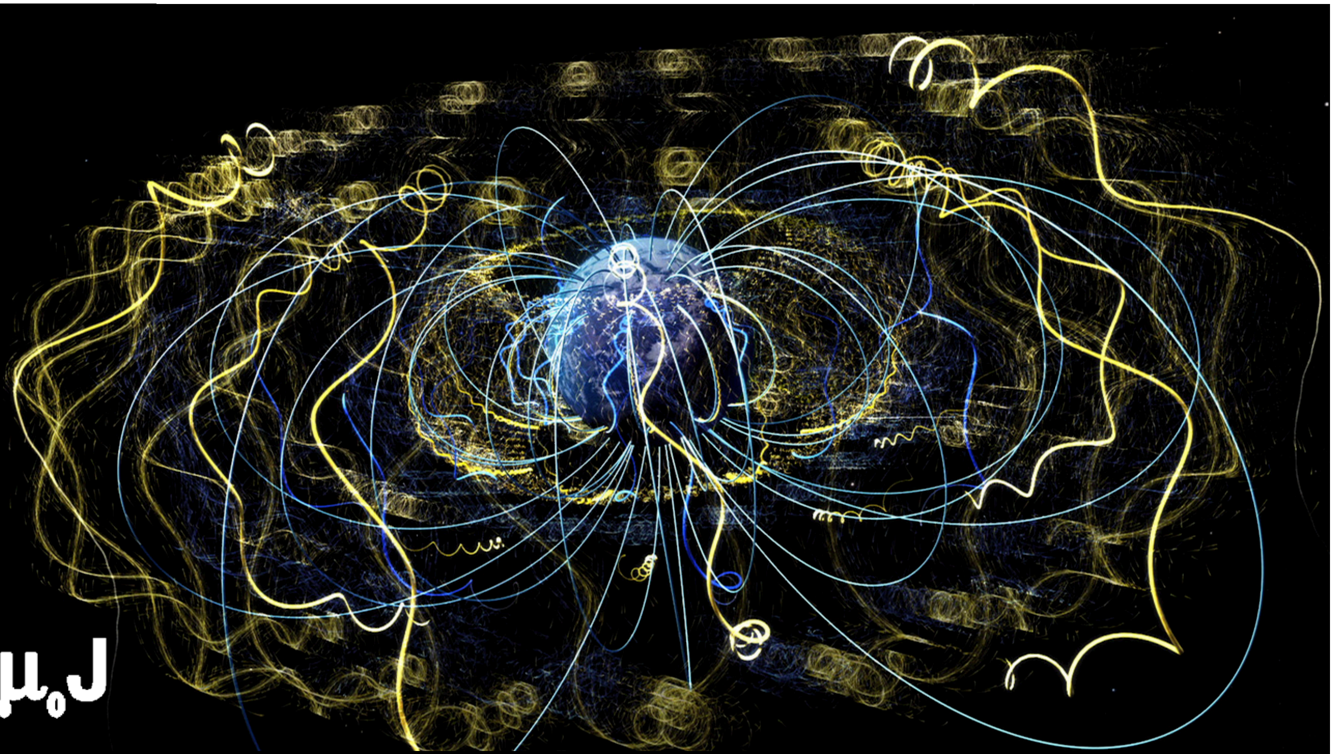
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism I: 3811

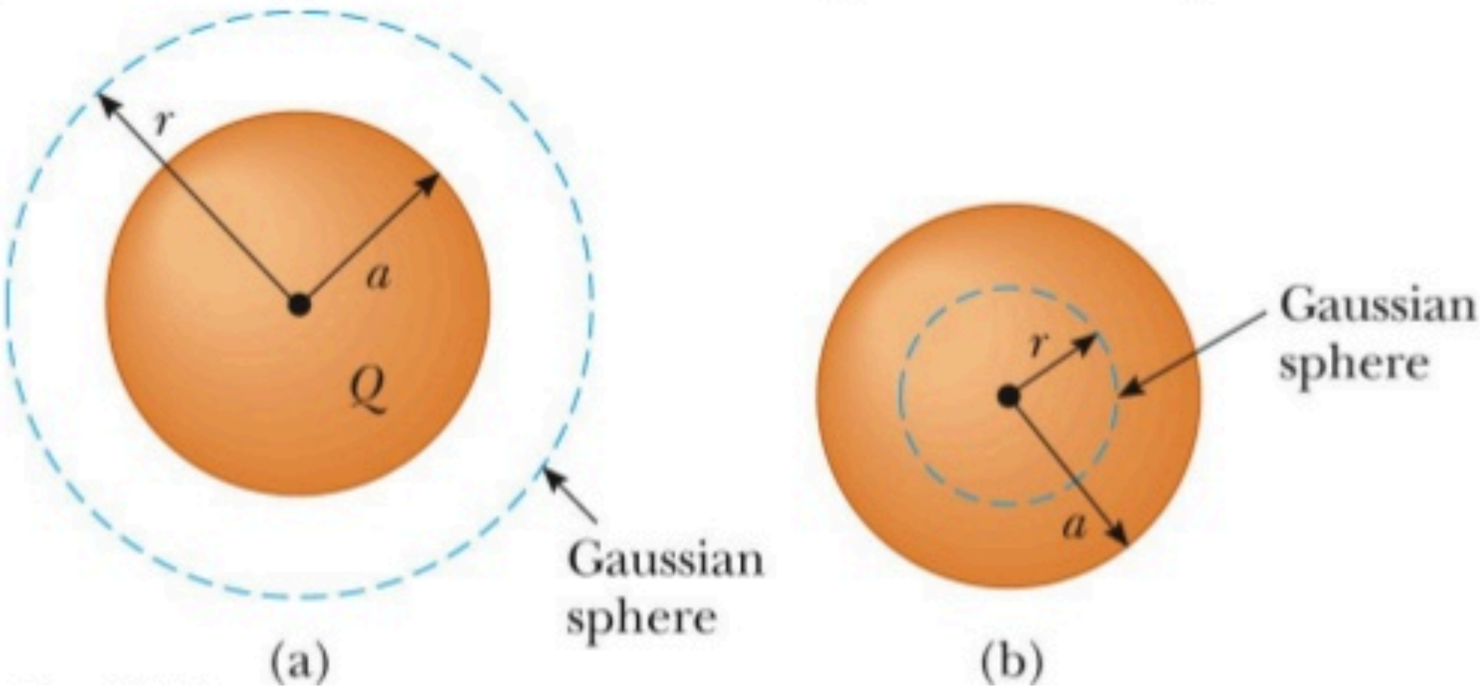
Professor Jasper Halekas
Van Allen 301
MWF 9:30-10:20 Lecture

Check Your Understanding 1

An insulating solid sphere of radius a has a uniform volume charge density ρ and carries total charge Q.

(A) Find the magnitude of the E-field at a point outside the sphere

(B) Find the magnitude of the E-field at a point inside the sphere



Q1: $\oint \vec{E} \cdot d\vec{a} = Q_{enc}/\varepsilon_0$

A. $E \cdot 4\pi r^2 = Q/\varepsilon_0$

$$\Rightarrow \boxed{E = Q/4\pi\varepsilon_0 r^2}$$

B. $E \cdot 4\pi r^2 = Q_{enc}/\varepsilon_0$

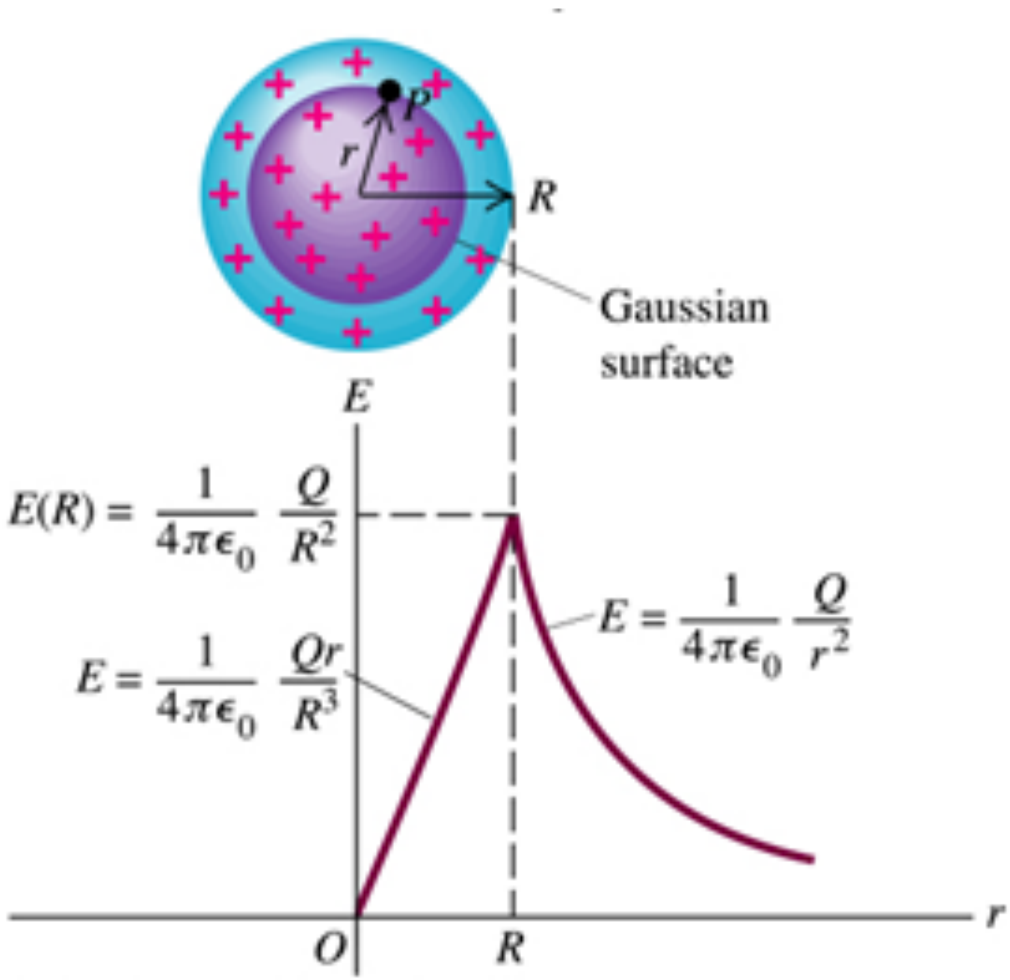
$$= \frac{1}{\varepsilon_0} \int \rho \, d\tau$$

$$= \frac{1}{\varepsilon_0} \int_0^r \int_0^\pi \int_0^{2\pi} \rho \, d\phi \sin\theta \, d\theta \, r'^2 dr'$$

$$= \rho/\varepsilon_0 \cdot 4/3 \pi r^3$$

$$\Rightarrow \boxed{E = \frac{\rho}{3\varepsilon_0} r = \frac{Qr}{4\pi\varepsilon_0 a^3}}$$

Check Your Understanding 2



For the charged sphere of the previous problem, find the electric potential V(o) at the center of the sphere, with respect to infinity.

Q2: $V(0) - V(\infty)$

$$= -\int_{\infty}^{0} \vec{E} \cdot d\vec{l}$$

$$= -\int_{\infty}^{0} E_r dr$$

$$= -\int_{\infty}^{R} \frac{Q}{4\pi\varepsilon_0 r^2} dr - \int_{R}^{0} \frac{Q r \, dr}{4\pi\varepsilon_0 R^3}$$

$$= Q/4\pi\varepsilon_0 r \Big|_{\infty}^{R} - \frac{Q r^2}{8\pi\varepsilon_0 R^3} \Big|_{R}^{0}$$

$$= Q/4\pi\varepsilon_0 R + Q/8\pi\varepsilon_0 R$$

$$= \boxed{3Q/8\pi\varepsilon_0 R}$$

Check Your Understanding 3

- A sphere of radius R has purely radial polarization that increases with distance from the center of the sphere: $P_r(r) = A\,r$
- Find all of the volume and surface bound charge density in and on the sphere.
  - Note $\nabla \cdot \vec{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r}$ for a purely radial field
- Find the electric field E(r) as a function of radius outside the sphere ($r > R$)

Q3: $\vec{P} = Ar\,\hat{r}$

$$\rho_b = -\nabla\cdot\vec{P}$$
$$= -\frac{1}{r^2}\,\partial/\partial r\,(r^2\cdot Ar)$$
$$= \boxed{-3A}$$

$$\sigma_b = \vec{P}\cdot\hat{n}\big|_R = Ar\,\hat{r}\cdot\hat{r}\,\big|_R$$
$$= \boxed{AR}$$

Total bound charge:

$$Q = \int \rho_b\,d\tau + \int \sigma_b\,da$$
$$= -3A\cdot 4/3\,\pi R^3 + AR\cdot 4\pi R^2$$
$$= 0 \quad \text{as it must be}$$

so $\boxed{E(r) = 0 \quad r > R}$

Check Your Understanding #4

- An infinitely long magnetized cylinder of radius R has purely azimuthal magnetization that increases with radius: $M_\phi(s) = A\,s$
- Find all the volume and surface bound current
  - Note $\nabla\times\vec{A} = \left(\frac{1}{s}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial (s\,A_\phi)}{\partial s} - \frac{\partial A_s}{\partial\phi}\right)\hat{z}$
- Find the magnetic field B(s) as a function of radius within the cylinder (s < R)

Q4: $\vec{M} = As\,\hat{\varphi}$

$$\vec{J}_b = \nabla \times \vec{M}$$

$$= \frac{1}{s}\,\partial/\partial s\,(s \cdot As)\,\hat{z}$$

$$= \boxed{2A\,\hat{z}}$$

$$\vec{K}_b = \vec{M} \times \hat{n}$$

$$= As\,\hat{\varphi} \times \hat{s}\,\Big|_R$$

$$= \boxed{-AR\,\hat{z}}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

$$= \mu_0 \int \vec{J} \cdot d\vec{a}$$

$$\Rightarrow B \cdot 2\pi s = \mu_0 \cdot 2A \cdot \pi s^2$$

$$\Rightarrow \boxed{\vec{B} = \mu_0 A s\,\hat{\varphi} \qquad s<R}$$

- Note $\vec{B} = \mu_0 \vec{M}$ so $\vec{H} = 0$

- Must be true since no free current present