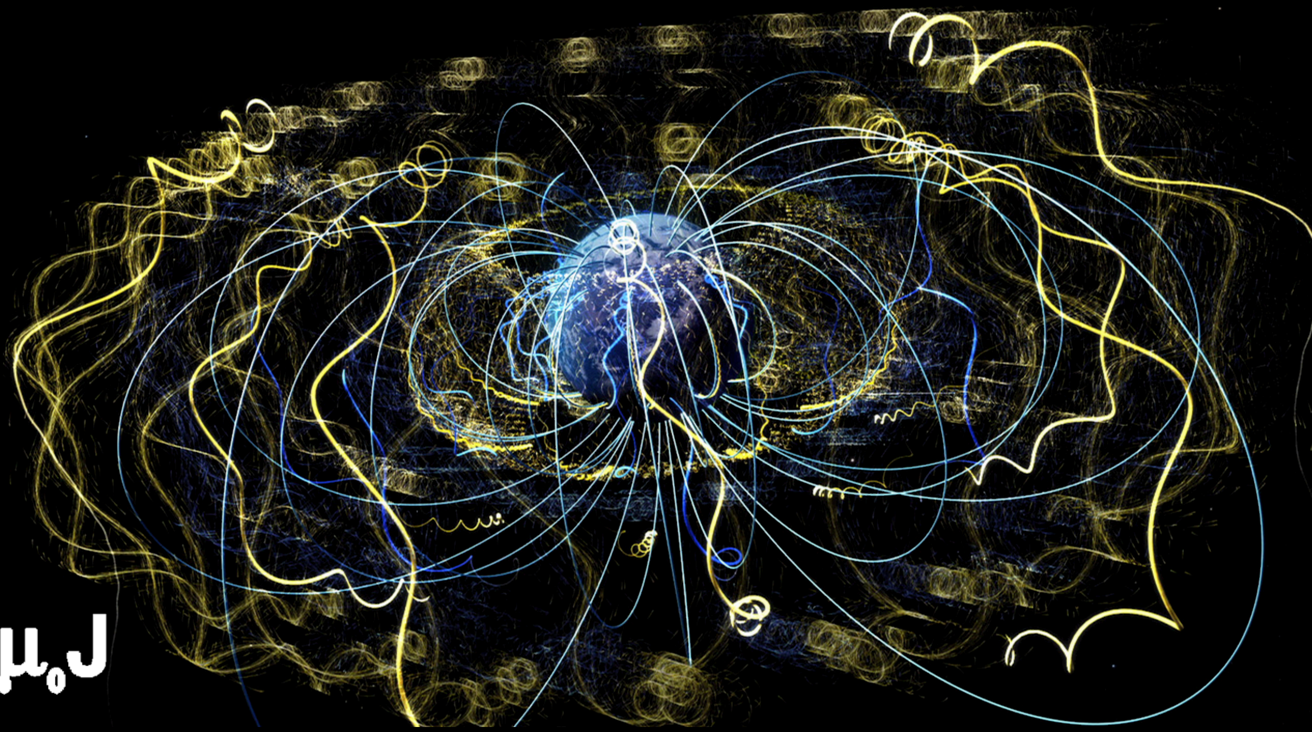
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism I: 3811

Professor Jasper Halekas

Van Allen 301

MWF 9:30-10:20 Lecture

Announcements

- Last year's final exam posted on course website
  - Solutions will follow next week
- Next Week
  - Monday: Guest lecture w/ Prof. Craig Kletzing
  - Wednesday: Problem session w/ Mr. Gian Andreone
  - Friday: Final Review (w/ me)
- Finals week
  - Additional office hours to be announced

Announcements II

- Course Evaluations are now open
  - Please take a few minutes to fill these out
  - The course evaluations are very valuable for me – and potentially for you as well
  - I read all evaluations carefully, and use them to improve my teaching

Boundary Conditions

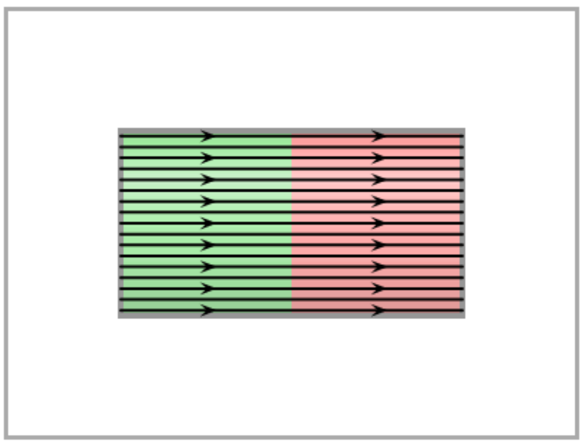
$\Delta B_\perp = 0$ $\qquad$ $\Delta H_\perp = -\Delta M_\perp$

$\Delta \vec{B}_\parallel = \mu_0 \vec{K} \times \hat{n}$ $\qquad$ $\Delta \vec{H}_\parallel = \vec{K}_f \times \hat{n}$

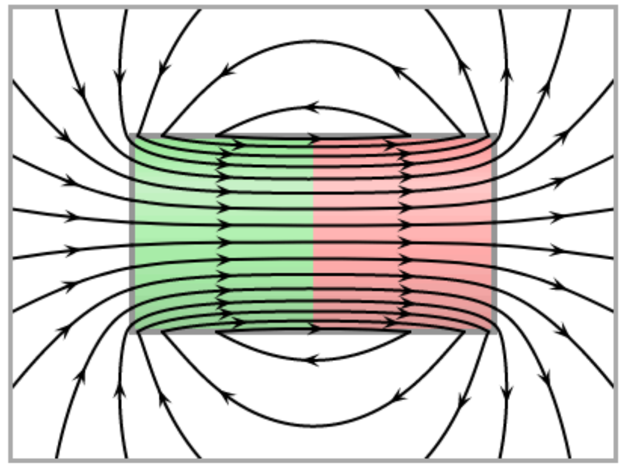
- B continuous
- H can have discontinuities at boundaries of $\vec{M}$

- B can change its tangential component at currents
- H can only change its tangential component at free currents

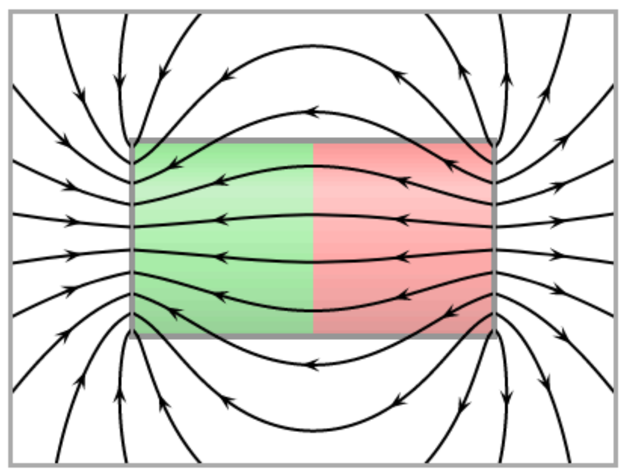
B, H, M



$\vec{M}$



$\vec{B}$



$\vec{H}$

Linear Media

$$\vec{M} = \chi_m \vec{H}$$

χ_m = "magnetic susceptibility"
positive = paramagnetic
negative = diamagnetic

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

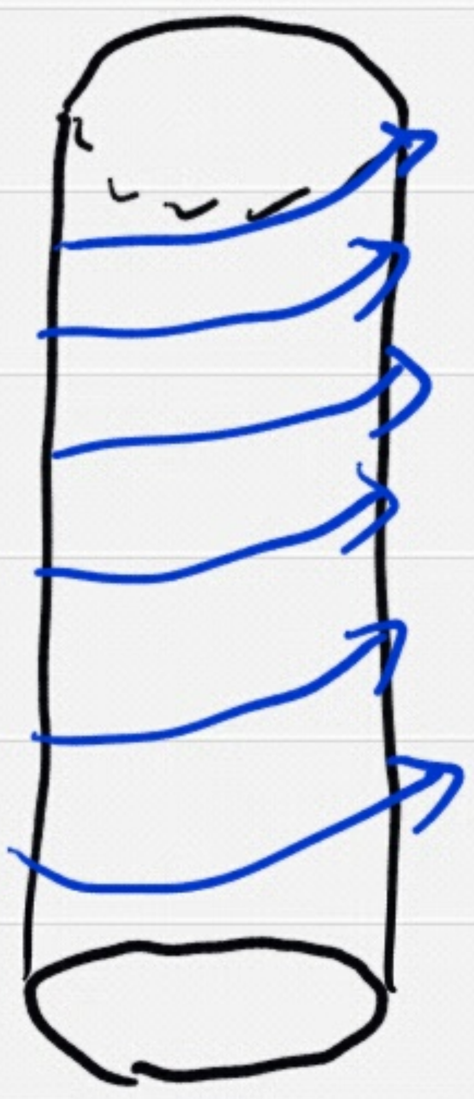
$$= \mu_0 (1 + \chi_m) \vec{H}$$

$$= \mu \vec{H}$$

w/ $\mu = \mu_0(1 + \chi_m)$ = "permeability"

Example

Solenoid w/ linear core



$\vec{H} = nI\hat{z}$
from Ampere's law

$\vec{B} = \mu \vec{H}$
$= \mu_0 (1+\chi_m) nI\hat{z}$

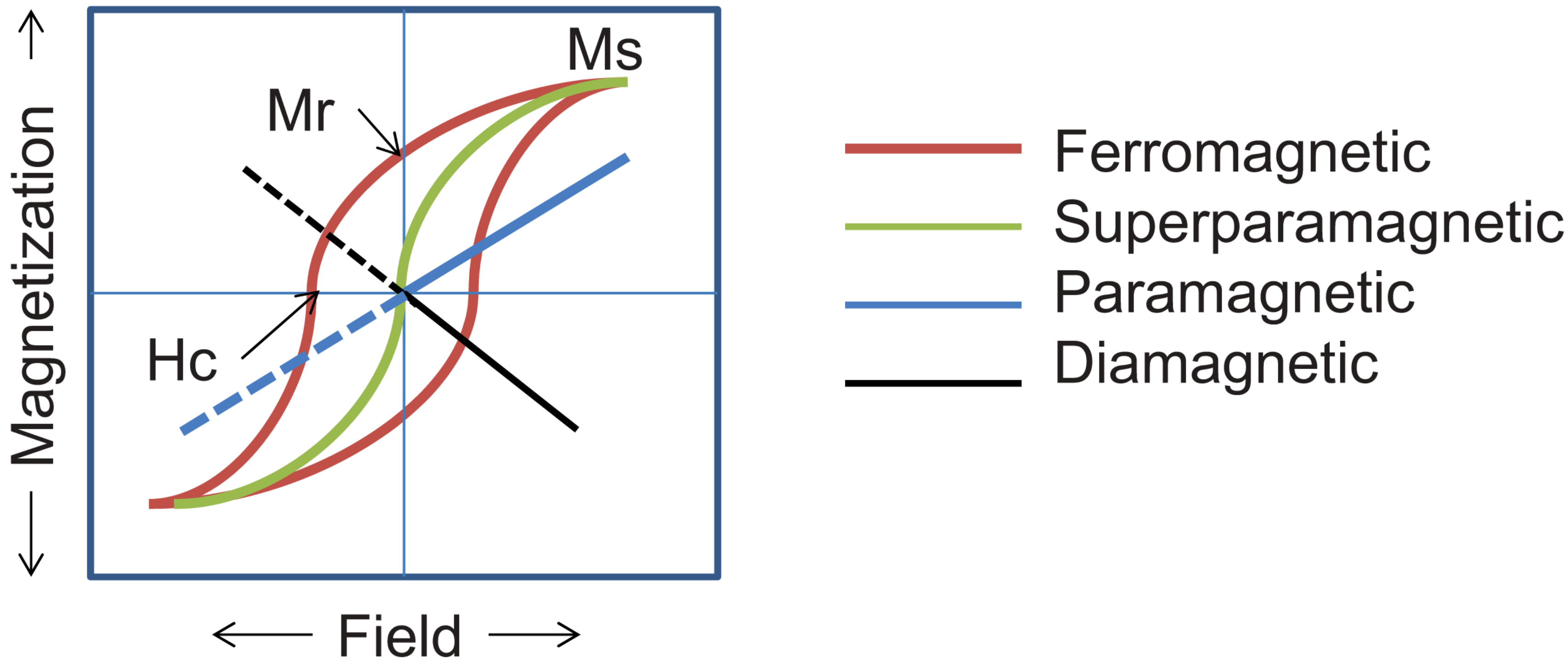
$\chi_m > 0$ increases $\vec{B}$
$\chi_m < 0$ decreases $\vec{B}$

$$\vec{K}_b = \vec{M} \times \hat{n} = \chi_m \vec{H} \times \hat{n}$$
$$= \chi_m n I \hat{\varphi}$$
$$= \chi_m \frac{NI}{L} \hat{\varphi}$$
$$= \chi_m \vec{K}_f$$

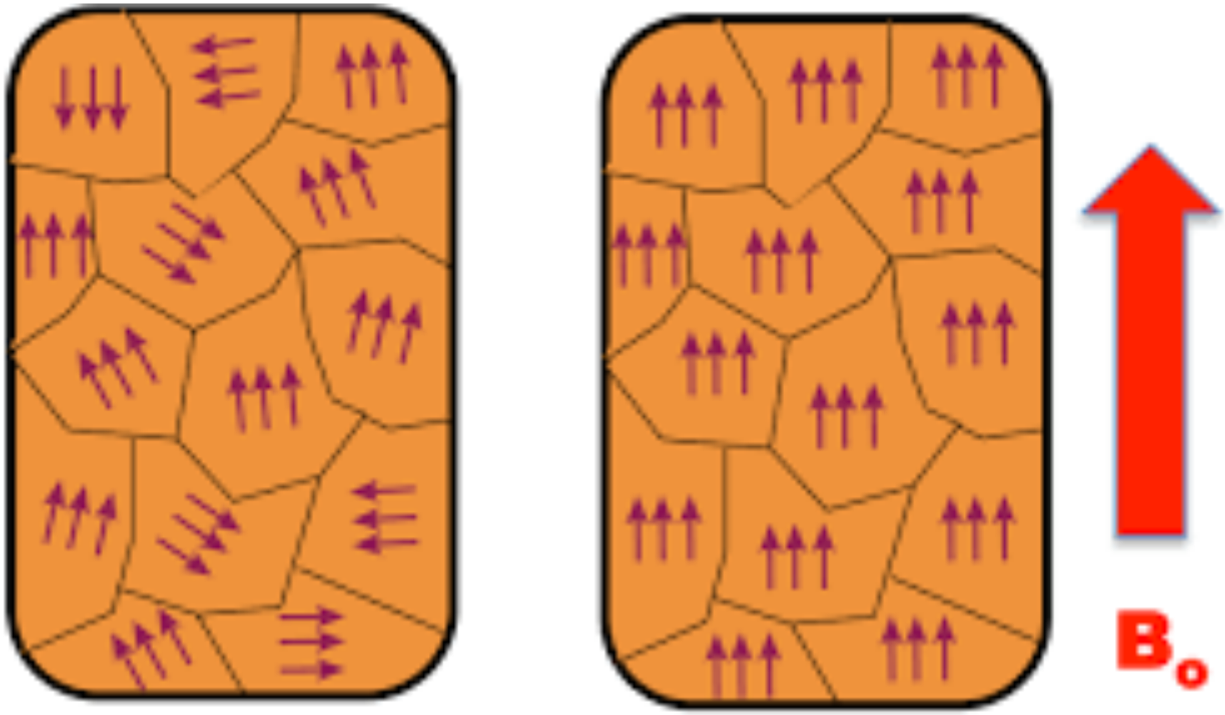
parallel to $\vec{K}_f$ for $\chi_m > 0$
anti-parallel for $\chi_m < 0$

- paramagnetic (or ferromagnetic) core very useful for transformers, inductors, etc.

Magnetic Classification



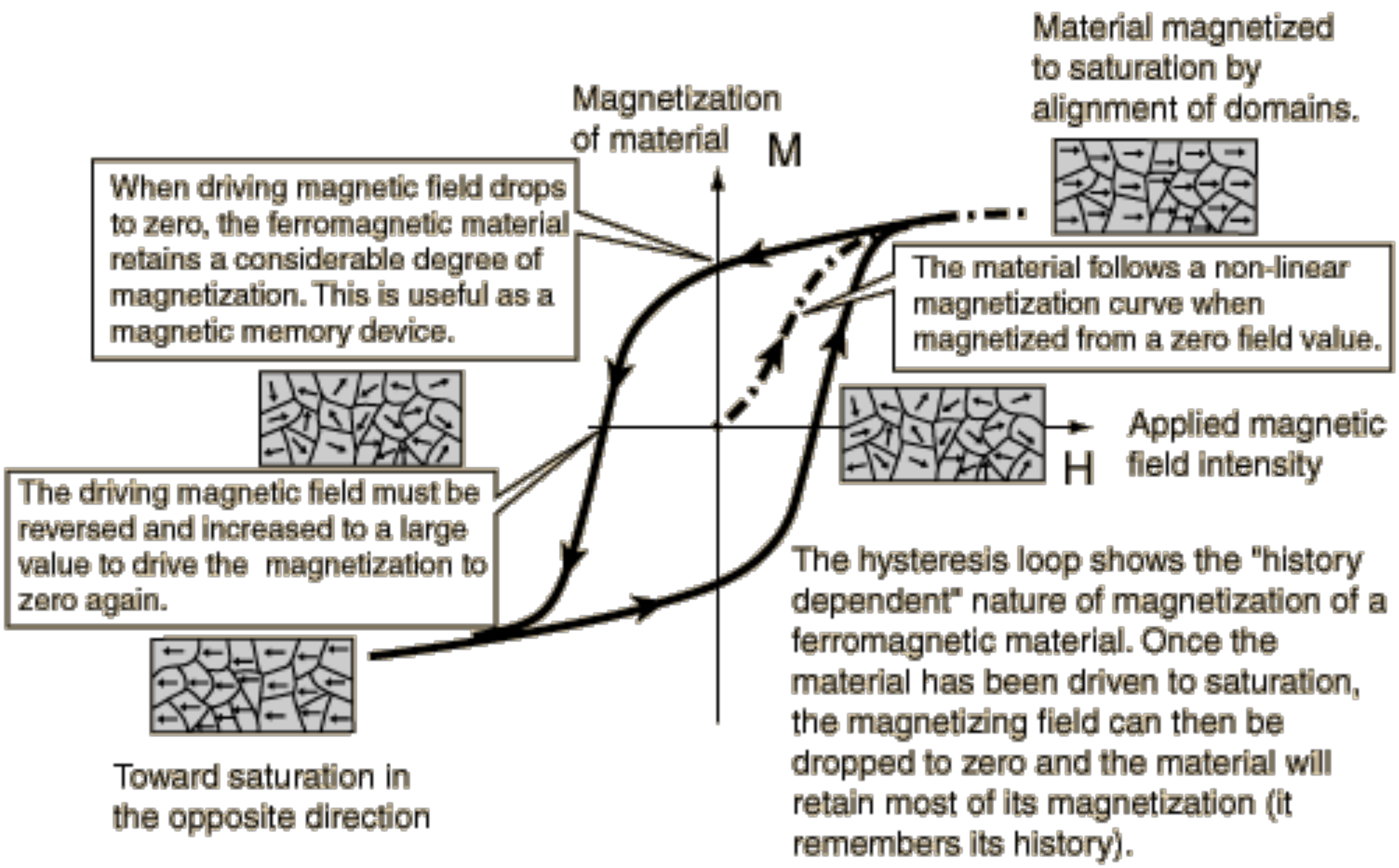
Ferromagnetism



Domains randomly aligned

Domains aligned with external field

Magnetic Hysteresis

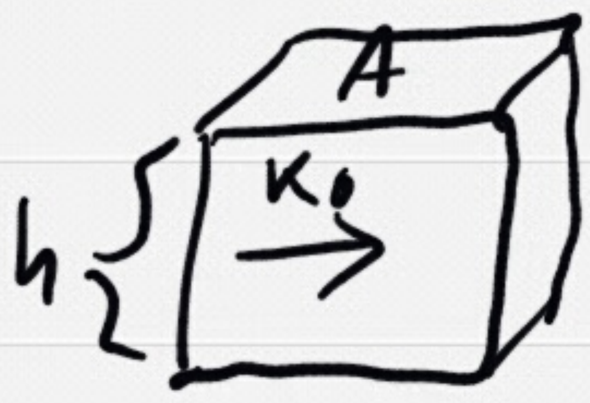


Surface Bound Current

$\vec{K}_0 \parallel \vec{M} \times \hat{n}$

Total current:

$I = K_0 h$

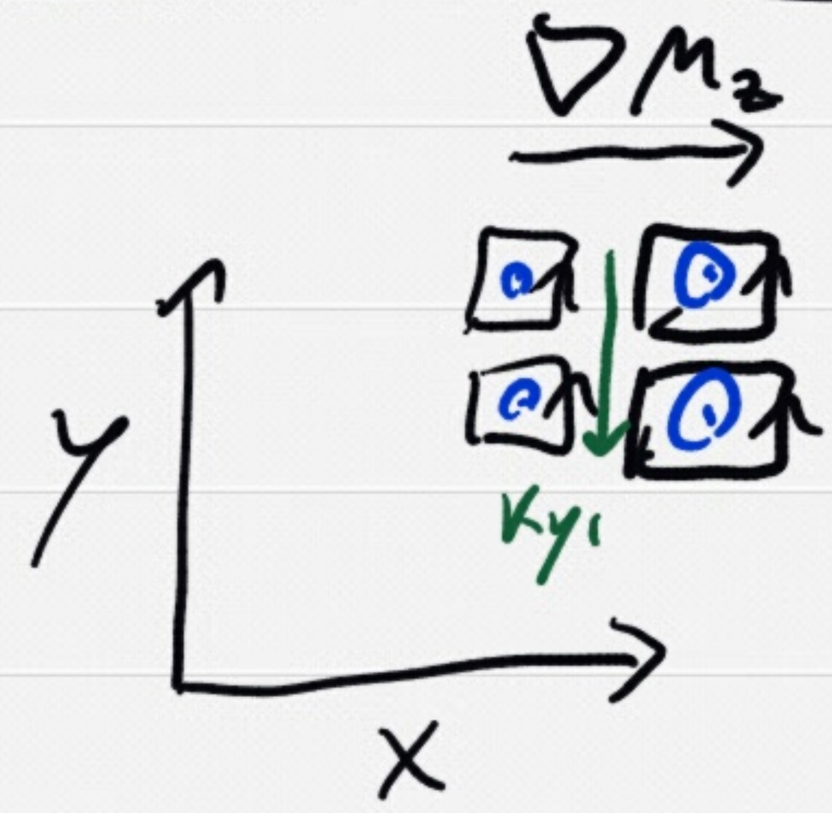


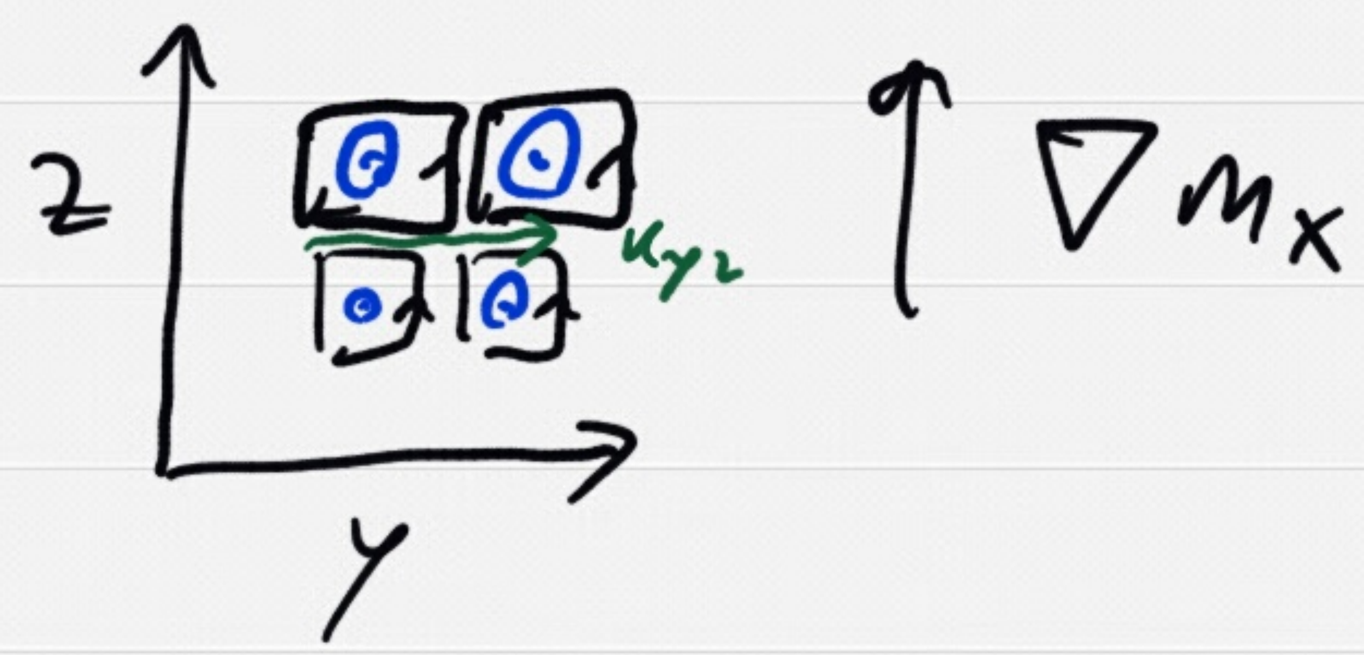
But $I = m/A$

$$= \frac{M \cdot Vol.}{A} = \frac{M \cdot A \cdot h}{A} = Mh$$

$$\Rightarrow |\vec{M}| = |\vec{K}_0| \quad //$$

Volume Bound Current





$$K_{y1} = \frac{I_{y1}}{\Delta z} = M_z(x) - M_z(x+\Delta x)$$

$$= -\partial M_z/\partial x \cdot \Delta x$$

$$K_{y2} = \frac{I_{y2}}{\Delta x} = M_x(z+\Delta z) - M_x(z)$$

$$= \partial M_z/\partial z \cdot \Delta z$$

$$K_y = \frac{I_{y1}}{\Delta z} + \frac{I_{y2}}{\Delta x}$$

$$J_y = \frac{I_{y1} + I_{y2}}{\Delta x \, \Delta z}$$

$$= \left(\partial M_x/\partial z - \partial M_z/\partial x\right)$$

$$= (\nabla \times \bar{M})_y$$

$$\Rightarrow \vec{J}_b = \nabla \times \vec{M} \; //$$