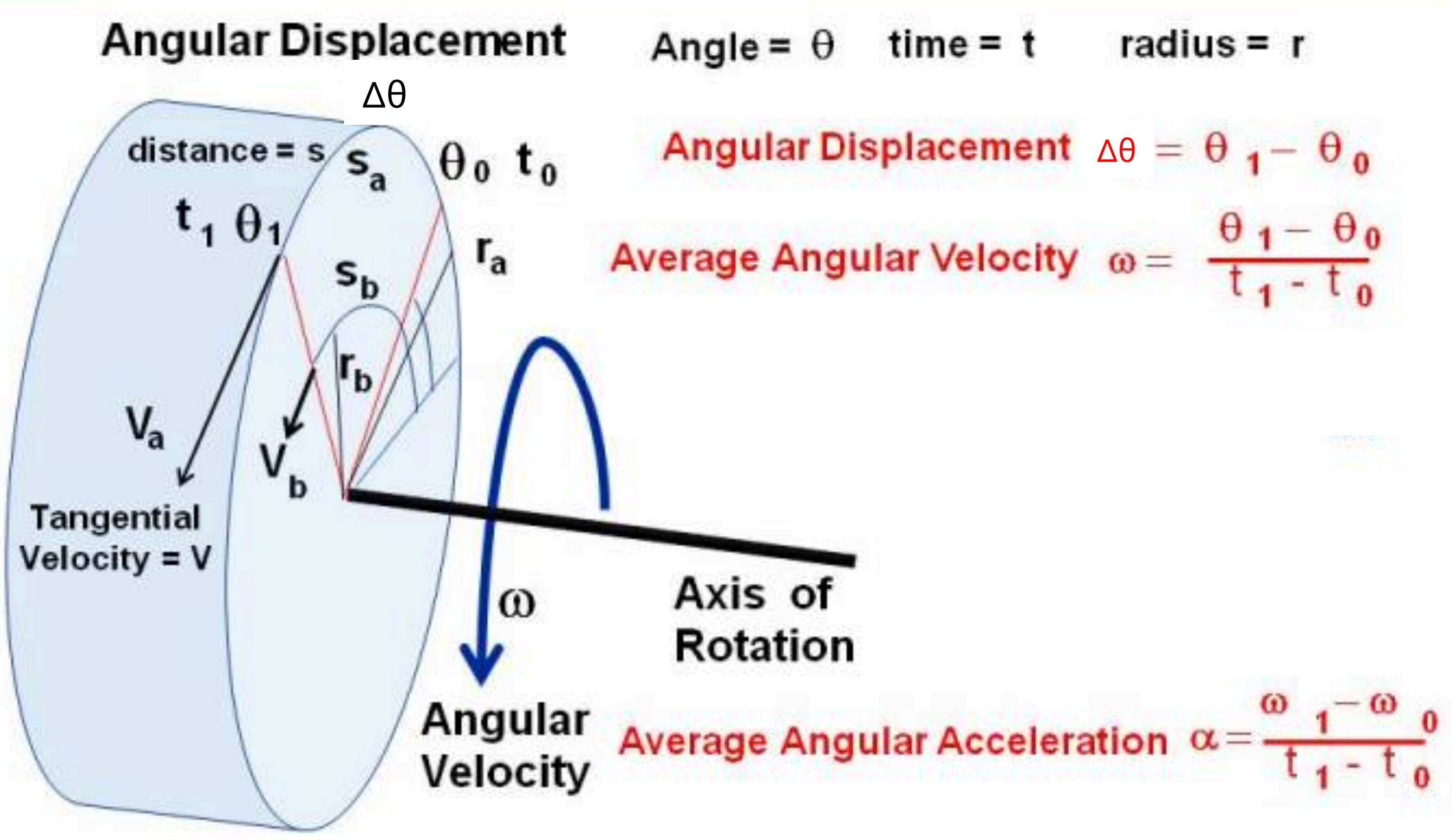
College Physics I: 1511 Mechanics & Thermodynamics

Professor Jasper Halekas
Van Allen Lecture Room 1
MWF 8:30-9:20 Lecture

Angular Kinematic Variables



Angular Vs. Tangential Variables

Linear and Rotational Quantities

Linear	Type	Rotational	Relation (θ in radians)
S = x	displacement	θ	S = $x = R\theta$
v	velocity	ω	$v = R\omega$
$a_{\tan}$	acceleration	α	$a_{\tan} = R\alpha$

$$\Delta x = s = v_0 t + \tfrac{1}{2} a t^2$$

$$s = r\Delta\theta$$
$$v = r\omega$$
$$a = r\alpha$$

$$r\Delta\theta = r\omega_0 t + \tfrac{1}{2} r\alpha t^2$$

cancel r's

$$\Delta\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2$$

- same equation in angular variables

- similarly

$$v^2 = v_0^2 + 2a\Delta x$$
$$(\omega r)^2 = (\omega_0 r)^2 + 2\alpha r \cdot r\Delta\theta$$
$$\omega^2 r^2 = \omega_0^2 r^2 + 2\alpha r^2 \Delta\theta$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

Angular Kinematic Equations

$$v = v_o + at$$

$$\omega = \omega_o + \alpha t$$

$$\Delta x = \tfrac{1}{2}\left(v_o + v\right) t$$

$$\Delta\theta = \tfrac{1}{2}\left(\omega_o + \omega\right) t$$

$$v^2 = v_o^2 + 2a\left(\Delta x\right)$$

$$\omega^2 = \omega_o^2 + 2\alpha\left(\Delta\theta\right)$$

$$\Delta x = v_o t + \tfrac{1}{2} a t^2$$

$$\Delta\theta = \omega_o t + \tfrac{1}{2}\alpha t^2$$

Concept Check

- A car slams on the brakes to avoid an accident, slowing its wheels from an initial angular velocity of 10 rad/s to rest. During this time, the wheels rotate 100 radians. What angular acceleration did the car wheels undergo?

A. $-1\ rad/s^2$
B. $-2\ rad/s^2$
C. $1\ rad/s^2$
D. $-0.5\ rad/s^2$
E. $0.5\ rad/s^2$

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Braking Car

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

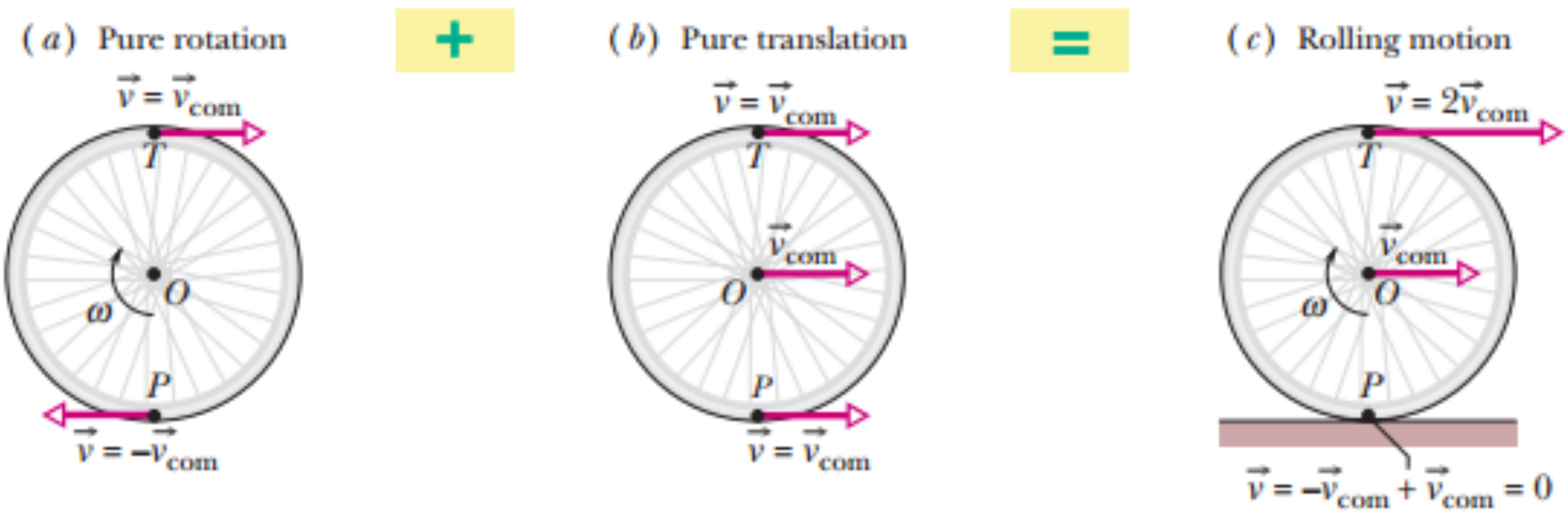
$$0 = 10^2 + 2\alpha \cdot 100$$

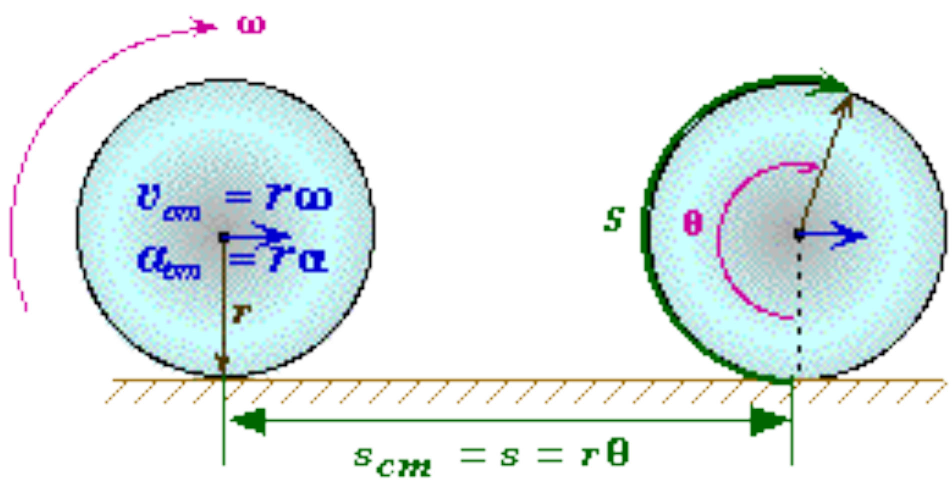
$$= 100 + 200\alpha$$

or $-100 = 200\alpha$

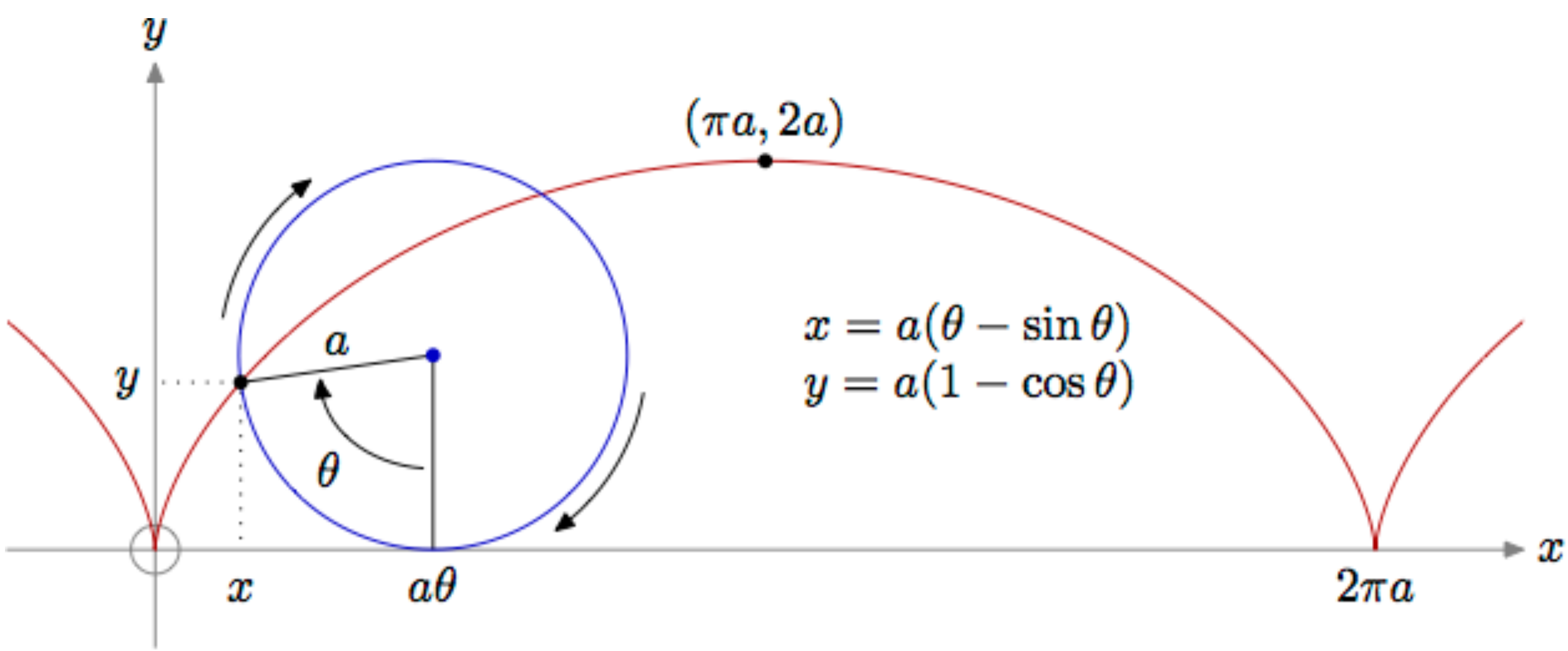
$$\Rightarrow \boxed{\alpha = -1/2 \ rad/s^2}$$

Rolling Motion & Center of Mass





Cycloid

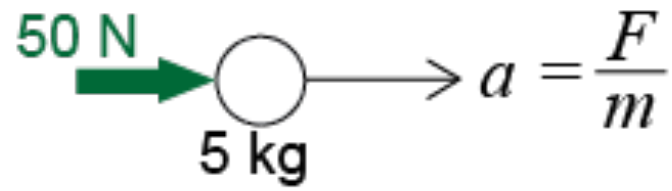


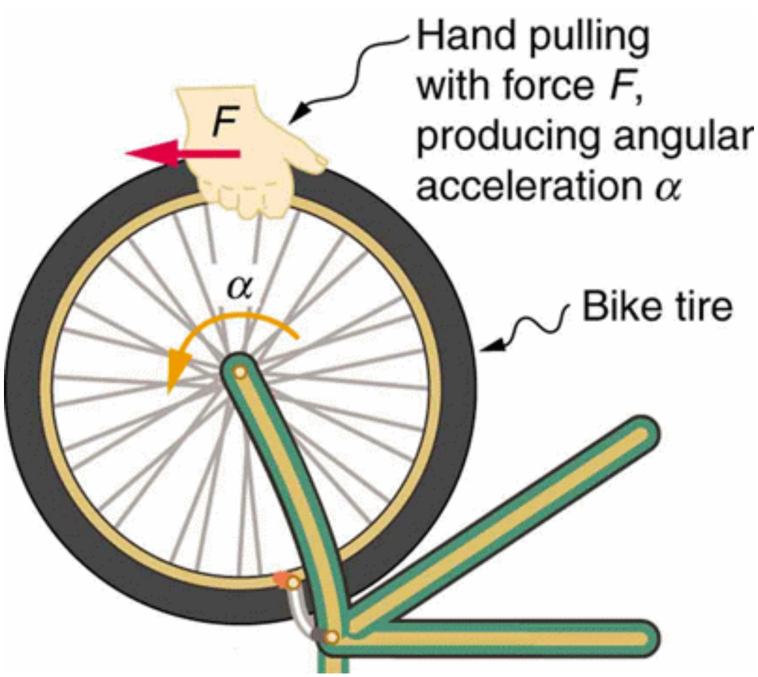
y
$(\pi a, 2a)$
a
y
θ
$x = a(\theta - \sin\theta)$
$y = a(1 - \cos\theta)$
x
$a\theta$
$2\pi a$
x

Newton's Second Law: Rotating Bodies

$$F_{\text{net external}} = ma$$

Net force on object = mass of object x acceleration

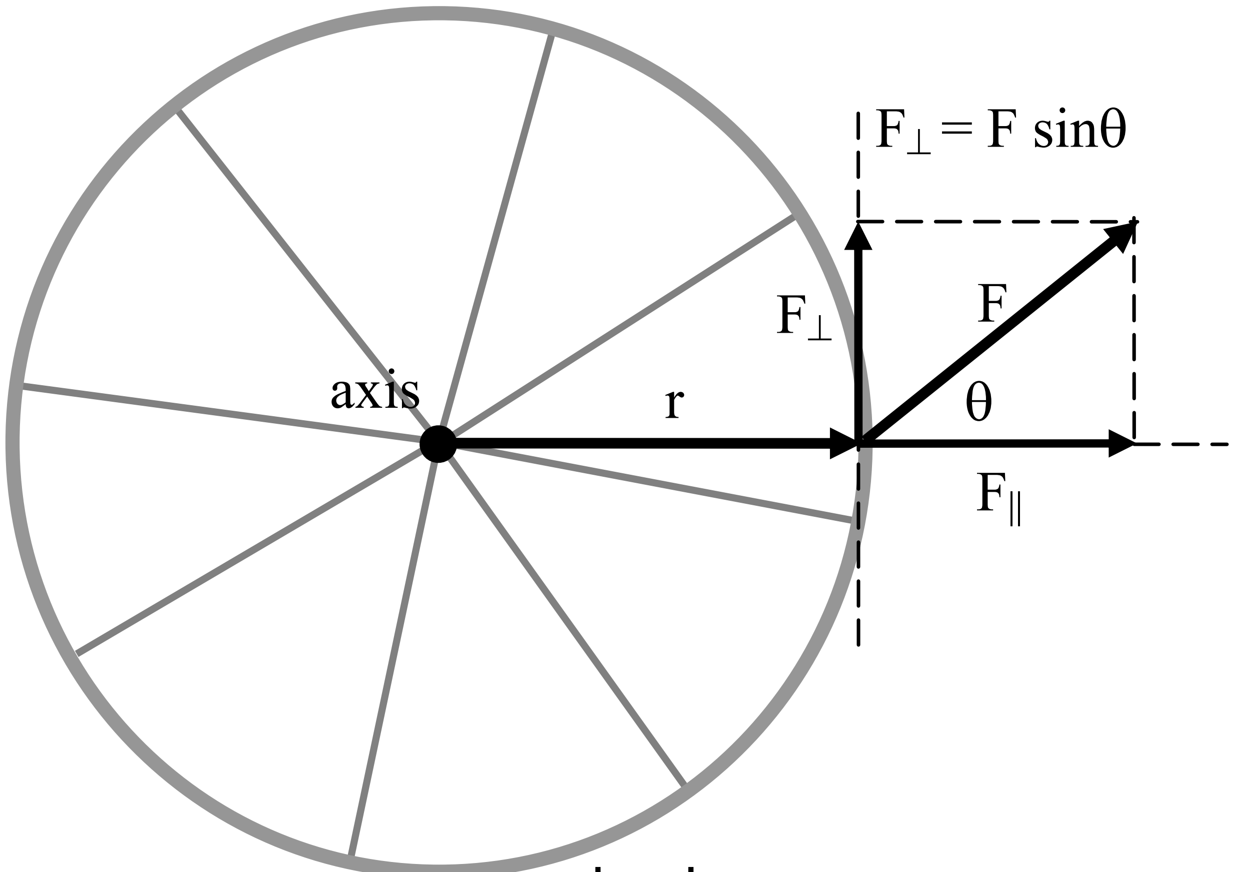




$$\alpha = \frac{\tau_{net}}{I}$$

Angular acceleration = Torque / Moment of Inertia

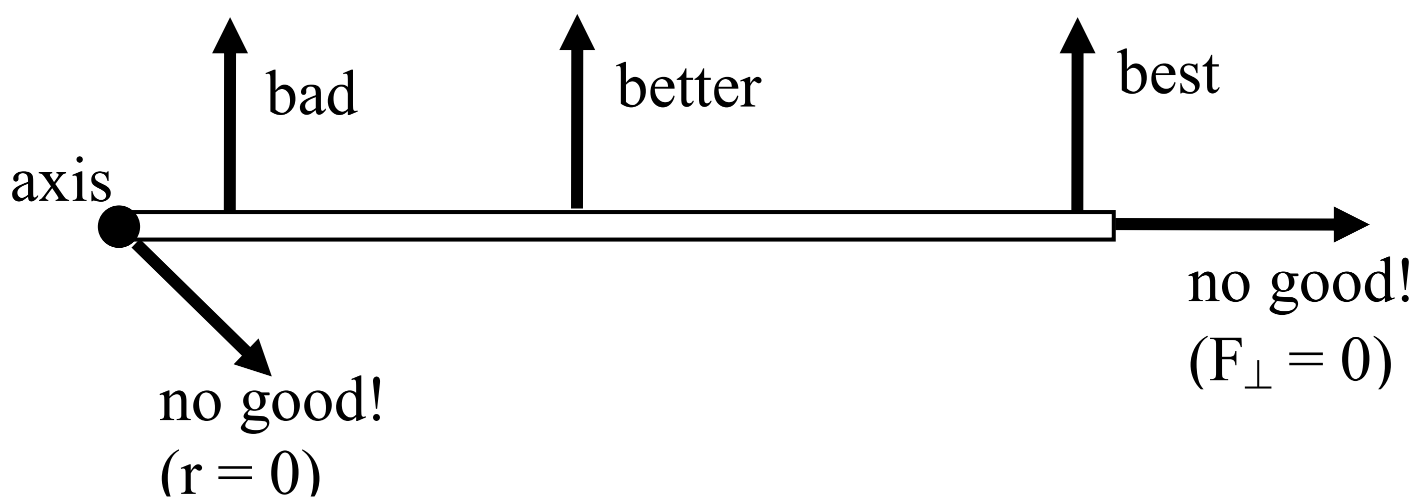
Rotational Quantities IV: Torque



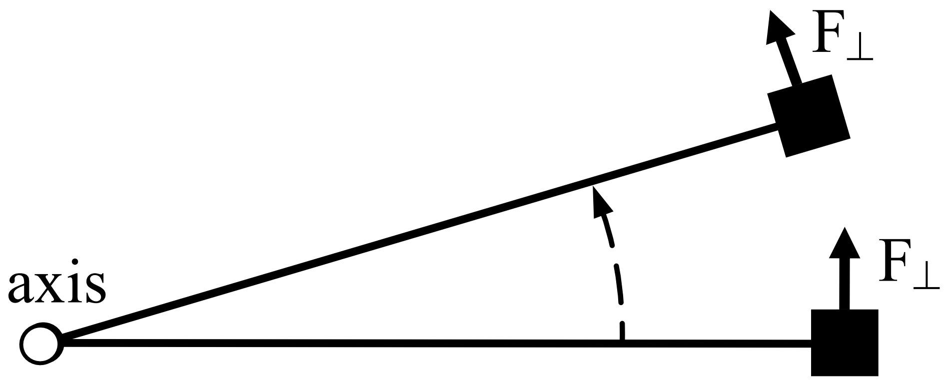
$$|\tau| \equiv r \cdot F_\perp = r F \sin\theta$$

Units of Torque

- SI Units [N][m]
- Why an extra factor of r compared to force?
- Because you can more easily rotate a wheel if you push farther from the center…



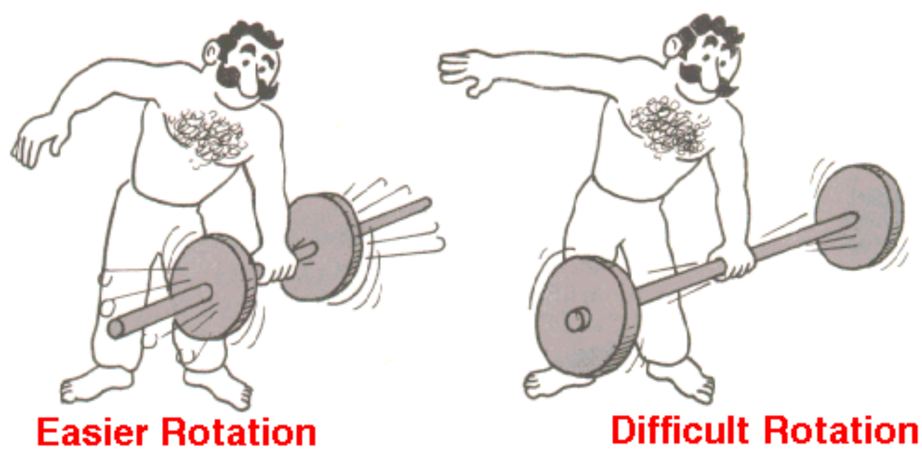
Rotational Quantities V: Moment of Inertia



$I = mr^2$ = "Moment of Inertia"

Units of Moment of Inertia

- SI Units [kg][m^2]
- Why an extra factor of r^2 compared to mass?
- Because it is much harder to move mass that is farther from the center…



$$F = ma$$

$$\tau = rF \Rightarrow F = \tau / r$$

$$I = mr^2 \Rightarrow m = I / r^2$$

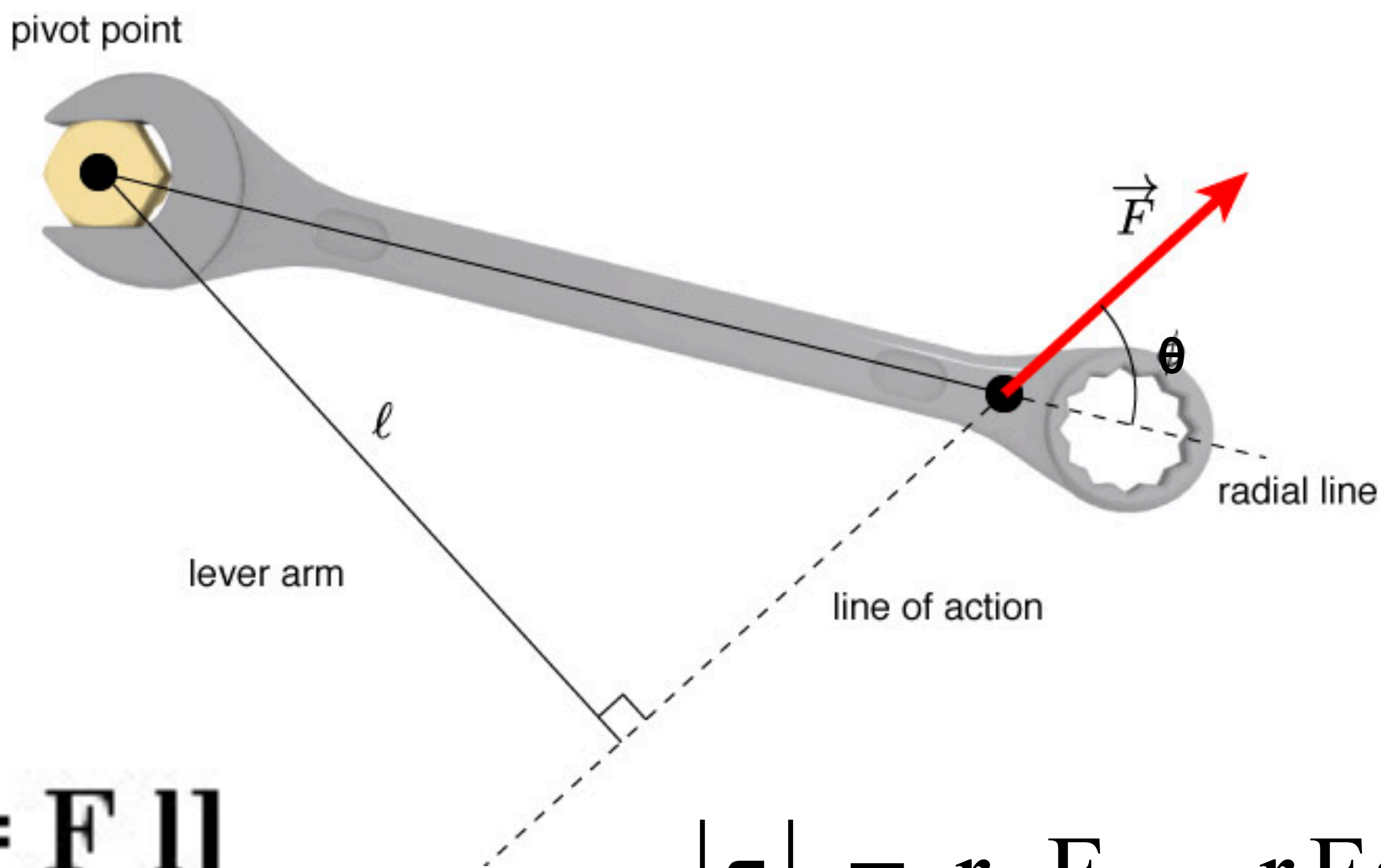
so

$$\tau / r = I / r^2 \, a$$
$$= I / r^2 \cdot r\alpha$$

$$\Rightarrow \tau = r \cdot I / r^2 \cdot r\alpha$$

$$\Rightarrow \boxed{\tau = I\alpha}$$

Line of Action & Lever Arm

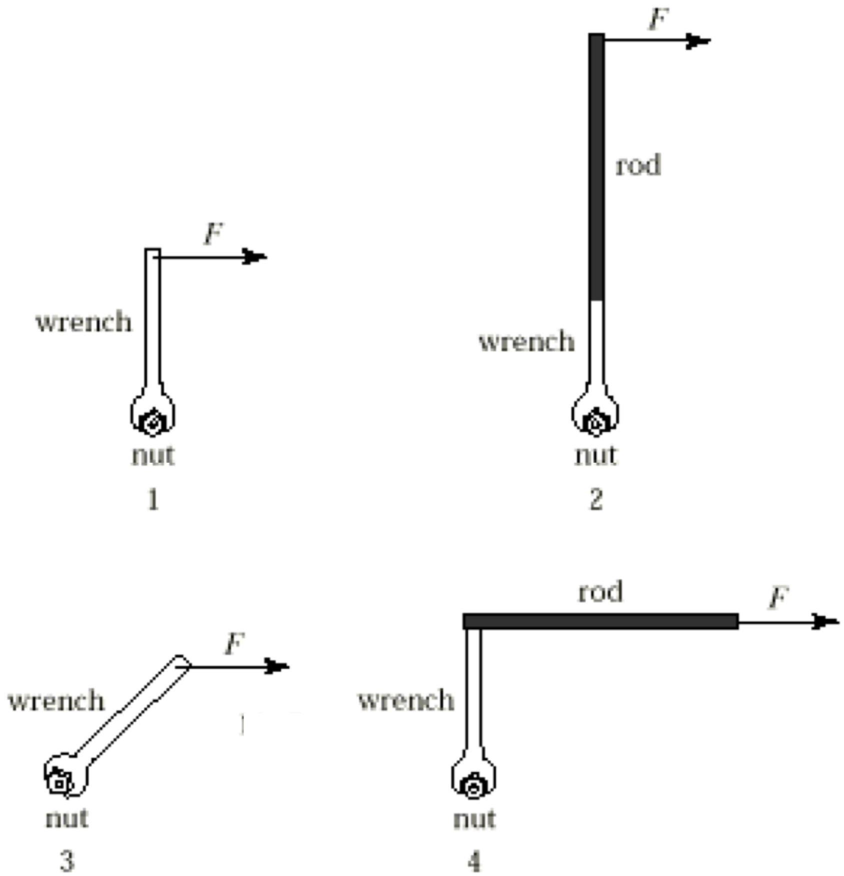


$$[\tau = F\, l]$$

$$|\tau| \equiv r \cdot F_{\perp} = r F \sin\theta$$

Concept Check

You are using a wrench and trying to loosen a rusty nut. Which of the arrangements shown is most effective in loosening the nut? (A=1, B=2, C=3, D=4)



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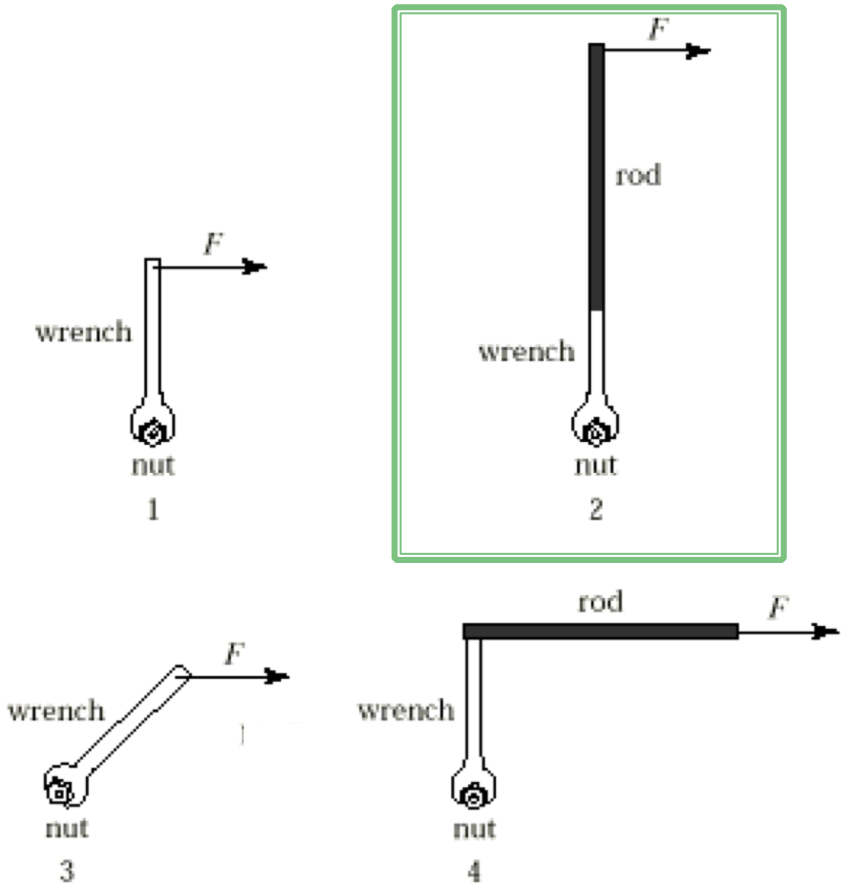

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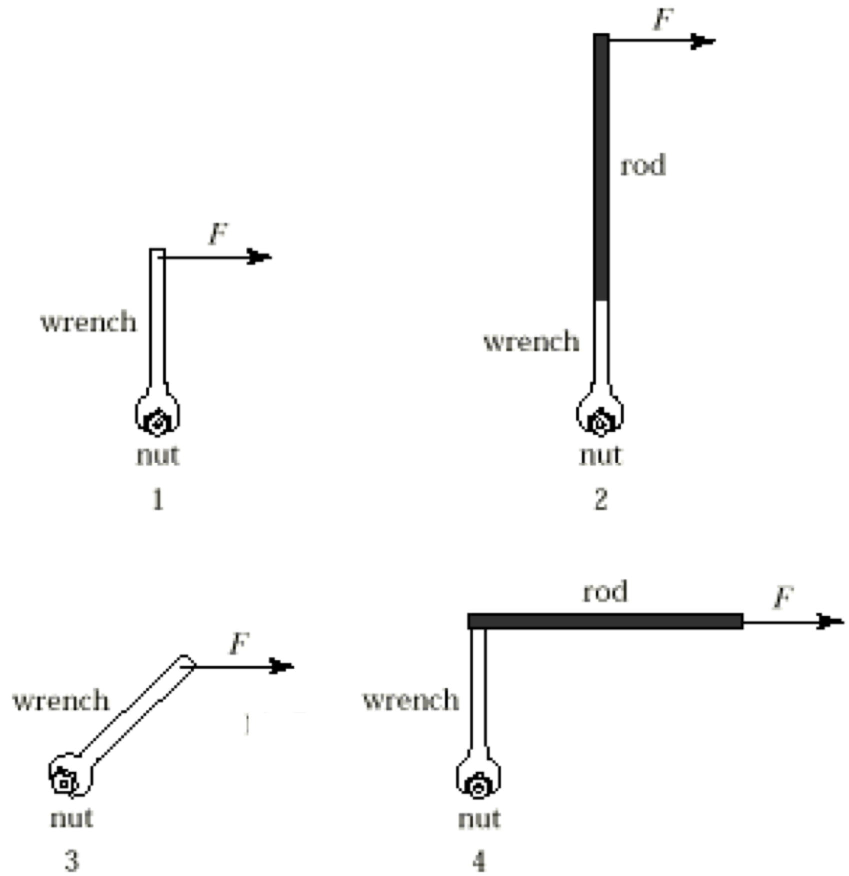

Which is least effective?

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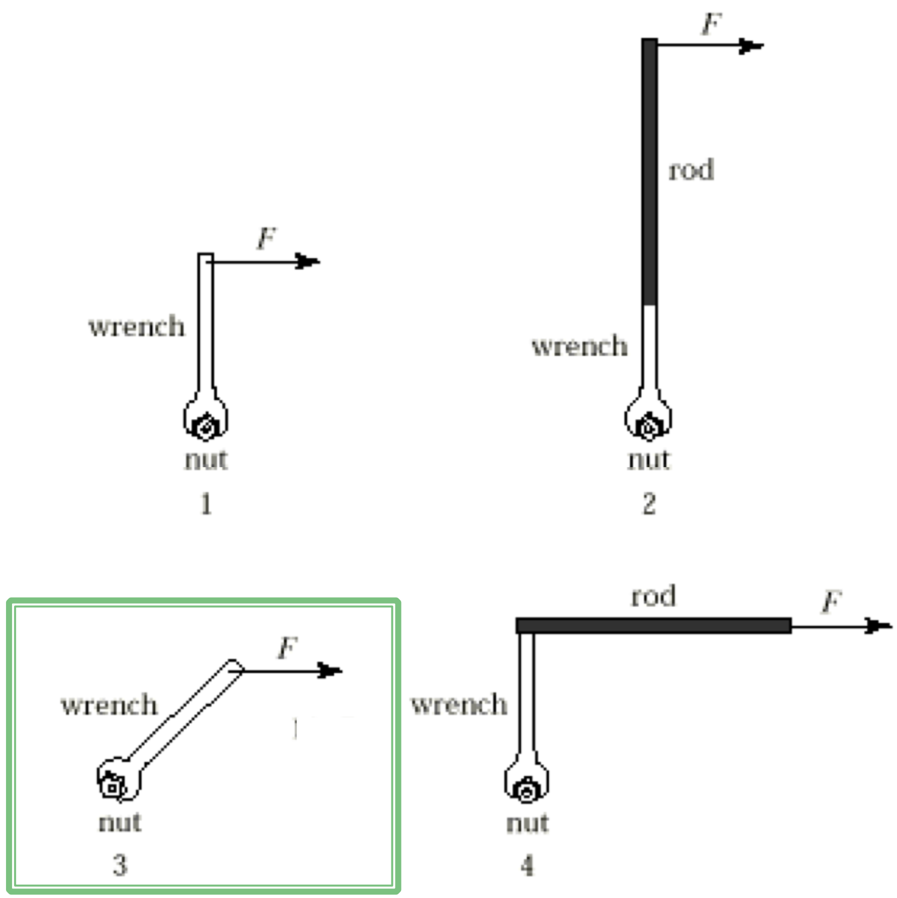
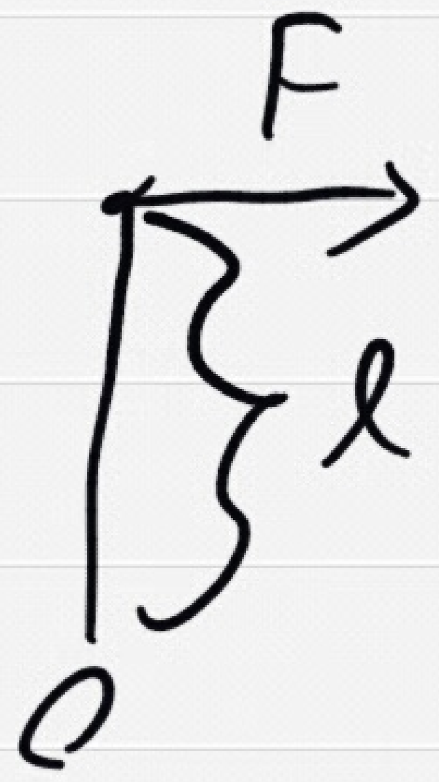
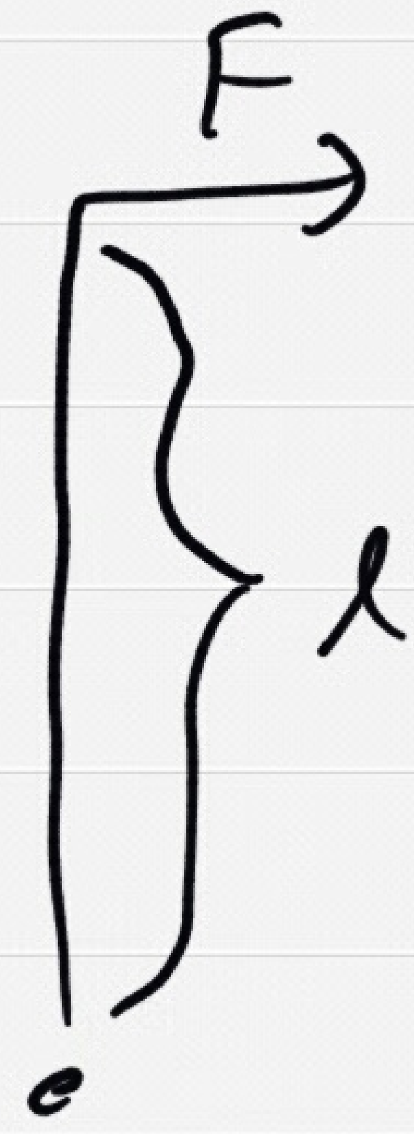


Which is least effective?

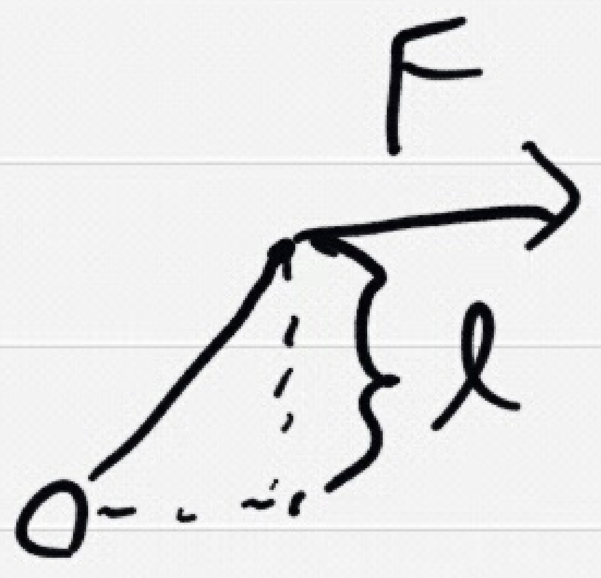
1.



2.



3.



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