of cluster properties on electroweak current.
- Models with confined quark degrees of freedom.