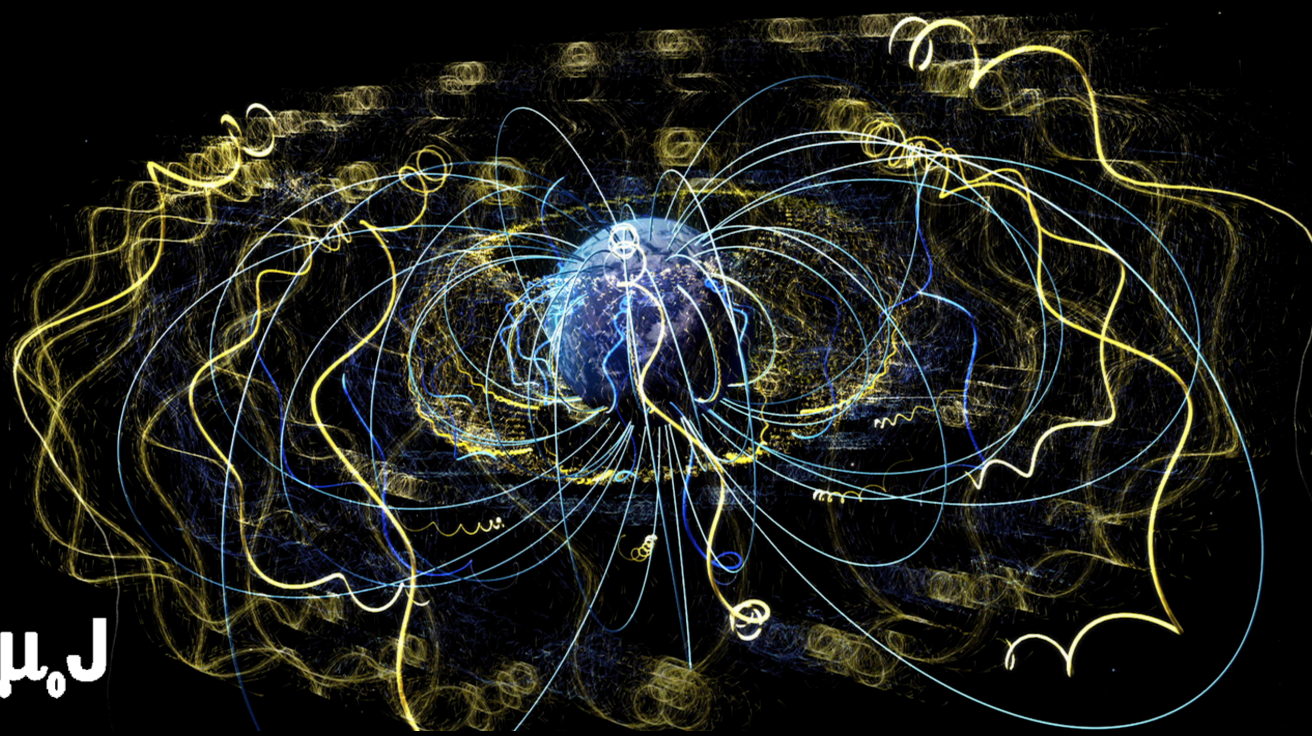
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$

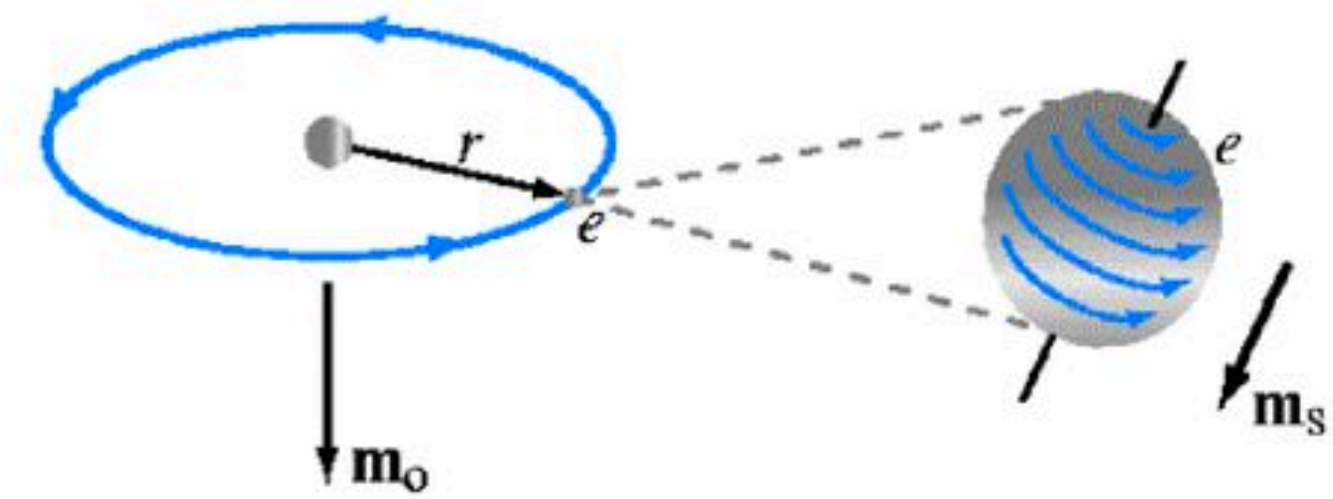


Electricity and Magnetism I: 3811

Professor Jasper Halekas
Van Allen 301
MWF 9:30-10:20 Lecture

Atomic Magnetic Moment

Warning: Classical Picture of Intrinsically Quantum-Mechanical Processes



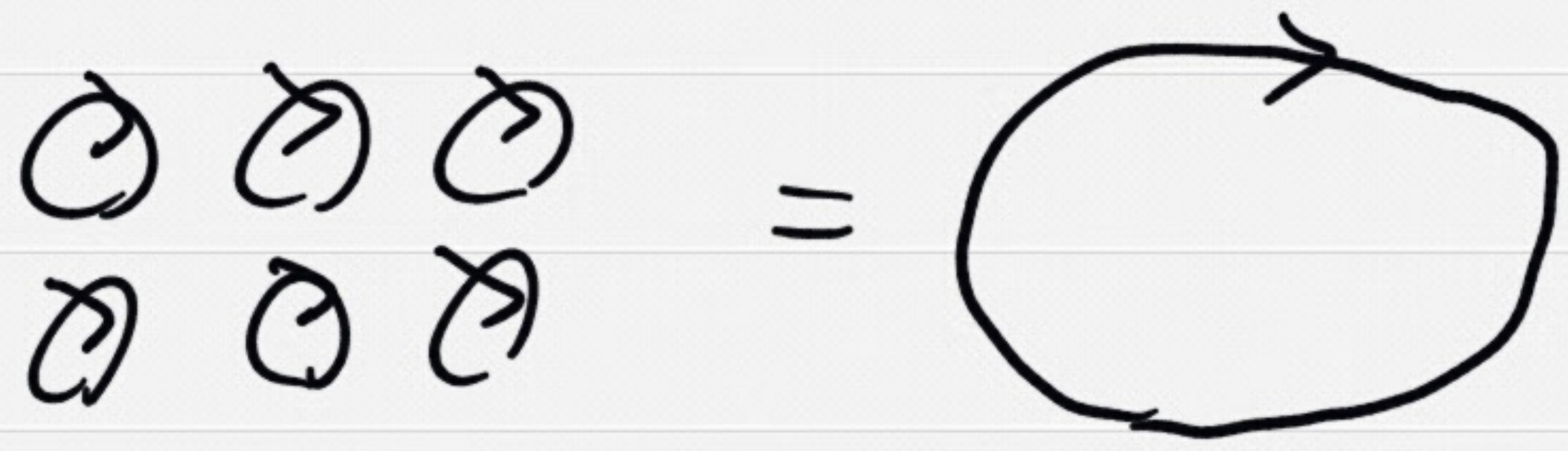
(a) Orbiting electron (b) Spinning electron

- Materials have magnetism due to atomic magnetic moments from electron orbital and spin angular momentum

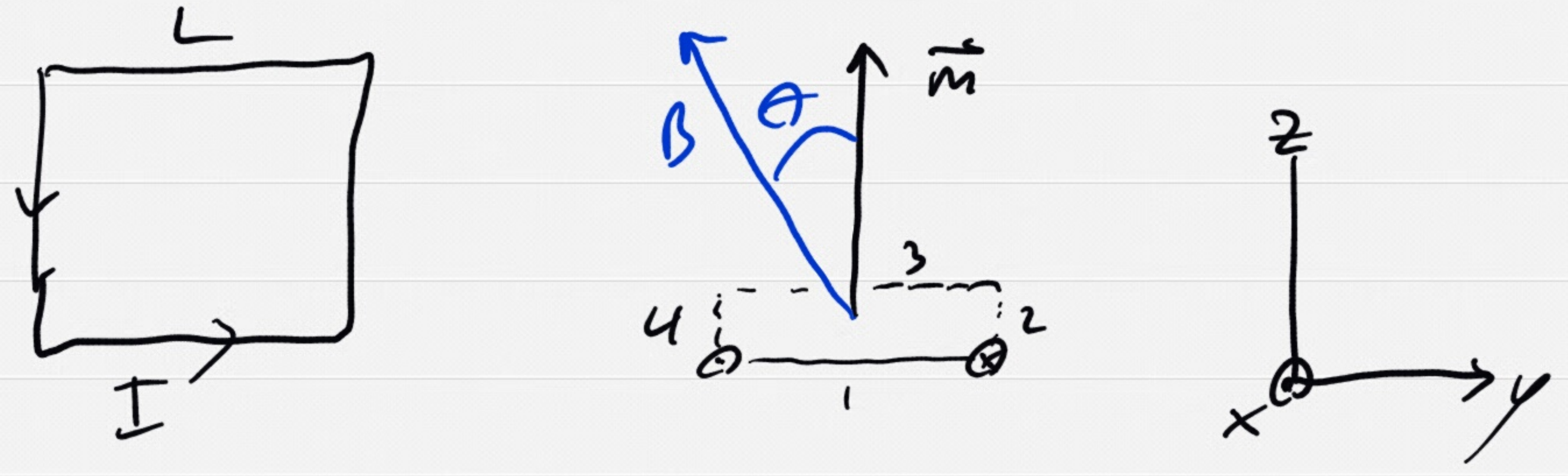
- Most materials have weak magnetism

randomly oriented current loops

- Some materials have intrinsic magnetism, or gain magnetism in external magnetic fields



Force and Torque on Current Loop



$$\vec{F}_1 = IL\hat{y} \times (B_1 \cos\theta\, \hat{z} - B_1 \sin\theta\, \hat{y})$$
$$= ILB_1 \cos\theta\, \hat{x}$$

$$\vec{F}_2 = ILB_2 (\cos\theta\, \hat{y} + \sin\theta\, \hat{z})$$

$$\vec{F}_3 = -ILB_3 \cos\theta\, \hat{x}$$

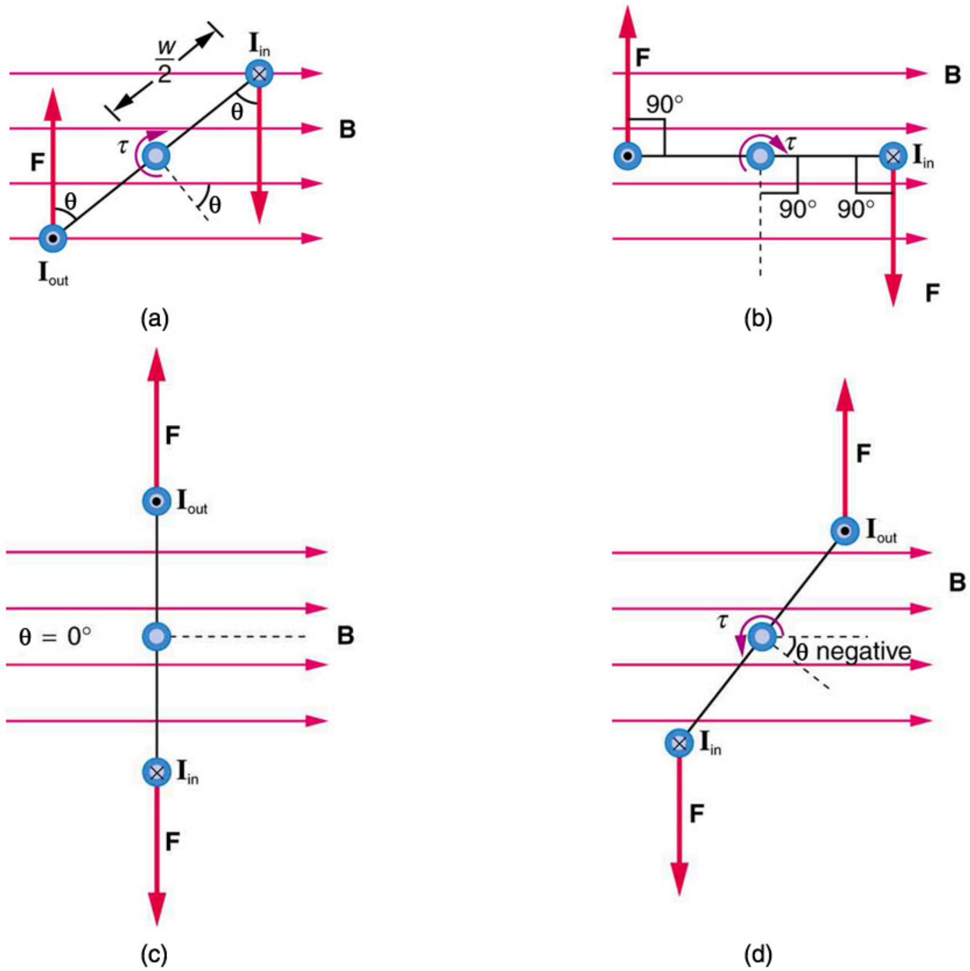
$$\vec{F}_4 = ILB_4 (-\cos\theta\, \hat{y} - \sin\theta\, \hat{z})$$

If $B_1 = B_2 = B_3 = B_4 \Rightarrow \vec{F}_{net} = 0$

$$\vec{\tau} = \sum \vec{r} \times \vec{F}$$
$$= L/2\, \hat{x} \times \vec{F}_1 + L/2\, \hat{y} \times \vec{F}_2 - L/2\, \hat{x} \times \vec{F}_3 - L/2\, \hat{y} \times \vec{F}_4$$
$$= 0 + L/2 \cdot ILB \sin\theta\, \hat{x} + 0 - L/2 \cdot -ILB \sin\theta\, \hat{x}$$
$$= IL^2 B \sin\theta\, \hat{x}$$
$$= \boxed{\vec{m} \times \vec{B}}$$

rotates $\vec{m}$ to align w/ $\vec{B}$

Force and Torque on Current Loop



If $\vec{B}$ non-uniform

$$\Rightarrow \vec{F}_{net} = IL\hat{y} \times \vec{B}_1 + IL \cdot -\hat{x} \times \vec{B}_2 + IL \cdot -\hat{y} \times \vec{B}_3 + IL \cdot \hat{x} \times \vec{B}_4$$

$$\cong IL\hat{y} \times \left(\vec{B}_0 + \partial\vec{B}/\partial x \cdot L/2\right) - IL\hat{x} \times \left(\vec{B}_0 + \partial\vec{B}/\partial y \cdot L/2\right) - IL\hat{y} \times \left(\vec{B}_0 - \partial\vec{B}/\partial x \cdot L/2\right) + IL\hat{x} \times \left(\vec{B}_0 - \partial\vec{B}/\partial y \cdot L/2\right)$$

$$= IL^2 \left(\hat{x} \times -\partial\vec{B}/\partial y + \hat{y} \times \partial\vec{B}/\partial x\right)$$

$$= m \left(-\partial B_y/\partial y\,\hat{z} + \partial B_z/\partial y\,\hat{y} - \partial B_x/\partial x\,\hat{z} + \partial B_z/\partial x\,\hat{x}\right)$$

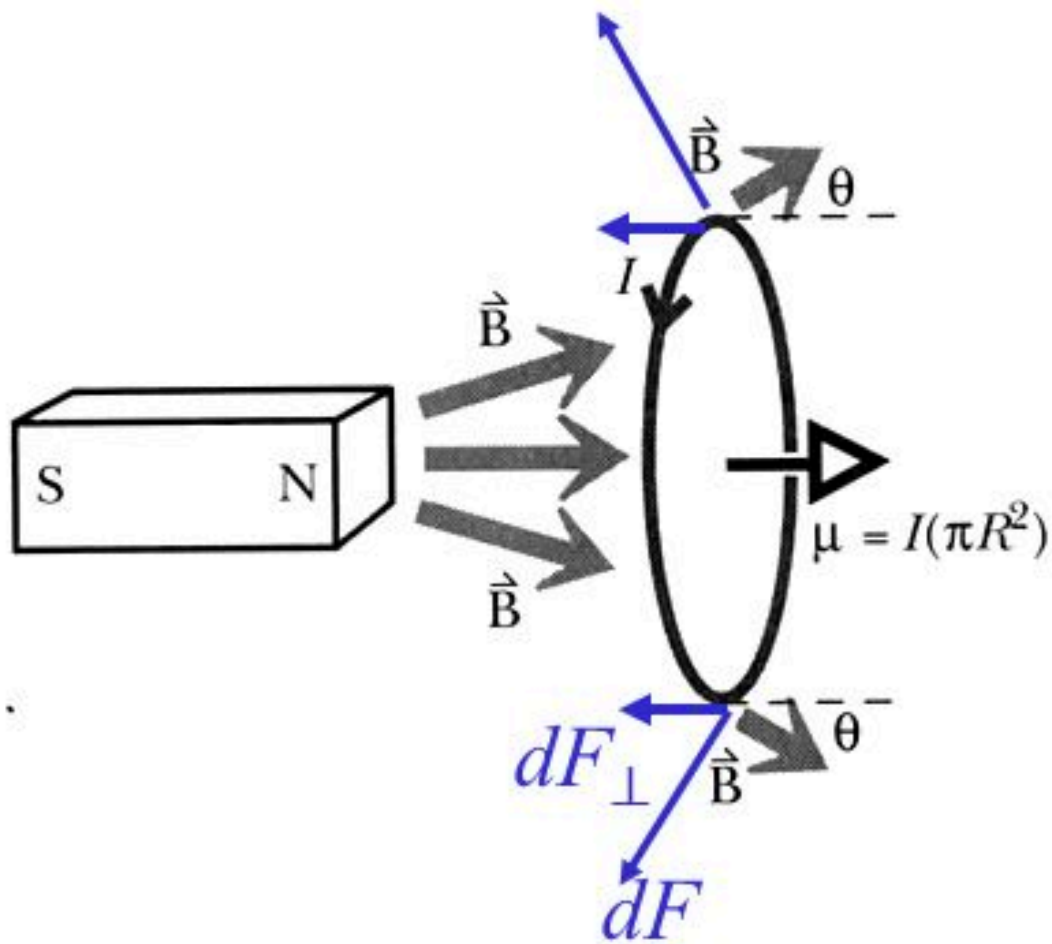
$$= m \left(\partial B_z/\partial x\,\hat{x} + \partial B_z/\partial y\,\hat{y} - \left(\partial B_x/\partial x + \partial B_y/\partial y\right)\hat{z}\right)$$

But $\nabla \cdot \vec{B} = \partial B_x/\partial x + \partial B_y/\partial y + \partial B_z/\partial z = 0$

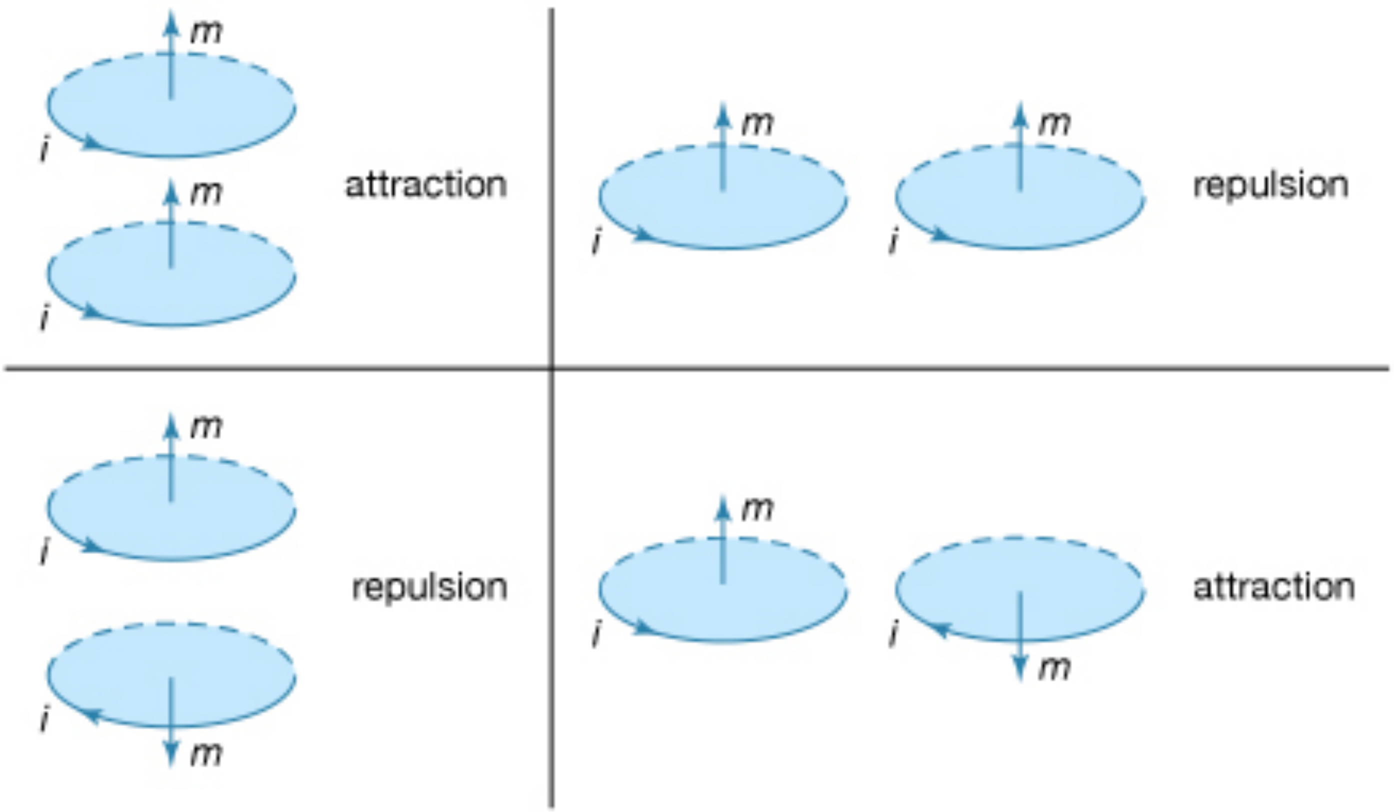
$$\Rightarrow \vec{F}_{net} = m \left(\partial B_z/\partial x\,\hat{x} + \partial B_z/\partial y\,\hat{y} + \partial B_z/\partial z\,\hat{z}\right) = \nabla(m B_z)$$

$$\Rightarrow \boxed{\vec{F} = \nabla(\vec{m} \cdot \vec{B})}$$

Force on Current Loop in Converging Magnetic Field

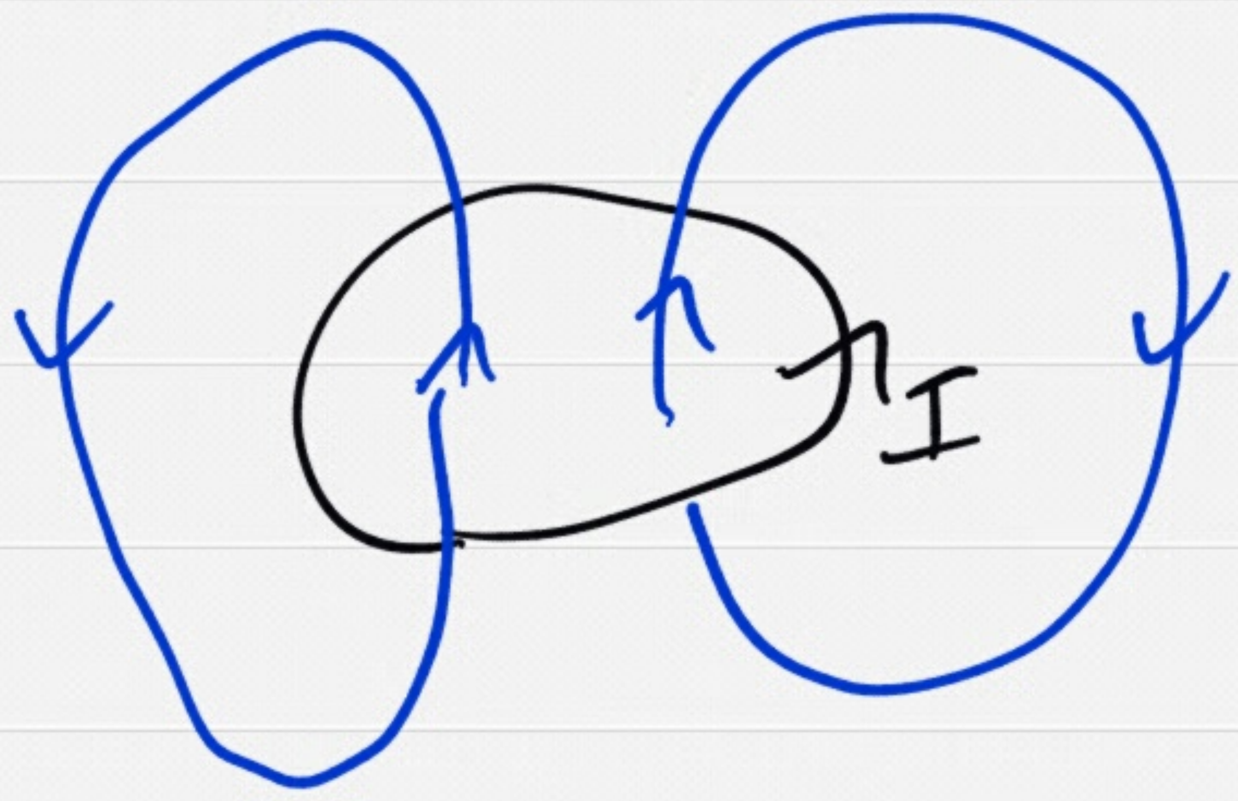


Force Between Dipoles



Paramagnetism

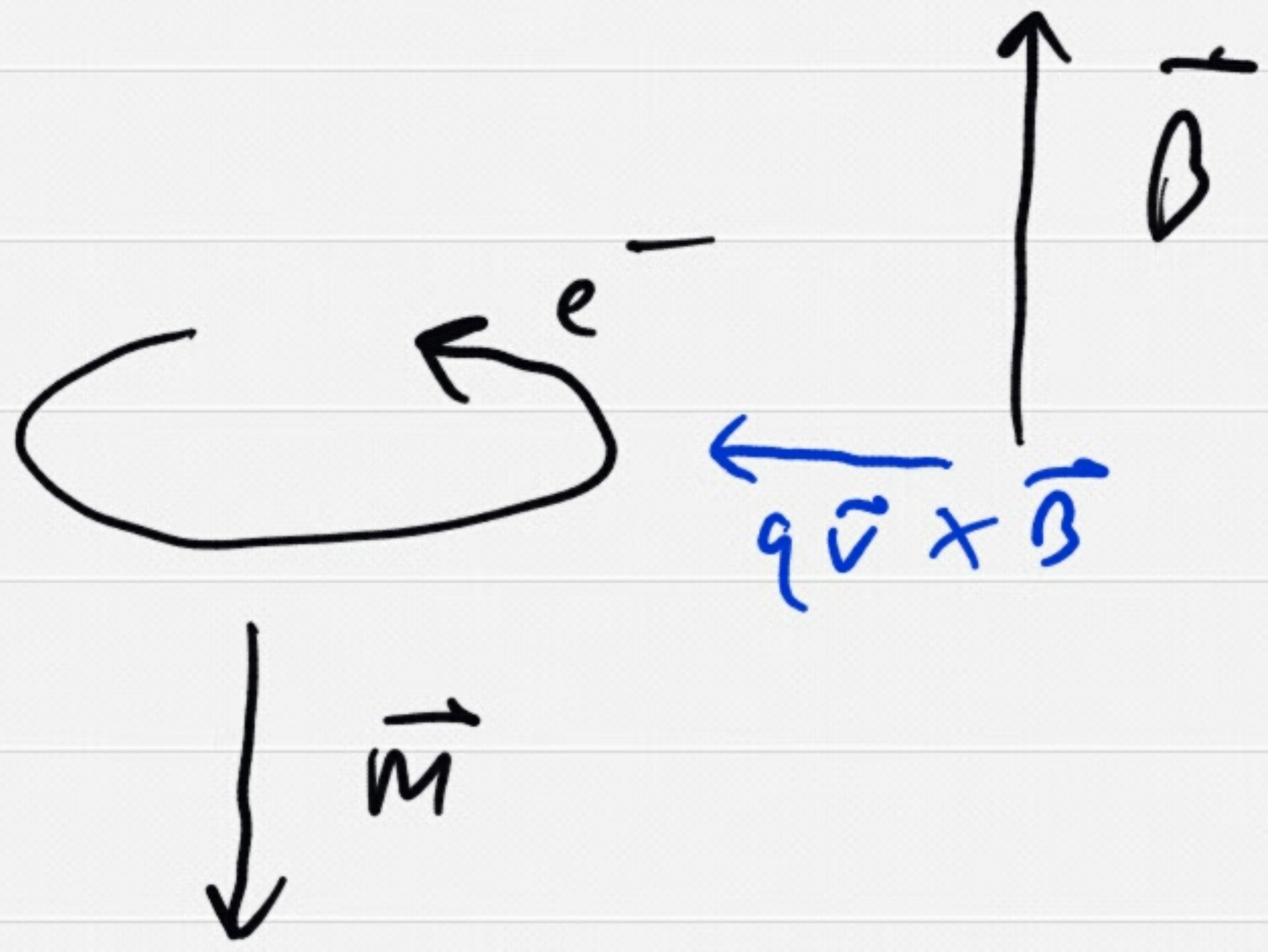
- External $\vec{B}$ causes atomic dipoles to align



- Reinforces $\vec{B}_{ext}$ inside
- Opposes it outside

Diamagnetism

- External magnetic field affects electron orbits



centripetal force increased
$\Rightarrow$ faster orbit
$\Rightarrow$ increased $\vec{m}$ opposite to $\vec{B}$

Magnetization