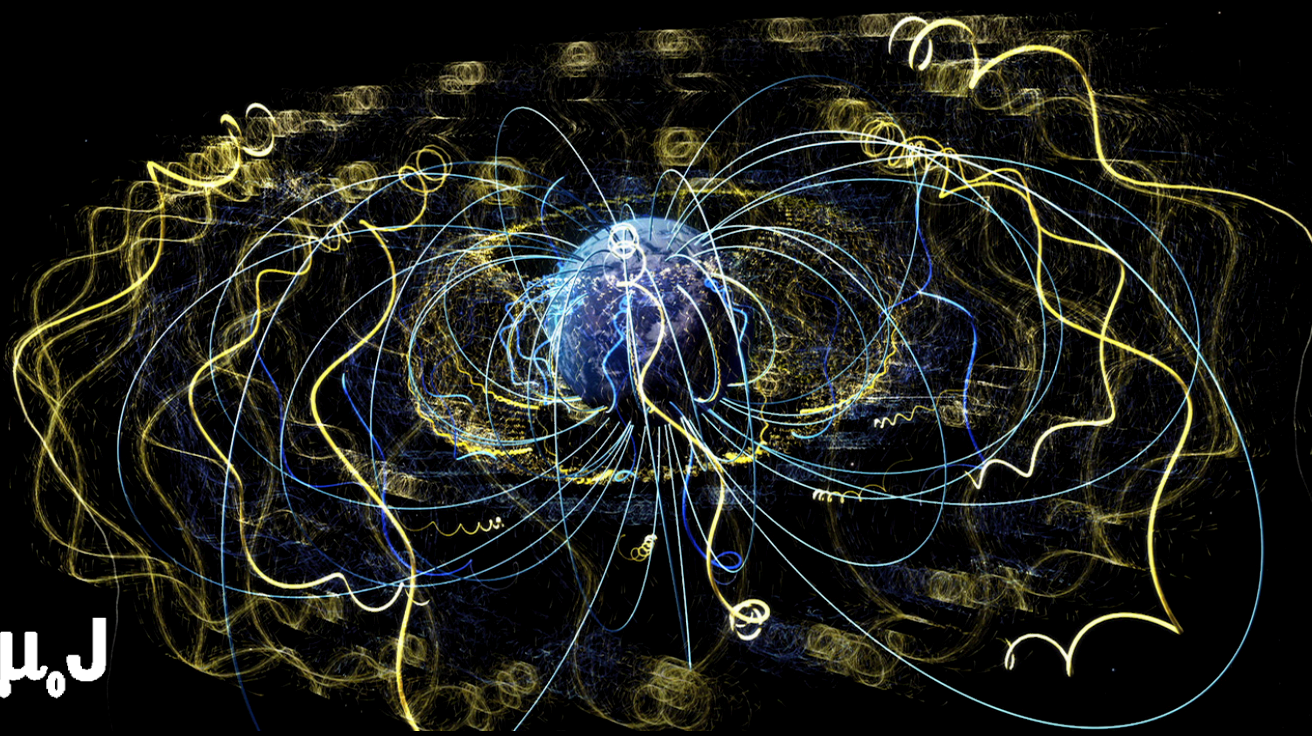
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism I: 3811

Professor Jasper Halekas
Van Allen 301
MWF 9:30-10:20 Lecture

Work to Charge Capacitor

$W = Q\Delta V$ for fixed Q, ΔV

$dW = \Delta V\, dq = q/C \, dq$ (since $C = Q/\Delta V$)

$W = \int dW = \int_0^Q q/C \, dq$

$= q^2/2C \Big|_0^Q$

$= Q^2/2C$

$= \frac{1}{2} C \cdot (Q/C)^2 = \frac{1}{2} C \Delta V^2$

For parallel-plate capacitor

$W = \frac{1}{2} Q^2/C = \frac{1}{2} Q^2 \cdot \frac{d}{\varepsilon_0 A}$

$= \frac{1}{2} \varepsilon_0 \cdot \left(\frac{Q}{\varepsilon_0 A}\right)^2 \cdot A \cdot d$

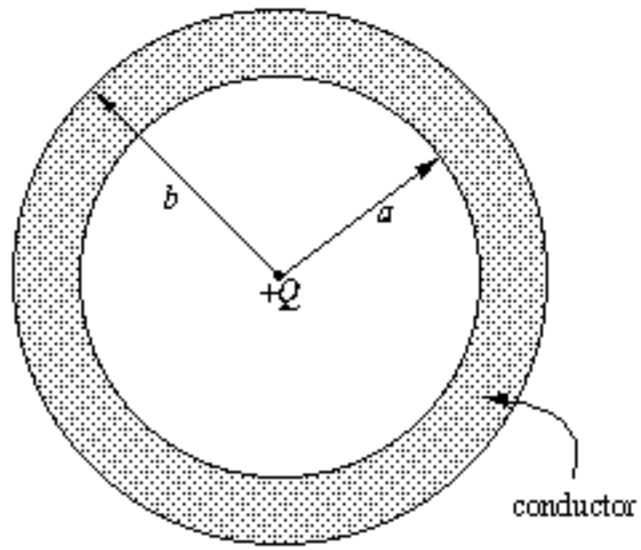
$= \frac{1}{2} \varepsilon_0 \cdot E^2 \cdot \text{Volume}$

$= \int \frac{1}{2} \varepsilon_0 E^2 \, d\tau$

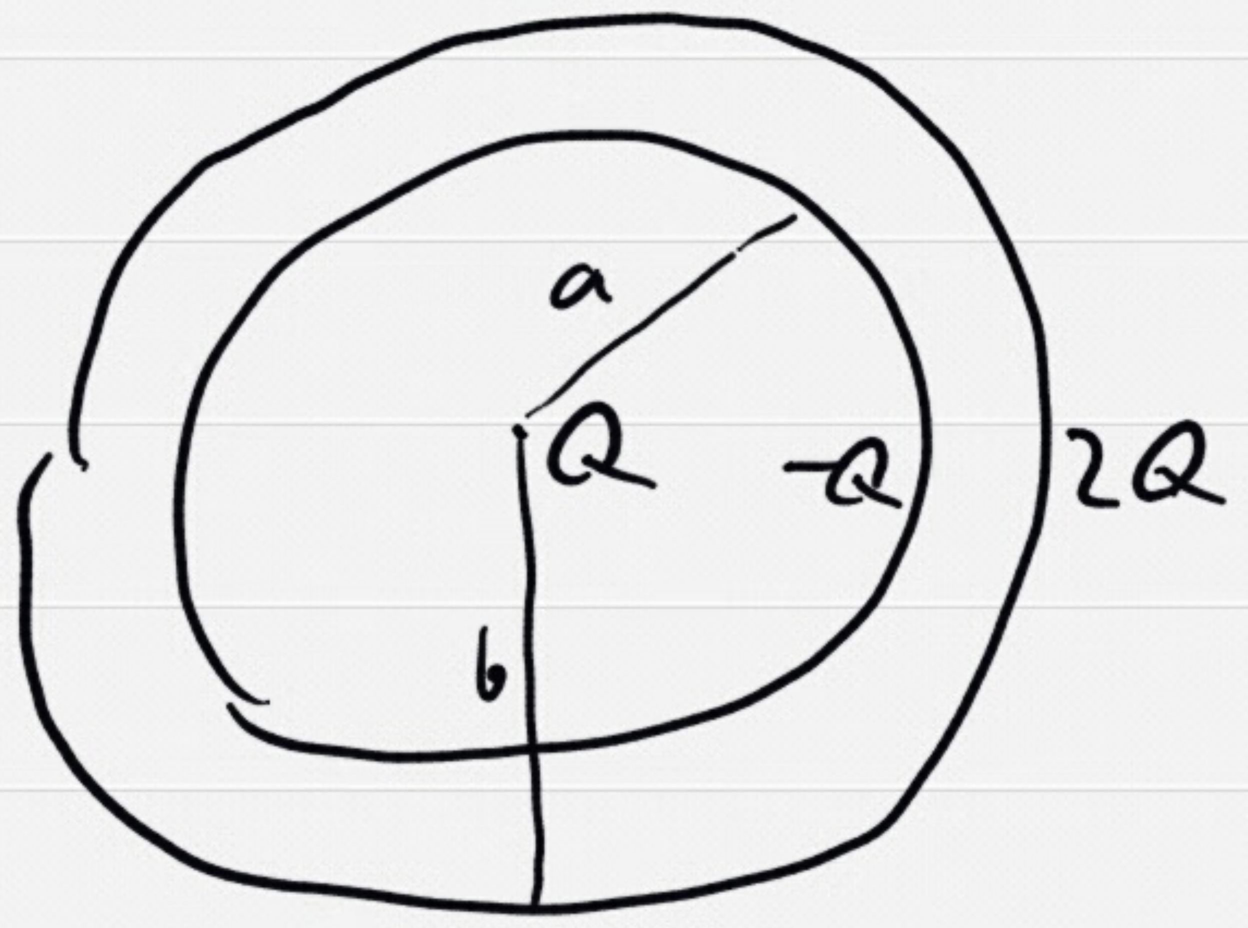
as expected

Check Your Understanding #1

- Consider an arrangement with a point charge Q at the origin, surrounded by a conducting shell with inner and outer radii a and b, that has a net charge of +Q
  - What is the electric field E(r) for r < a, a < r < b, and r > b?



Q1.



Hard Way: $\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{r}\,')}{\Delta r^2}\hat{\Delta r}\, d\tau'$

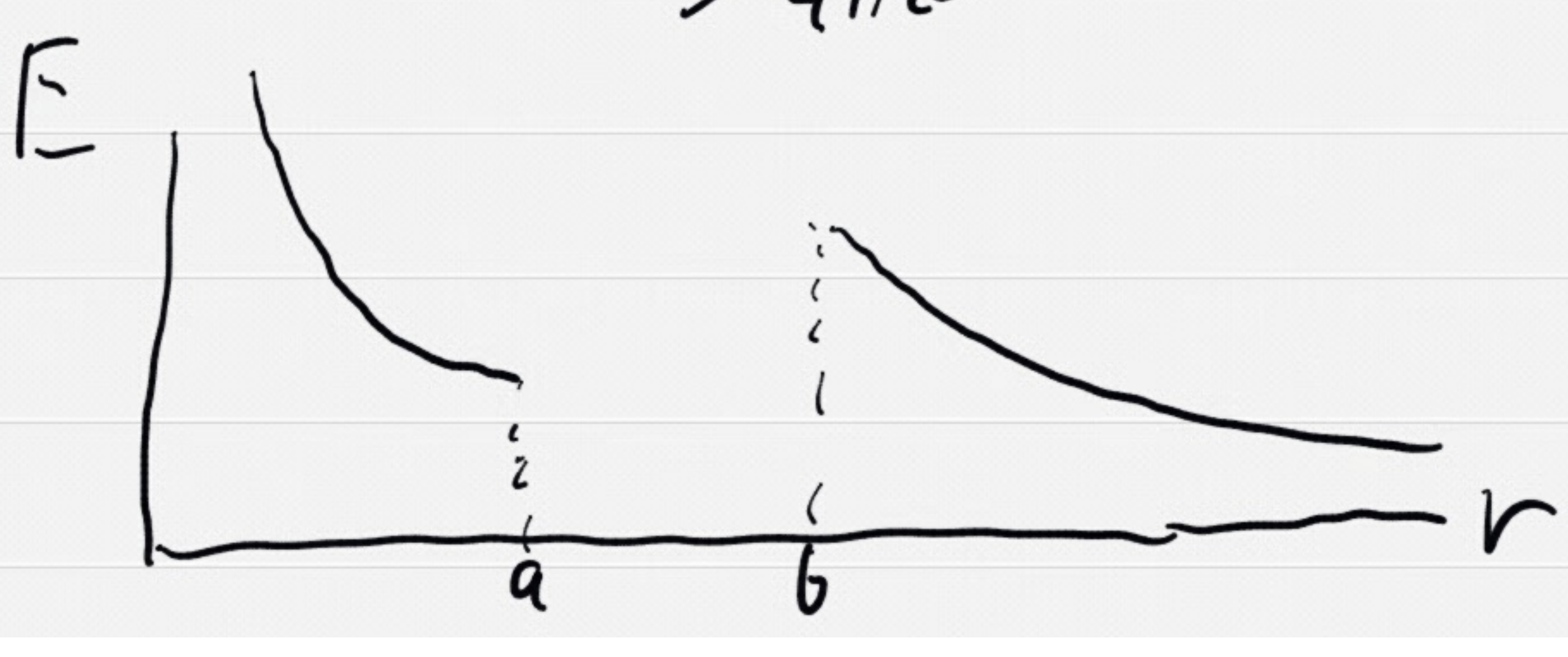
Easier Way:

$$\oint \vec{E}\cdot d\vec{a} = Q_{enc}/\varepsilon_0$$

$$\oint \vec{E}\cdot d\vec{a} = E\cdot 4\pi r^2 \quad \text{by symmetry}$$

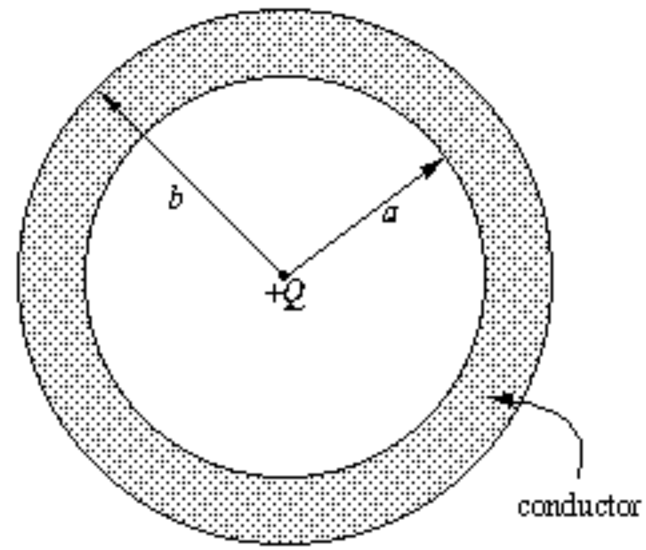
$$Q_{enc} = \begin{cases} +Q & r<a \\ 0 & a<r<b \\ +2Q & r>b \end{cases}$$

$$\Rightarrow \vec{E} = \begin{cases} \frac{Q\hat{r}}{4\pi\varepsilon_0 r^2} & r<a \\ 0 & a<r<b \\ 2Q\hat{r}/4\pi\varepsilon_0 r^2 & r>b \end{cases}$$



Check Your Understanding #2

- Consider an arrangement with a point charge Q at the origin, surrounded by a conducting shell with inner and outer radii a and b, that has a net charge of +Q



  - What is the electric potential V(r) for r < a, a < r < b, and r > b (with respect to infinity)?

Q 2. $V(r) = -\int_{\infty}^{r} \vec{E} \cdot d\vec{l}$

$$= -\int_{\infty}^{r} E_r \, dr'$$

$$= -\int_{\infty}^{r} \frac{2Q}{4\pi\varepsilon_0 r'^2} dr' \quad r > b$$

$$= \frac{2Q}{4\pi\varepsilon_0 r'}\Big|_{\infty}^{r} \quad r > b$$

$$= \boxed{\frac{2Q}{4\pi\varepsilon_0 r} \quad r > b}$$

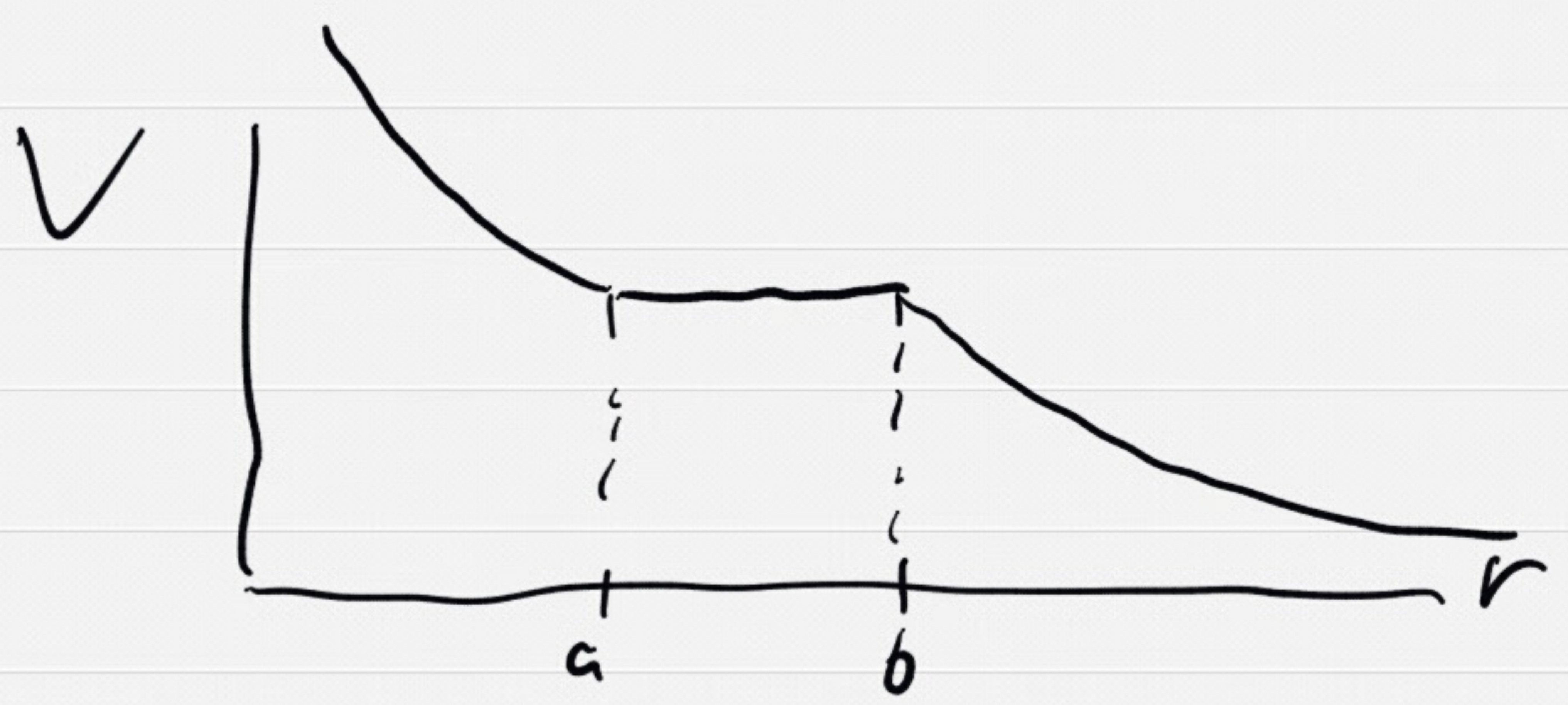
$$= -\int_{\infty}^{b} \frac{2Q}{4\pi\varepsilon_0 r'^2} dr' - \int_{b}^{r} 0 \, dr' \quad a < r < b$$

$$= \boxed{2Q/4\pi\varepsilon_0 b \quad a < r < b}$$

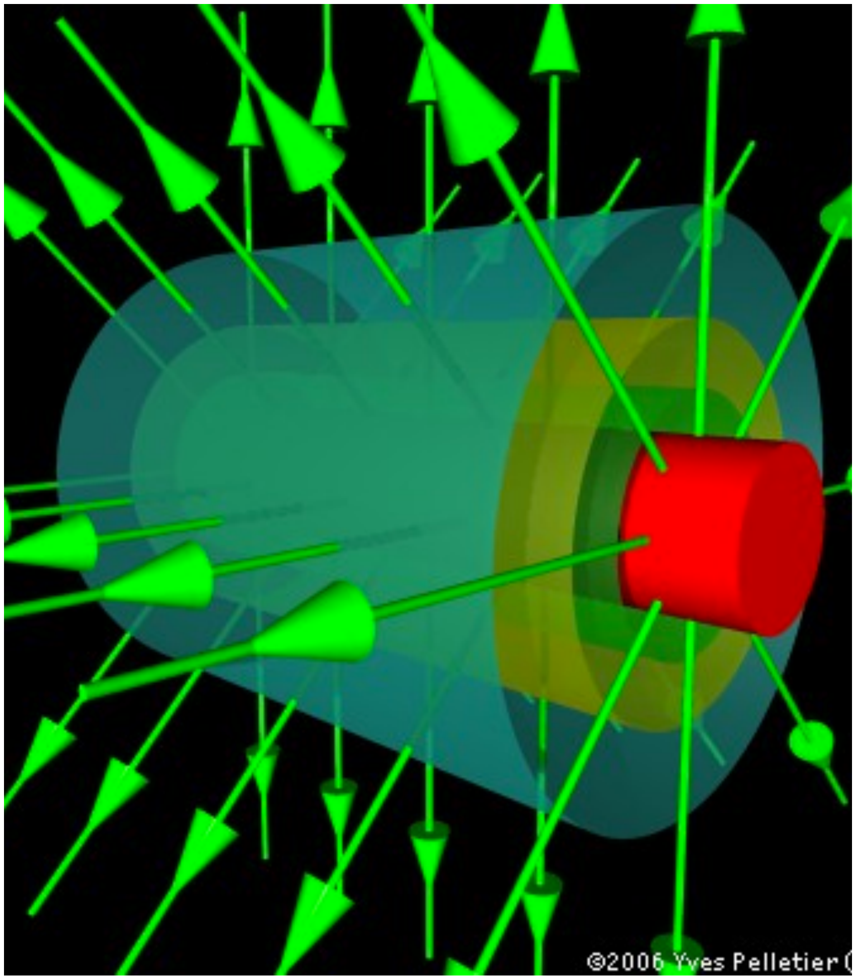
$$= -\int_{\infty}^{b} \frac{2Q}{4\pi\varepsilon_0 r'^2} dr' - \int_{b}^{a} 0 \, dr' - \int_{a}^{r} \frac{Q}{4\pi\varepsilon_0 r'^2} dr'$$

$$= 2Q/4\pi\varepsilon_0 b + \frac{Q}{4\pi\varepsilon_0 r'}\Big|_{a}^{r} \quad r < a$$

$$= \boxed{\frac{2Q}{4\pi\varepsilon_0 b} + \frac{Q}{4\pi\varepsilon_0 r} - \frac{Q}{4\pi\varepsilon_0 a} \quad r < a}$$

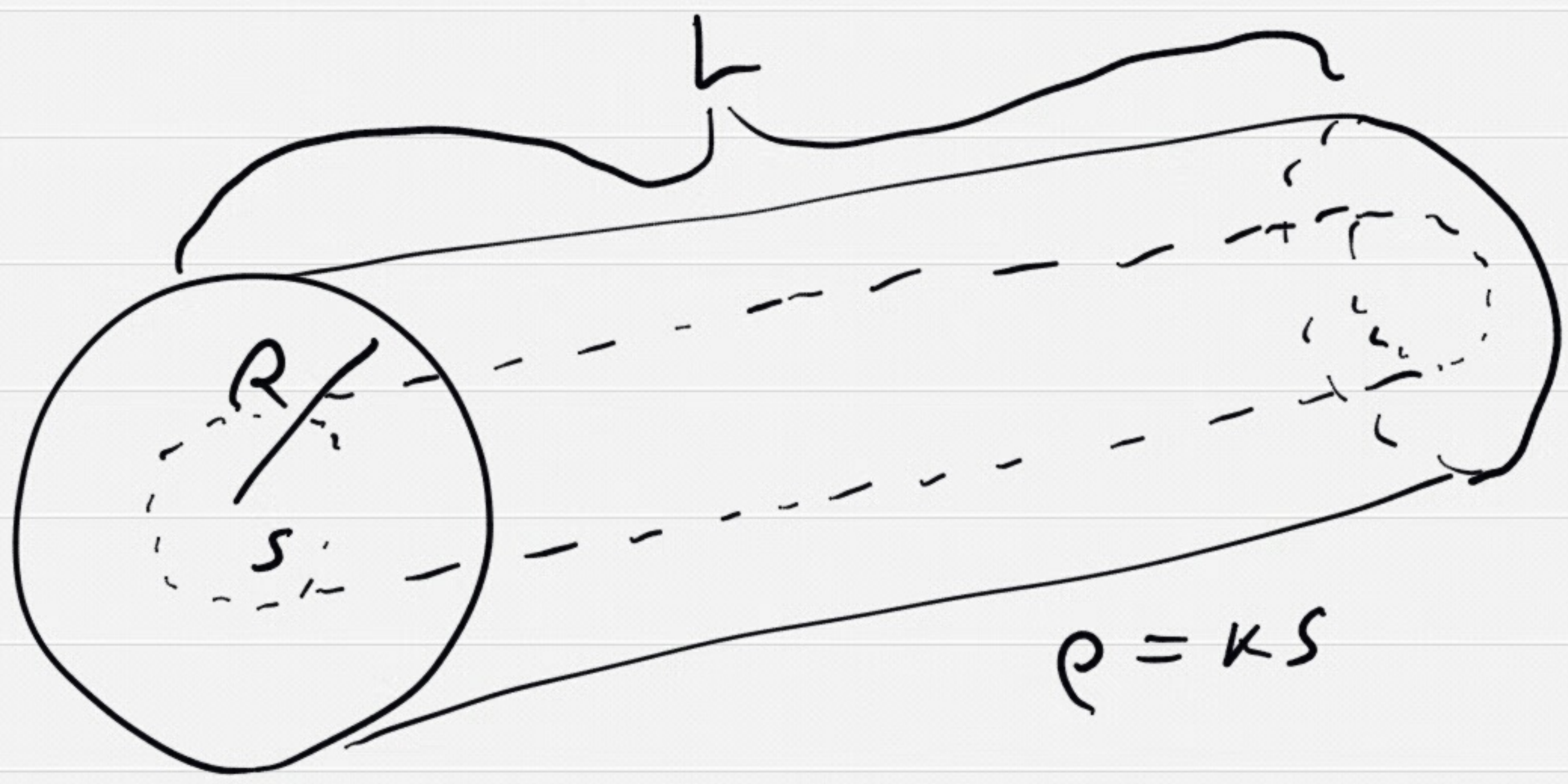


Check Your Understanding #3



- What is the electric field inside and outside of an infinitely long charged cylinder of radius R, whose volume charge density varies linearly with radius - i.e. $\rho = ks$.

Q3:



$$\oint \vec{E} \cdot d\vec{a} = 2\pi s \cdot L \cdot E$$

$$Q_{enc} = \int \rho \, d\tau$$

$$= \int_0^L \int_0^{2\pi} \int_0^s ks' \cdot s' ds' d\phi \, dz \qquad s < R$$

$$= 2\pi L \cdot \int_0^s ks'^2 ds' \qquad s < R$$

$$= 2\pi L \cdot \frac{ks^3}{3} \qquad s < R$$

$$= 2\pi L \cdot \frac{kR^3}{3} \qquad s > R$$

$$\Rightarrow \vec{E} = \frac{ks^2}{3\varepsilon_0} \hat{s} \qquad s < R$$

$$= \frac{kR^3}{3\varepsilon_0 s} \hat{s} \qquad s > R$$