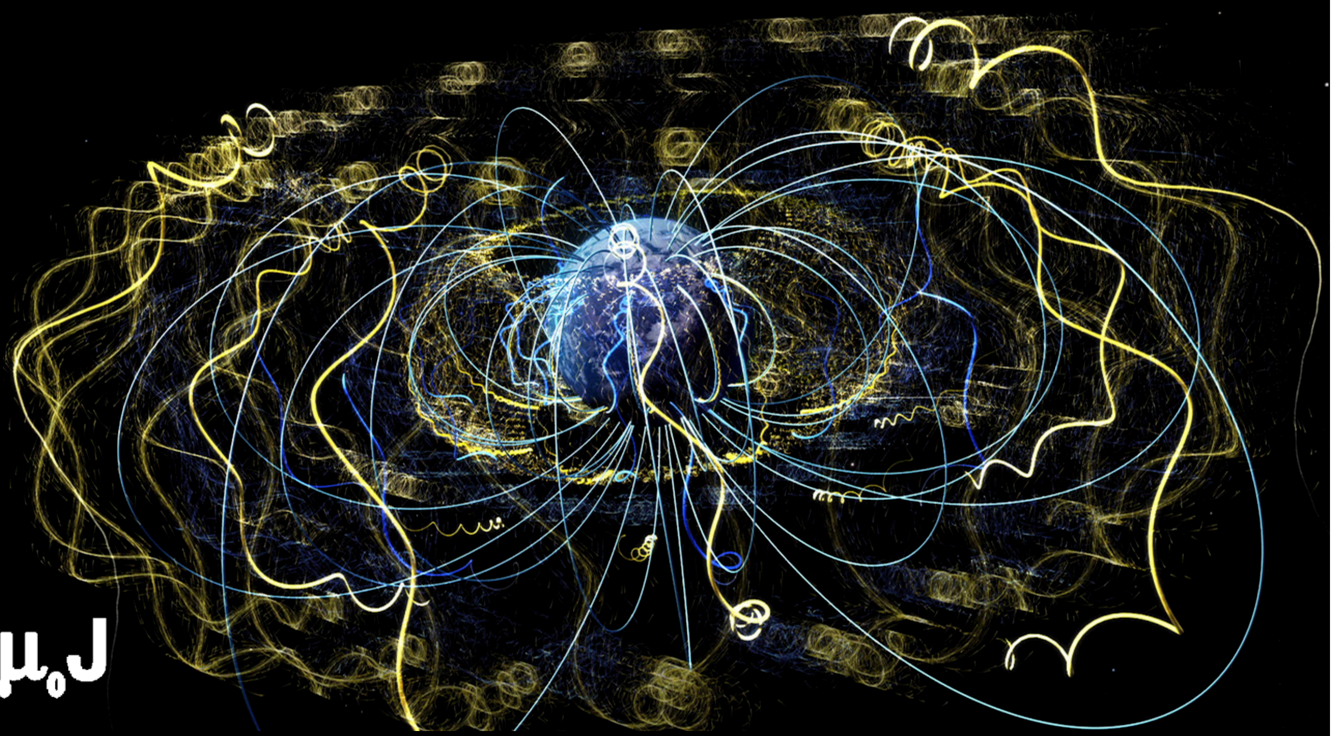
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$

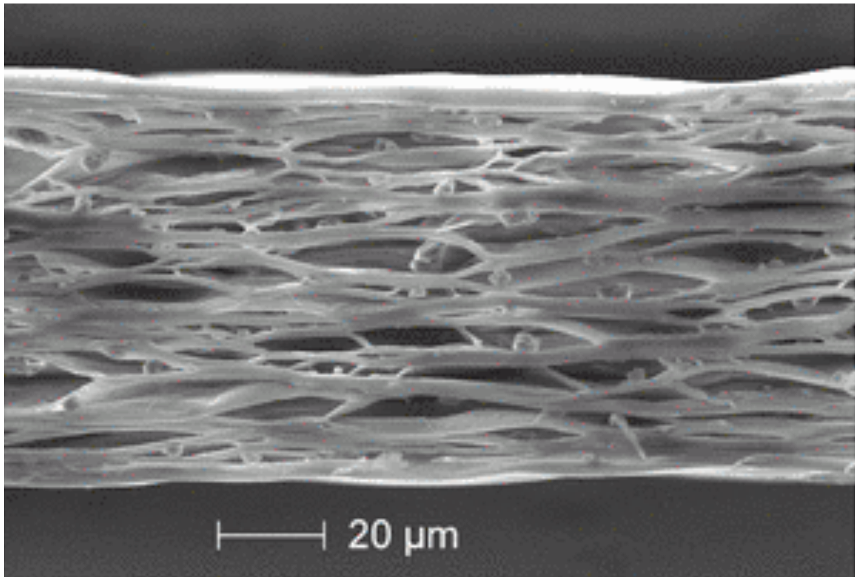


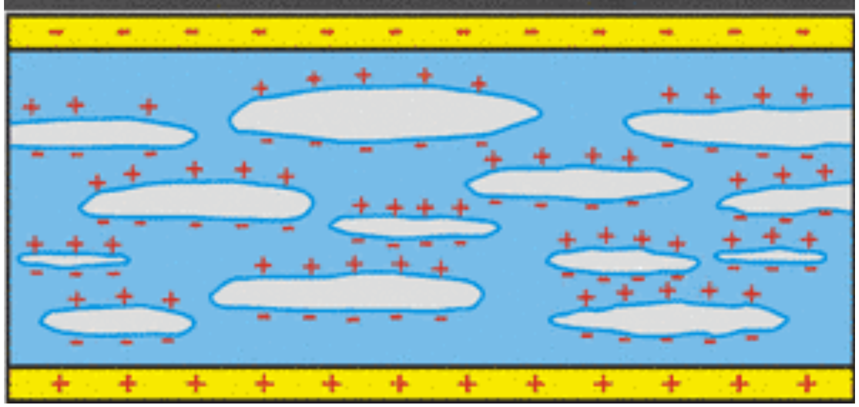
# Electricity and Magnetism I: 3811


Professor Jasper Halekas
Van Allen 301

MWF 9:30-10:20 Lecture

# Electrets



**To Fabricate:**

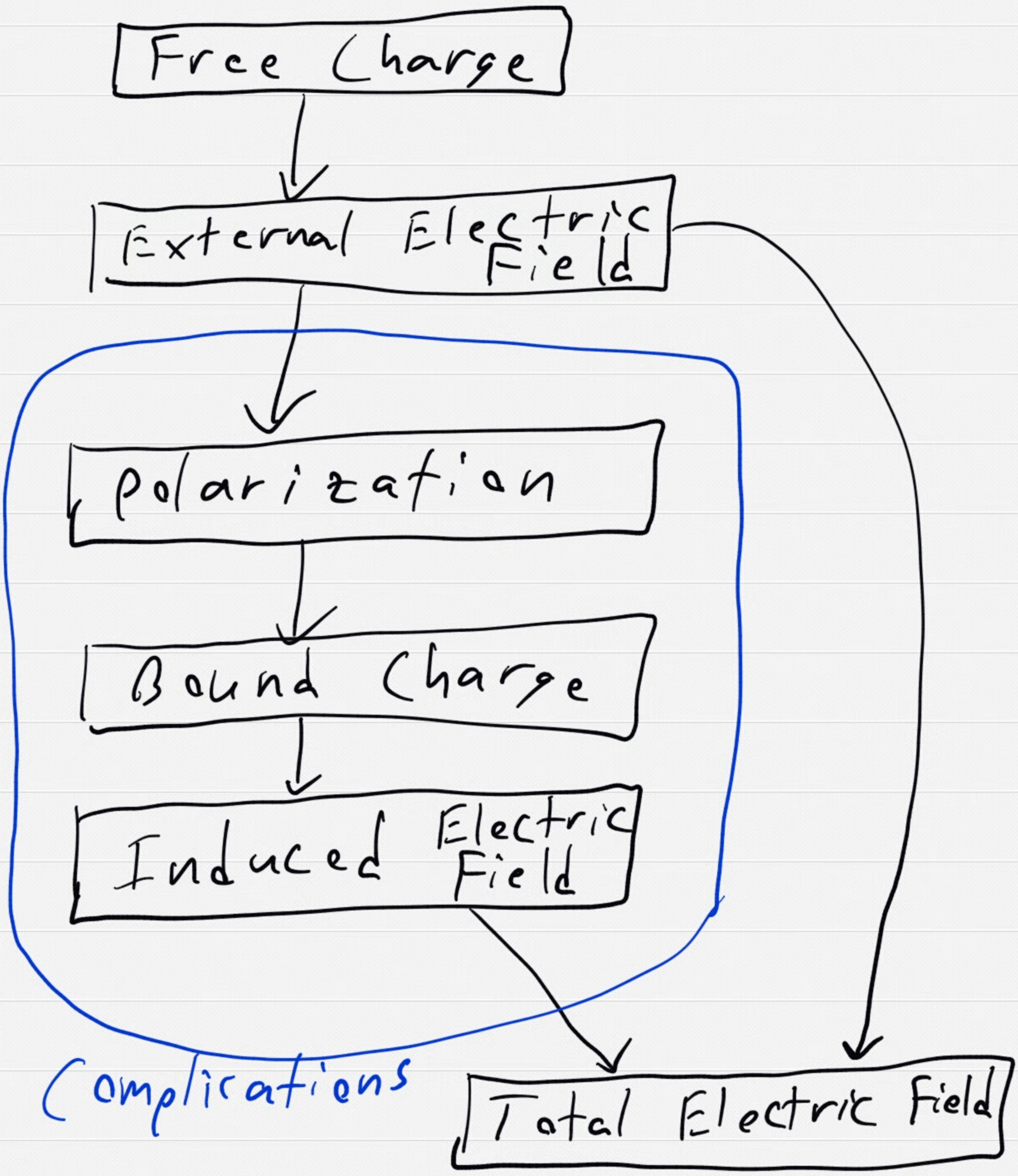
1. Make stretched polymer substance
2. Expose to corona discharge
3. Electric breakdown in voids
4. Charge deposited on void surfaces

**Other Methods:**

- Melt dielectric material and let cool in strong electric field

- Implant charge w/ electron beam

# Dielectrics



# Electric Displacement

$$\rho = \rho_b + \rho_f$$

$$\nabla \cdot \vec{E} = \rho / \varepsilon_0 = (\rho_b + \rho_f)/\varepsilon_0$$

$$\varepsilon_0 \nabla \cdot \vec{E} = (\rho_f - \nabla \cdot \vec{P})$$

$$\Rightarrow \nabla \cdot (\varepsilon_0 \vec{E} + \vec{P}) = \rho_f$$

Define $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

= "electric displacement"

$$\nabla \cdot \vec{D} = \rho_f$$

$$\Rightarrow \int \nabla \cdot \vec{D} \, d\tau = \int \rho_f \, d\tau = Q_f$$

Divergence Theorem

$$\Rightarrow \oint \vec{D} \cdot d\vec{a} = Q_{f\,enc}$$

- Just like Gauss's law, but w/ only free charge

- Bound charge swept under the rug

# New flow chart

$\boxed{\text{Free Charge}}$

$\downarrow$

$\boxed{\text{Electric Displacement}}$

---

## Limitations of $\vec{D}$

① Can only get $\vec{E}$ if $\vec{P}$ is known

② $\nabla \times \vec{D} = \varepsilon_0 \nabla \times \vec{E} + \nabla \times \vec{P}$

$= \nabla \times \vec{P}$

not always zero

$\Rightarrow \oint \vec{D} \cdot d\vec{l} \neq 0$

$\vec{D} \neq -\nabla f$ no potential formulation

$\vec{D}(\vec{r}) \neq \frac{1}{4\pi} \int \rho_f(\vec{r}\,') \frac{\hat{r}}{r^2} d\tau'$

no Coulomb's law

# Boundary Conditions

$$\Delta E_\perp = \sigma/\varepsilon_0$$
$$\Delta \vec{E}_\parallel = 0$$

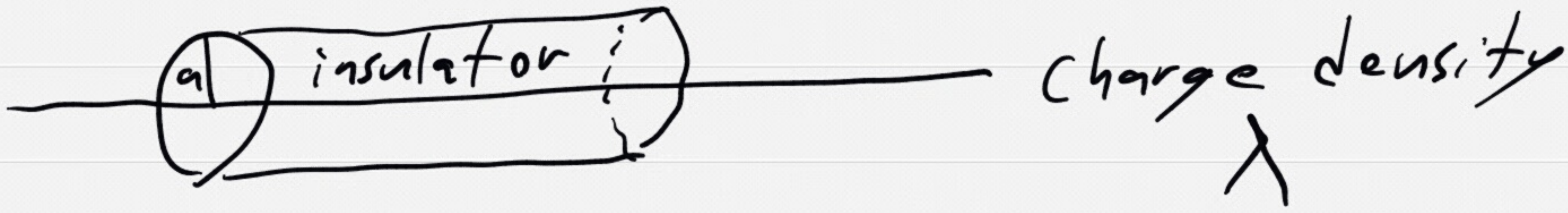
Gauss's law for $\vec{D}$

$$\Rightarrow \Delta D_\perp = \sigma_f$$

$$\Delta \vec{D}_\parallel = \Delta \vec{P}_\parallel$$

D inherits any tangential discontinuity in polarization

# Example




Gauss's law: $\oint \vec{D} \cdot d\vec{a} = Q_{f\,enc}$

$$D \cdot 2\pi s L = \lambda L$$

$$\Rightarrow \vec{D} = \frac{\lambda}{2\pi s}\hat{s} \quad \text{everywhere}$$

$s < a$: $\vec{P} = ? \Rightarrow \vec{E} = ?$

$s > a$: $\vec{P} = 0 \Rightarrow \vec{E} = \frac{\lambda}{2\pi\varepsilon_0 s}\hat{s}$

- same as w/o insulator outside. Makes sense since total bound charge is zero

# Linear Dielectrics

In many materials, $\vec{P}$ is proportional to $\vec{E}$
(since $\vec{E}$ responsible for polarizing dielectric)

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}$$

w/ $\chi_e$ = "electric susceptibility"

$$\Rightarrow \vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$
$$= \varepsilon_0 \vec{E} + \varepsilon_0 \chi_e \vec{E}$$
$$= \varepsilon_0 (1 + \chi_e) \vec{E}$$

$$= \varepsilon \vec{E}$$

$$\varepsilon = \varepsilon_0 (1 + \chi_e)$$
= "permittivity"

$$\varepsilon_r = \varepsilon / \varepsilon_0 = 1 + \chi_e$$
= "relative permittivity"
= "dielectric constant"

$\varepsilon_r$ often written as $\kappa$

# Electric Displacement

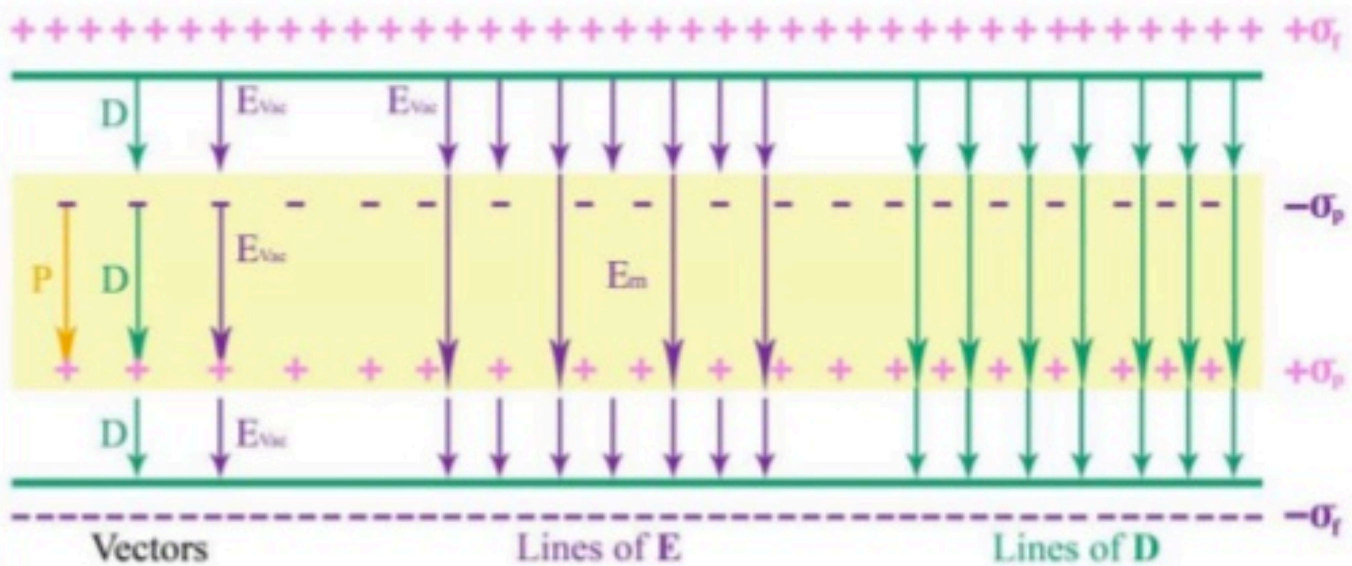



Fig. Relation between **D**, **E** and **P**

Fig. Representation of the relation between three Electric Vectors $D = \varepsilon_0 E + P$