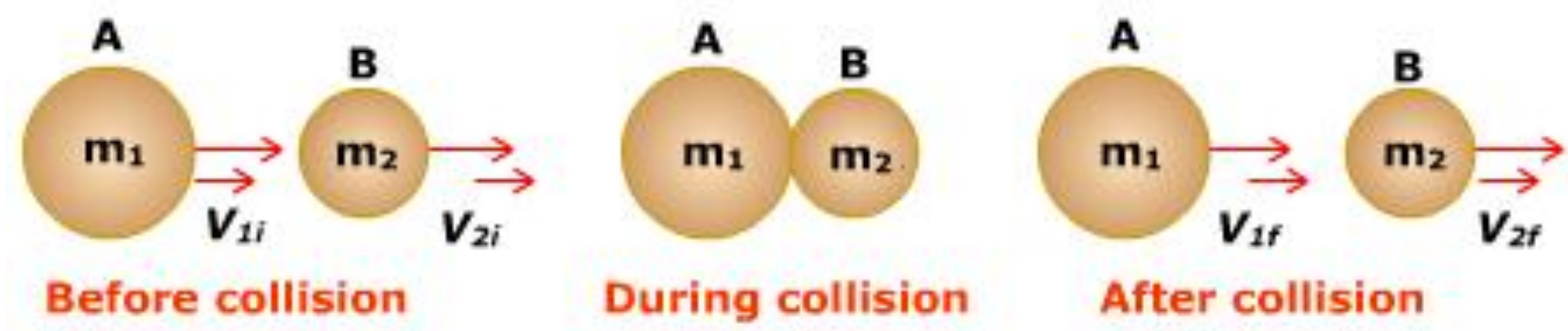
College Physics I: 1511 Mechanics & Thermodynamics

Professor Jasper Halekas
Van Allen Lecture Room 1
MWF 8:30-9:20 Lecture

1-d Elastic Collisions



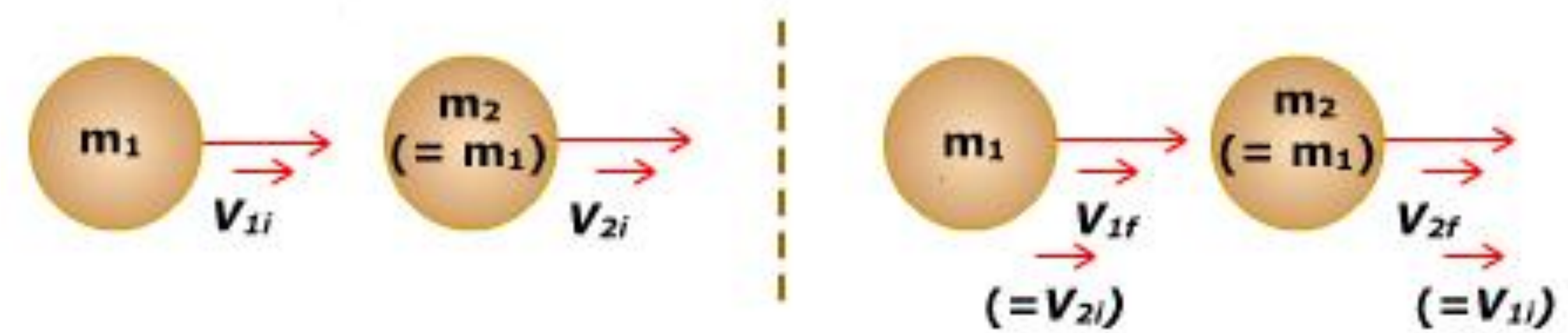
Final Velocities in 1-d Elastic Collisions

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i}$$

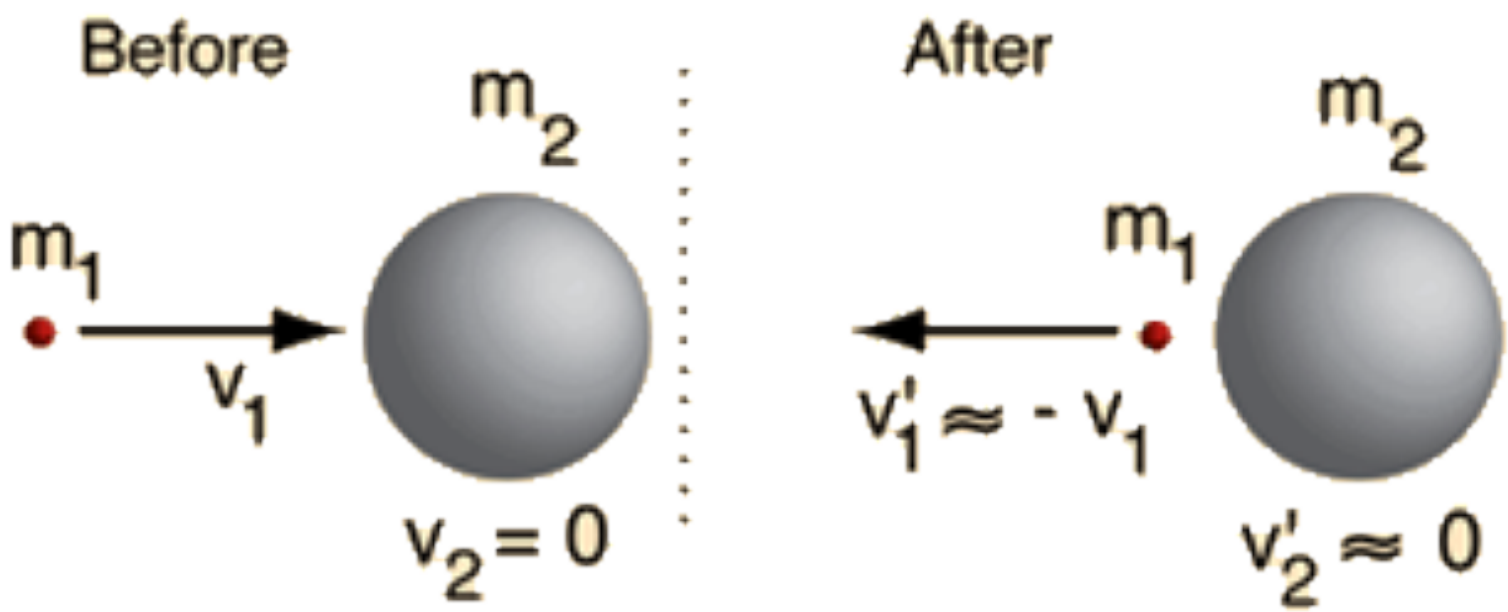
$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2i}$$

Final Velocities For Equal Mass

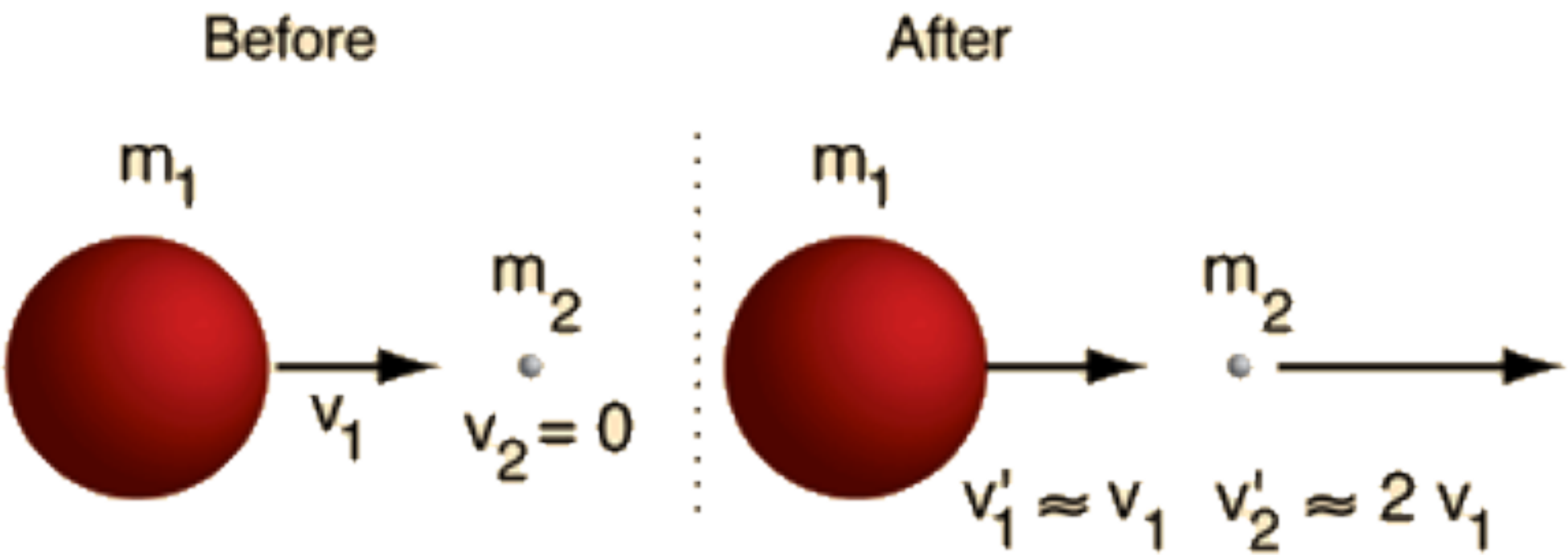
- For a 1-d elastic collision between two objects with equal mass, the two trade velocities



Final Velocities For Highly Unequal Mass: Small Mass Moving

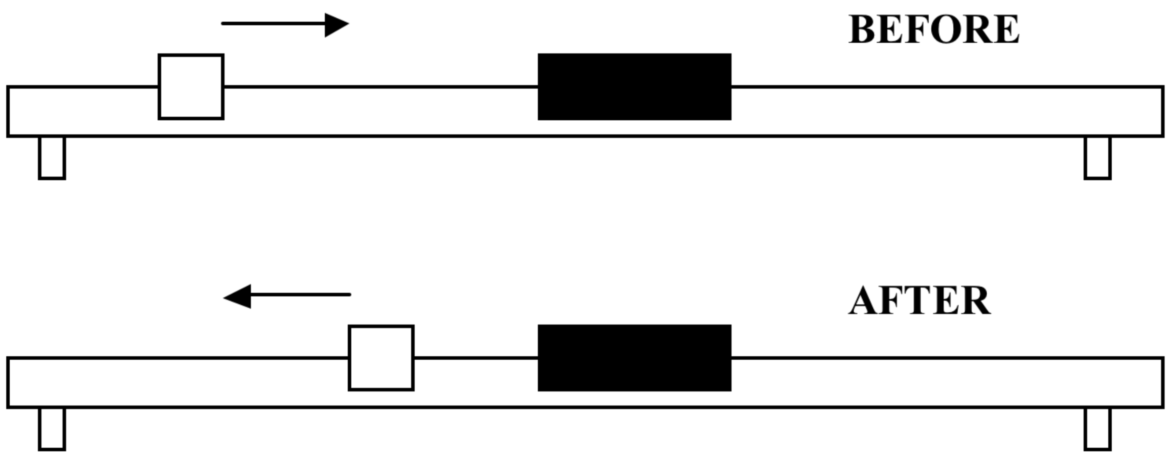


Final Velocities For Highly Unequal Mass: Large Mass Moving



Concept Check

A small glider is gliding along an air track at some initial speed, hits a much larger glider, and is observed to bounce back with a final speed nearly equal to its initial speed.



The speed of the large glider after the collision is…

A) less than the initial speed of the small glider.
B) larger than the initial speed of the small glider.
C) necessarily zero.

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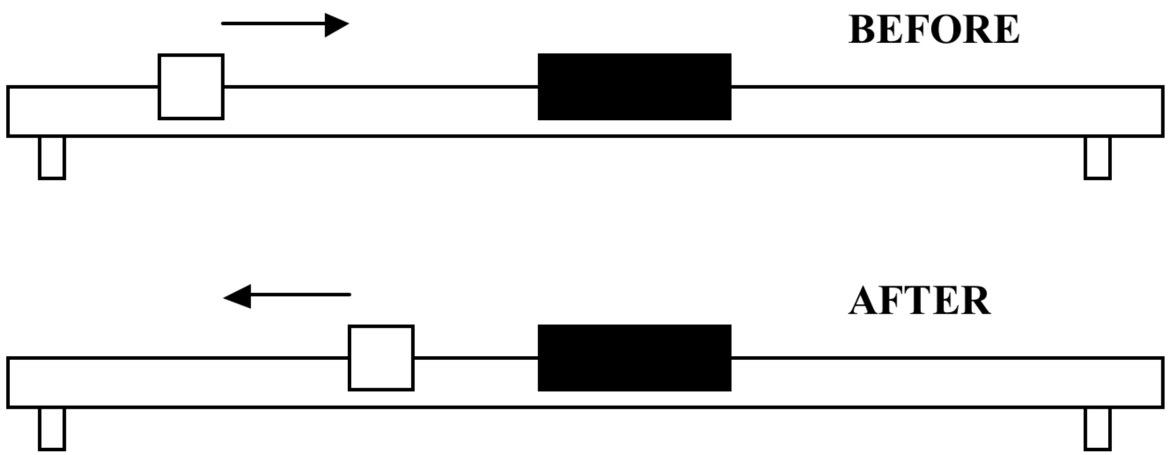

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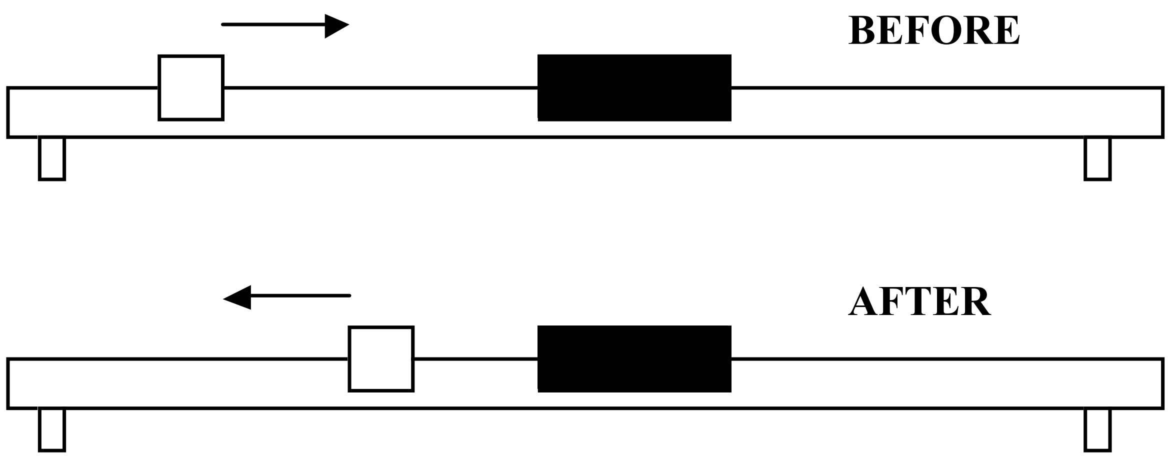

The magnitude of the momentum of the large glider after the collision is…

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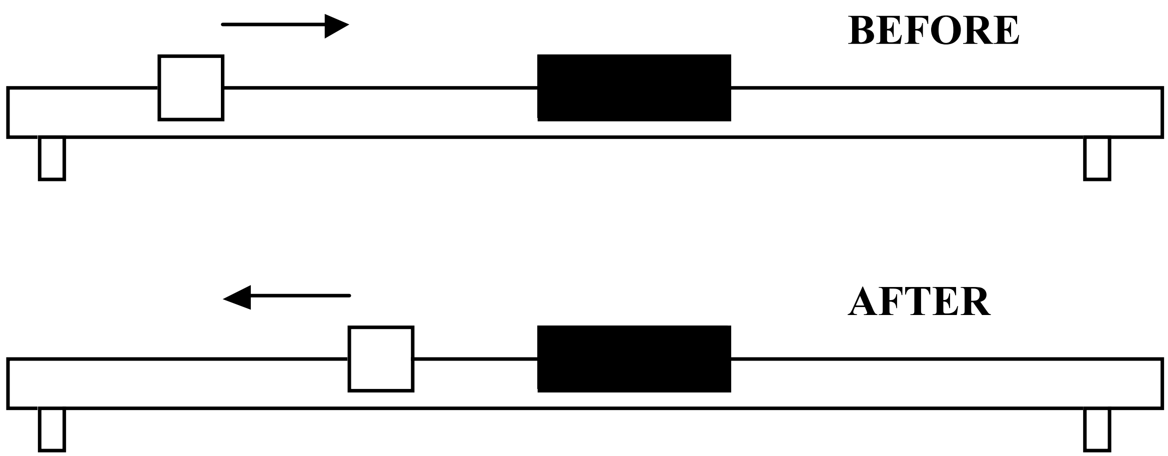

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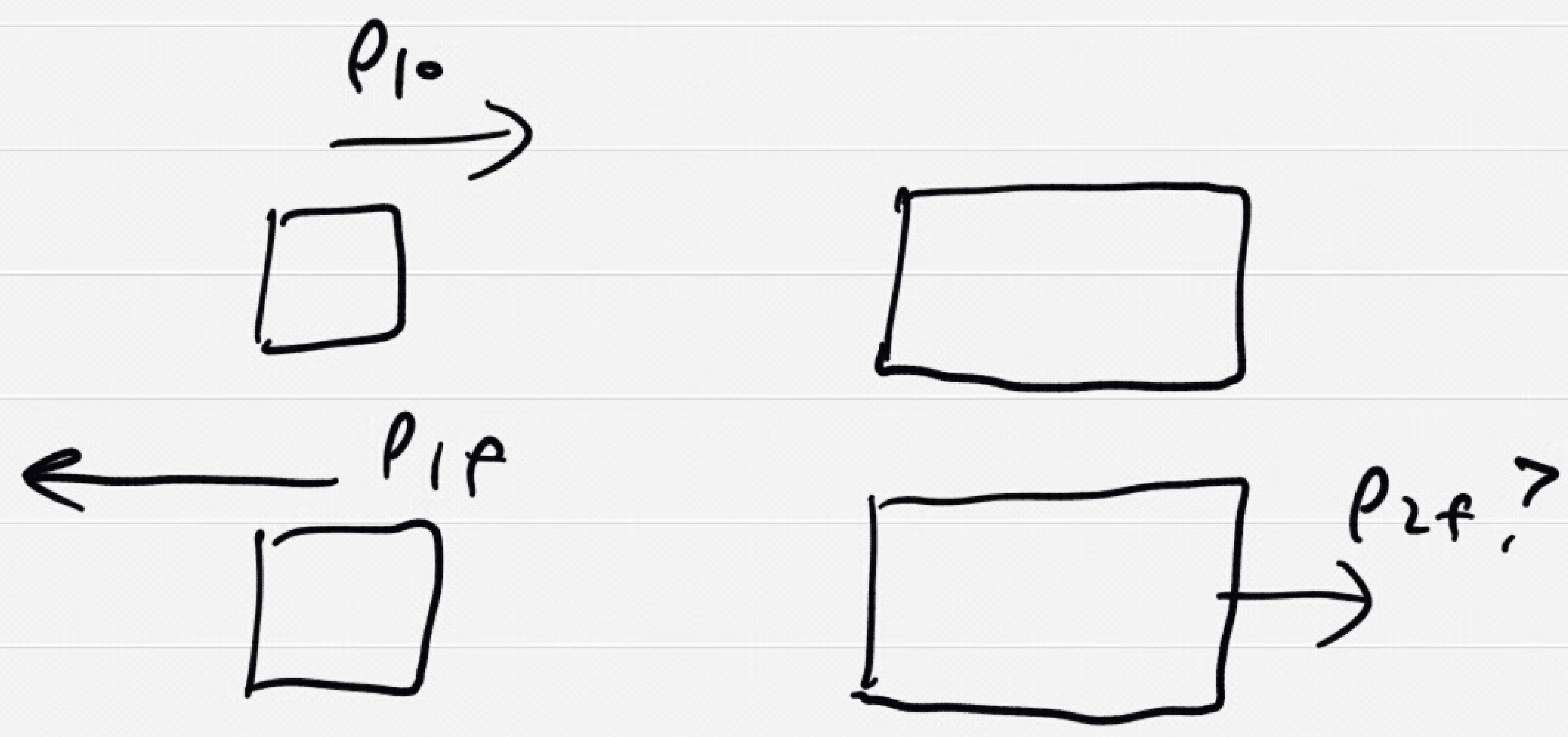

$p_{tot} = p_{1o}$

$= p_{1f} + p_{2f}$

$p_{1f} \sim -p_{1o}$

$\Rightarrow p_{1o} \sim -p_{1o} + p_{2f}$

$\Rightarrow p_{2f} \sim 2p_{1o}$

small velocity, but big mass so still a lot of momentum

Concept Check

- A person attempts to knock down a large wooden bowling pin by throwing a ball at it. The person has two balls of equal size and mass, one made of rubber and the other of putty. The rubber ball bounces back, while the ball of putty sticks to the pin.

- Which ball is most likely to topple the bowling pin?

- A: the rubber ball
- B: the ball of putty
- C: makes no difference
- D: need more information

Concept Check

- A person attempts to knock down a large wooden bowling pin by throwing a ball at it. The person has two balls of equal size and mass, one made of rubber and the other of putty. The rubber ball bounces back, while the ball of putty sticks to the pin.

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Putty Ball:

$$p_0 = mv_0$$
$$p_f \sim 0$$
$$\Delta p = -mv_0 = F\Delta t$$

Rubber Ball:

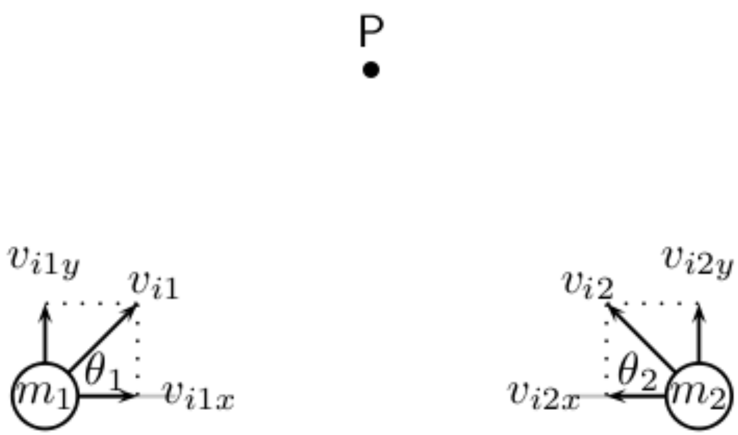
$$p_0 = mv_0$$
$$p_f = -mv_0$$
$$\Delta p = -mv_0 - mv_0$$
$$= -2mv_0 = F\Delta t$$

- Bigger impulse to ball in elastic collision
- By Newton's 3rd this means biggest impulse to bowling pin as well

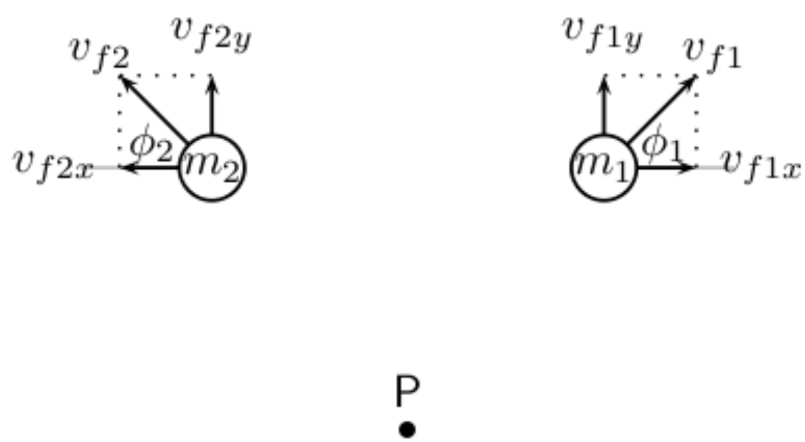
Impulse in Collisions

- Impulse = force*time = change in momentum
  - By Newton's 3rd $\mathbf{F}_{12} = -\mathbf{F}_{21}$
    - Since time interval the same, impulse to body 1 is equal and opposite to impulse to body 2
- Biggest impulse exerted on objects in totally elastic collisions
- Least impulse exerted on objects in totally inelastic collision
  - This is because mechanical energy is lost to heat and/ or deformation of the objects

3-d Conservation of Momentum



(a) Before the collision



(b) After the collision

$$m_1 v_{x1i} + m_2 v_{x2i} = m_1 v_{x1f} + m_2 v_{x2f}$$

$$m_1 v_{y1i} + m_2 v_{y2i} = m_1 v_{y1f} + m_2 v_{y2f}$$

$$m_1 v_{z1i} + m_2 v_{z2i} = m_1 v_{z1f} + m_2 v_{z2f}$$

Two-D Collisions

Two-Dimensional Collisions

- Conservation of momentum holds in multiple dimensions
- In “Head-On” collisions, if you correctly orient coordinates, you can solve as a 1-d problem
- In “Glancing” collisions, you need to know something about one of the final velocities

Concept Check

- In a glancing collision between two equal mass objects, both are observed to have a final velocity magnitude of 4 m/s. One object moves away at an angle θ = 30° above the horizontal. What angle ϕ (below the horizontal) does the other object's trajectory follow?

A. 0°
B. 30°
C. 45°
D. 60°
E. 90°

(a) Before the collision

(b) After the collision

Concept Check

- In a glancing collision between two equal mass objects, both are observed to have a final velocity magnitude of 4 m/s. One object moves away at an angle θ = 30° above the horizontal. What angle ϕ (below the horizontal) does the other object's trajectory follow?

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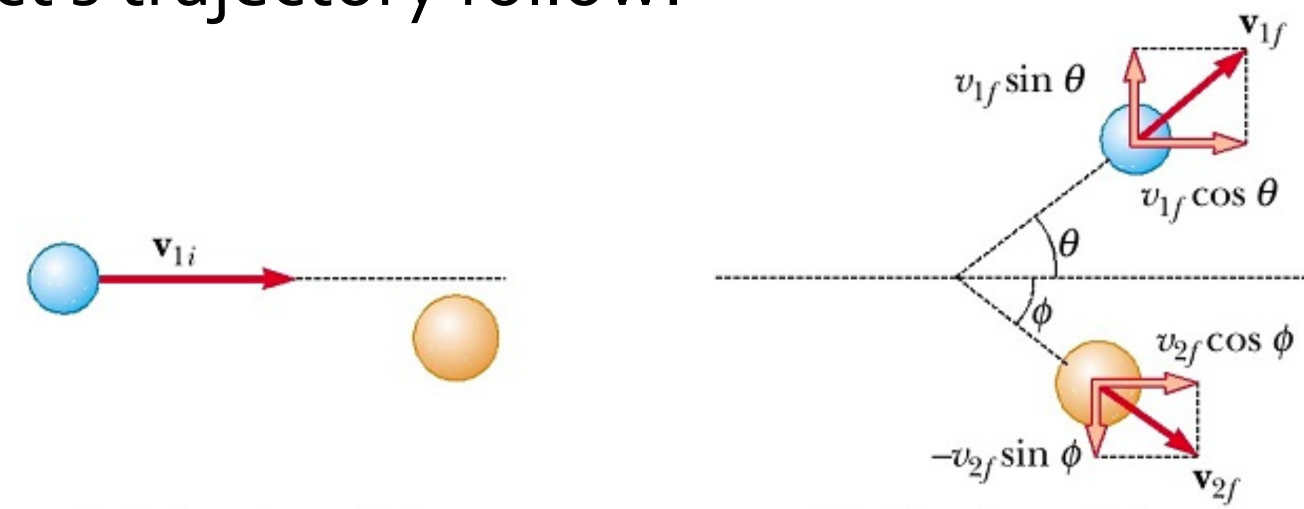


(a) Before the collision (b) After the collision

Conservation of momentum

$$P_{y\text{-}tot} = P_{y1} + P_{y2}$$

$$= m_1 v_{y1i} + m_2 v_{y2i}$$

$$= 0$$

$$= m_1 v_{y1f} + m_2 v_{y2f}$$

$$\Rightarrow \quad m_1 v_{y1f} = -m_2 v_{y2f}$$

$$mv \sin\theta = mv \sin\varphi$$

$$\sin\theta = \sin\varphi$$

$$\theta = \varphi$$

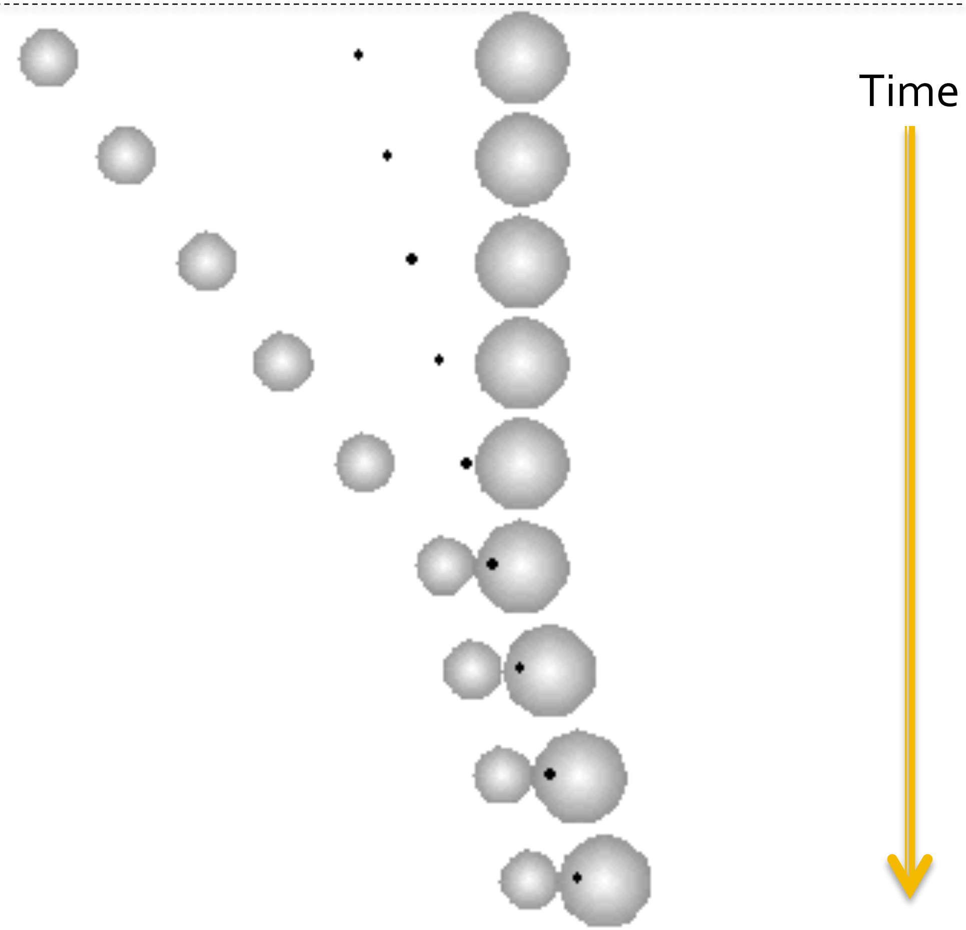
Center of Mass

- A useful concept in analyzing collisions is the "center of mass" velocity
  - This is the weighted average of the velocities

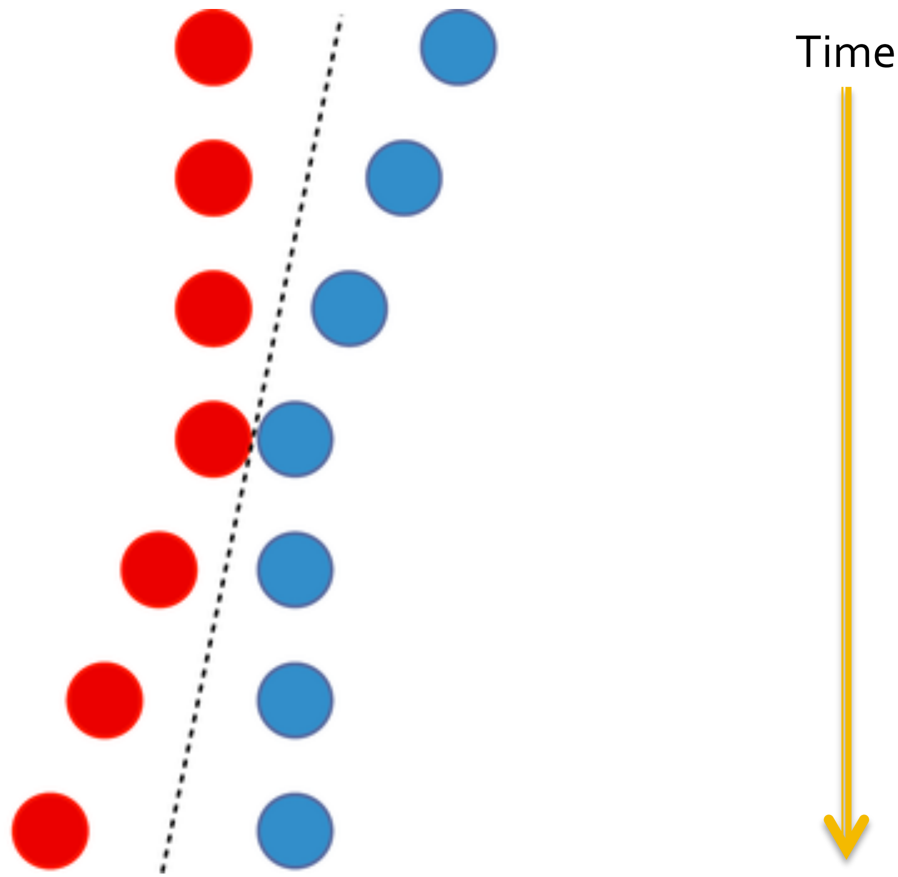
- $\mathbf{v}_{cm} = (m_1\mathbf{v}_1 + m_2\mathbf{v}_2)/(m_1 + m_2)$

- If the total momentum $m_1\mathbf{v}_1 + m_2\mathbf{v}_2$ is constant, then so is $\mathbf{v}_{cm}$
  - True in both elastic and inelastic collisions

Totally Inelastic Collision: CM Frame



Elastic Collision: CM Frame



$V_{2f} \sim V$

$V_{1f} \sim 2V$

$V_{2f} \sim 0$

$V_{1f} \sim -V$

$V_{cm} = V$

$V_{cm} = 0$

- Transform to CM frame
- Subtract V_{cm} from both

$V_{2i_{cm}} = V - V = 0$

$V_{1i_{cm}} = 0 - V = -V$

$V_{2f_{cm}} = V - V = 0$

$V_{1f_{cm}} = 2V - V = V$

In CM frame, small ball just bounces