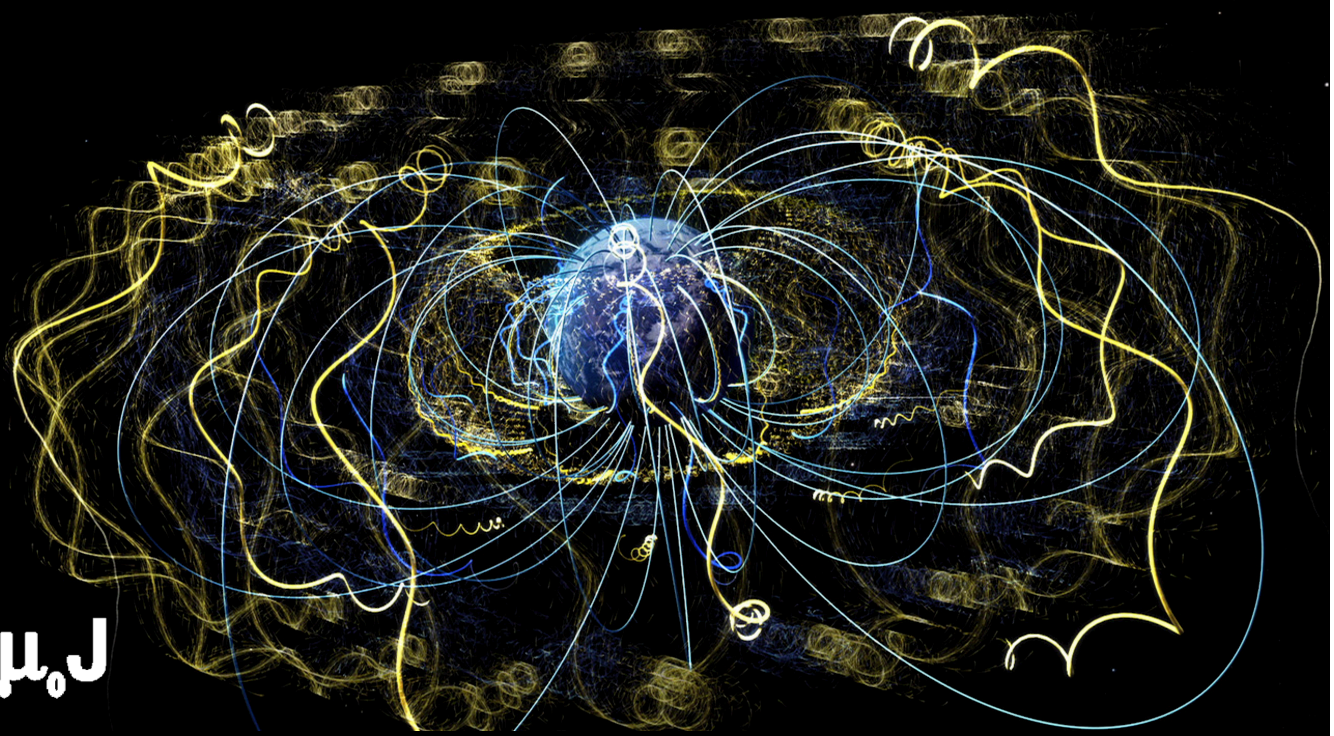
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

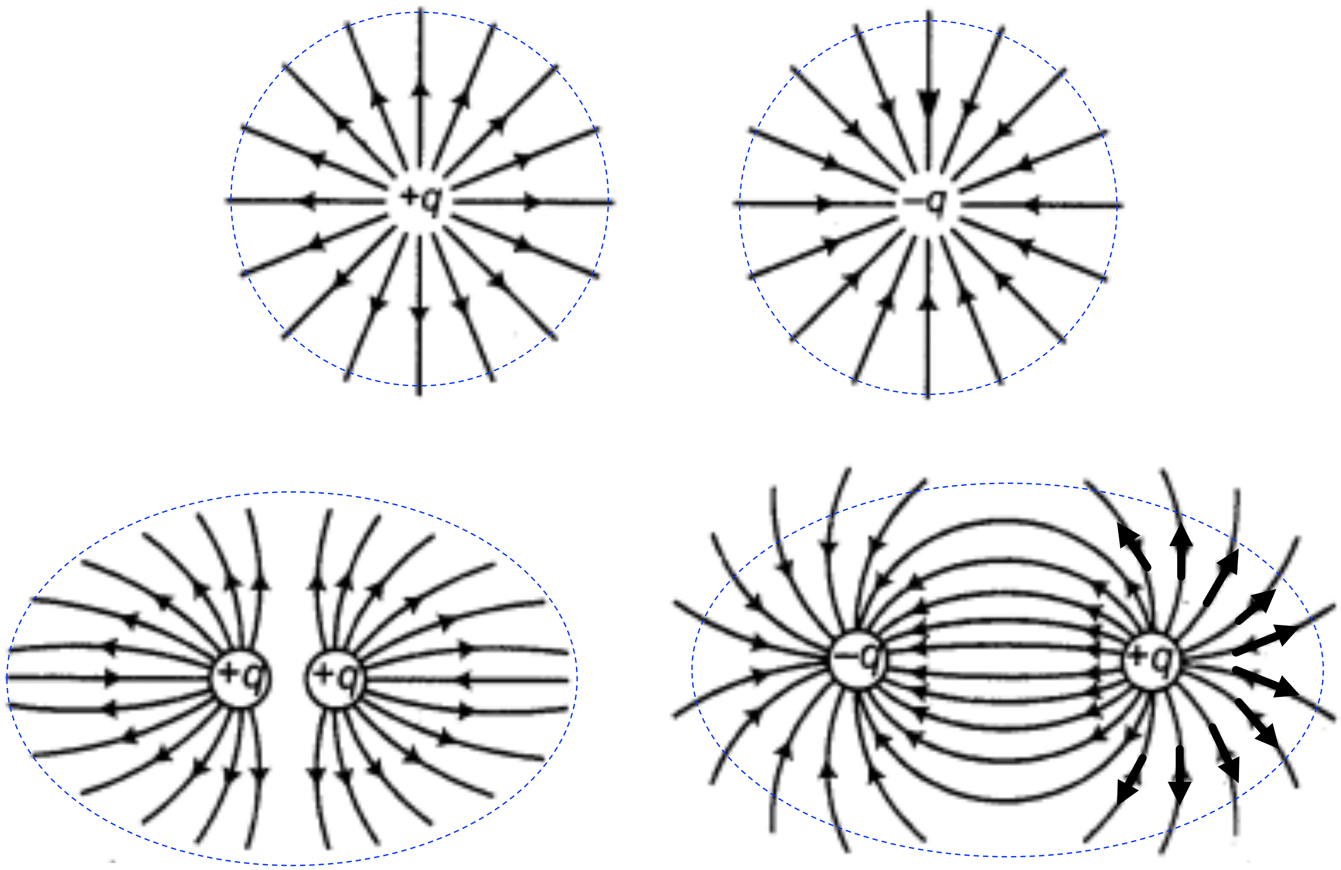
$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism I: 3811

Professor Jasper Halekas
Van Allen 301
MWF 9:30-10:20 Lecture

Electric Field Lines and Gauss's Law



Charge enclosed -> electric field lines outward -> electric flux through surface

Gauss's Law

Point charge at origin

$$\oint \vec{E} \cdot d\vec{a} = \oint \frac{1}{4\pi\varepsilon_0} \frac{q\hat{r}}{r^2} \cdot d\vec{a}$$

Sphere of radius R

$$d\vec{a} = R^2 \sin\theta \, d\theta \, d\varphi \, \hat{r}$$

$$\Rightarrow \oint \vec{E} \cdot d\vec{a} = \oint \frac{1}{4\pi\varepsilon_0} \cdot q \cdot \frac{R^2}{R^2} \sin\theta \, d\theta \, d\varphi$$

$$= q/\varepsilon_0$$ independent of R

- Gauss's Law says this is true for any distribution of charge or surface

$$\oint \vec{E} \cdot d\vec{a} = Q_{enc}/\varepsilon_0$$

Rigorous Derivation

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\hat{\Delta r}}{\Delta r^2} \rho(\vec{r}\,') \, d\tau'$$

$$\nabla \cdot \vec{E} = \frac{1}{4\pi\varepsilon_0} \nabla \cdot \int \frac{\hat{\Delta r}}{\Delta r^2} \rho(\vec{r}\,') \, d\tau'$$

$$= \frac{1}{4\pi\varepsilon_0} \int \nabla \cdot \left(\frac{\hat{\Delta r}}{\Delta r^2} \rho(\vec{r}\,') \right) d\tau'$$

$$\nabla \cdot \left(\frac{\hat{\Delta r}}{\Delta r^2} \rho(\vec{r}\,') \right)$$

$$= \rho(\vec{r}\,') \nabla \cdot \left(\frac{\hat{\Delta r}}{\Delta r^2} \right) + \frac{\hat{\Delta r}}{\Delta r^2} \cdot \nabla \rho(\vec{r}\,')$$

But $\nabla \rho(\vec{r}\,') = 0$ since $\rho(\vec{r}\,')$ doesn't depend on $\vec{r}$

Meanwhile $\nabla \cdot \left(\frac{\hat{\Delta r}}{\Delta r^2} \right) = 4\pi \delta^3(\vec{\Delta r})$

$$\Rightarrow \nabla \cdot \vec{E} = \frac{1}{4\pi\varepsilon_0} \int \rho(\vec{r}\,') \cdot 4\pi \delta^3(\vec{\Delta r}) \, d\tau'$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot 4\pi \cdot \rho(\vec{r})$$

$$\boxed{\nabla \cdot \vec{E} = \rho(\vec{r}) / \varepsilon_0}$$

Divergence Theorem

$$\int_V \nabla \cdot \vec{E} \, d\tau = \int_V \rho(\vec{r}) / \varepsilon_0 \, d\tau$$

$$\Rightarrow \boxed{\oint \vec{E} \cdot d\vec{a} = Q_{enc} / \varepsilon_0}$$

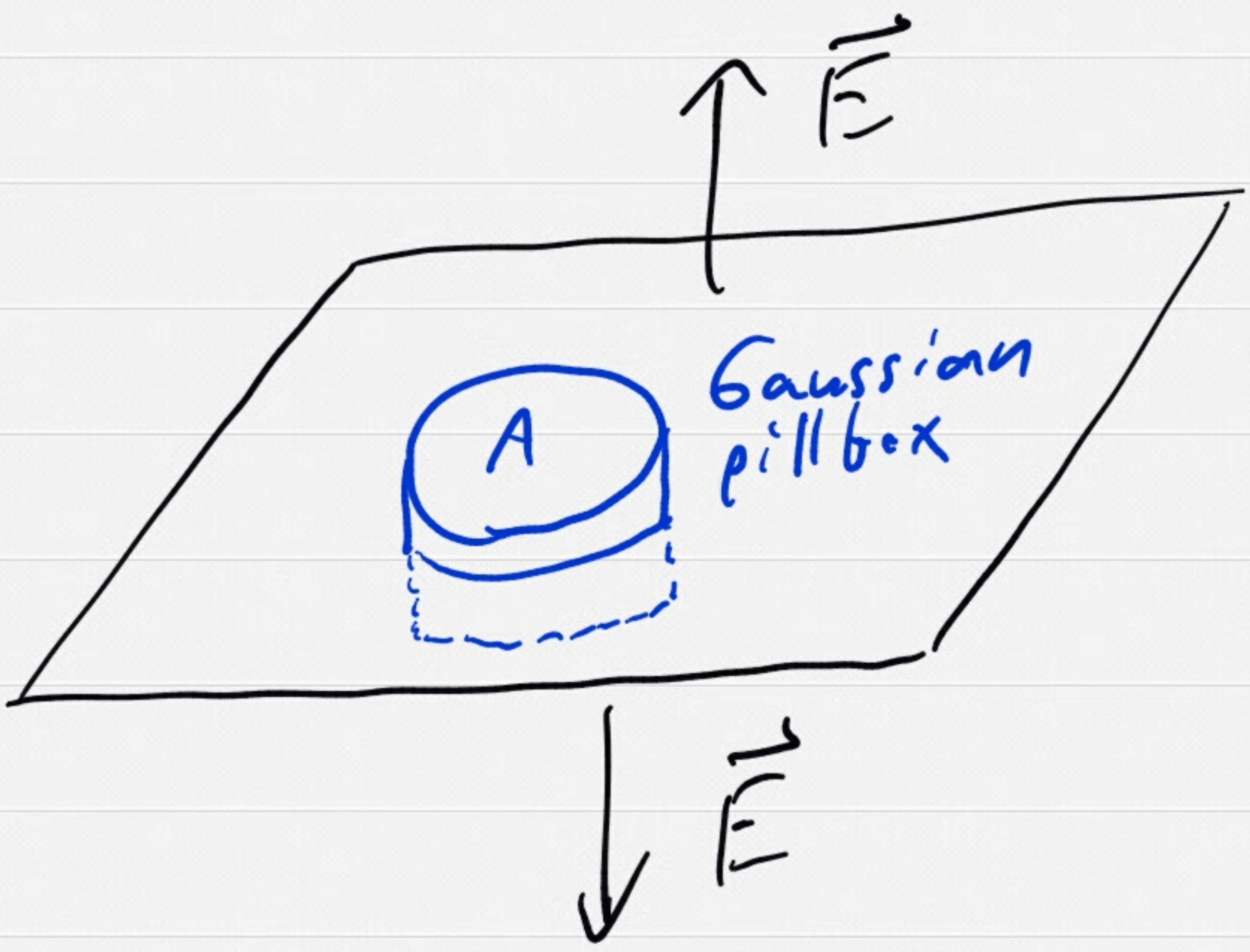
Gauss's Law

Always true!

Only sometimes useful

Need symmetry to exploit it.

Infinite Plane



Charge density σ
$= Q/A$

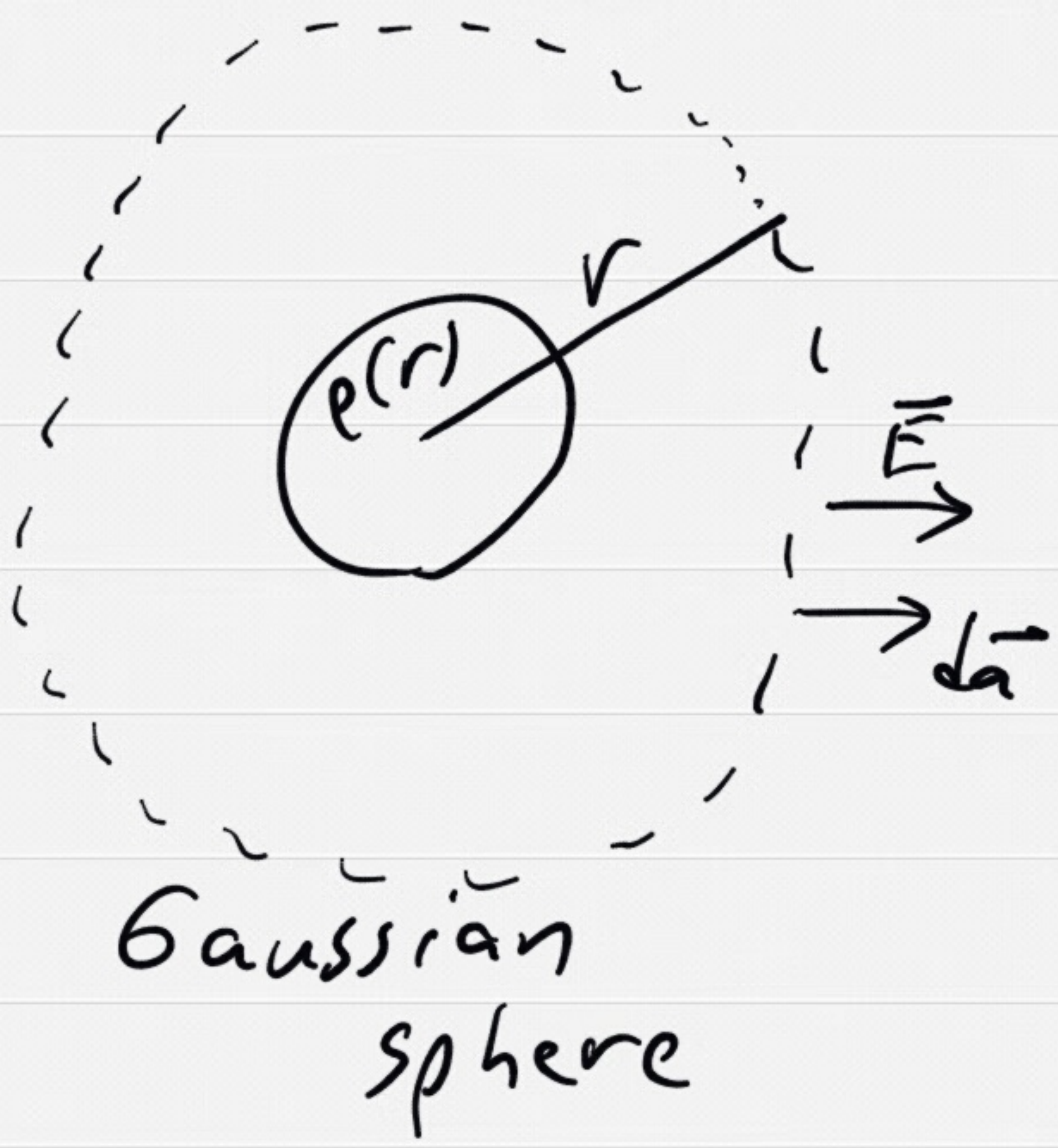
$$\oint \vec{E} \cdot d\vec{a} = \int_{top} \vec{E} \cdot d\vec{a} + \int_{bottom} \vec{E} \cdot d\vec{a} + \int_{sides} \vec{E} \cdot d\vec{a}$$

$$= EA + EA$$

$$= Q_{enc}/\varepsilon_0 = \sigma A/\varepsilon_0$$

$$\Rightarrow \boxed{E = \sigma/2\varepsilon_0}$$

Spherical Symmetry



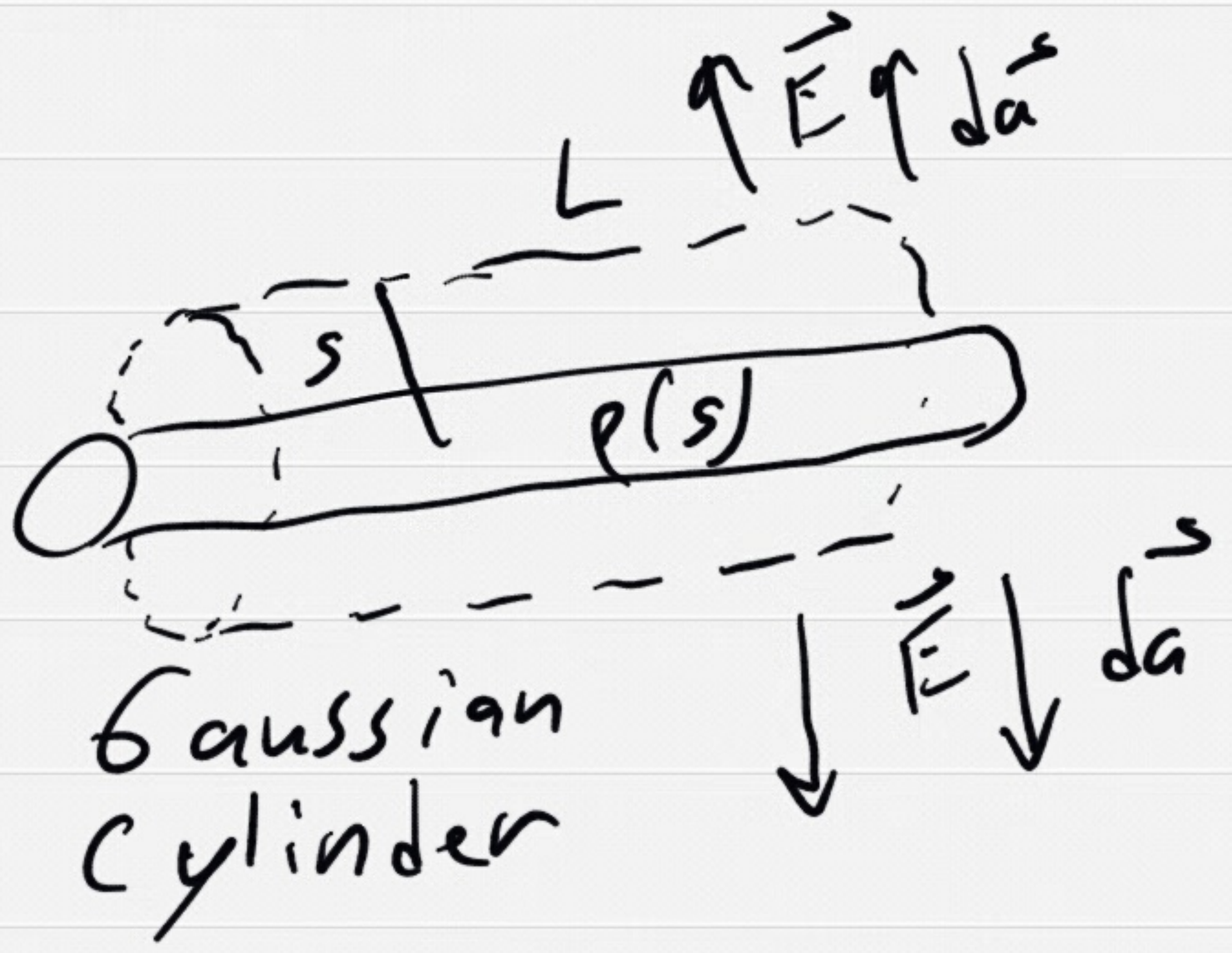
$$\oint \vec{E} \cdot d\vec{a}$$
$$= \int E \cdot r^2 \sin\theta \, d\theta \, d\phi$$
$$= E \cdot 4\pi r^2$$
$$= Q/\epsilon_0 \quad \text{w/} \quad Q = \int_V \rho(r) \, d\tau$$

$$\Rightarrow \boxed{\vec{E}(r) = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}}$$

- Doesn't matter how Q is distributed as long as it's spherically symmetric so that E is radial

- Any spherically symmetric charge distribution looks like a point charge from outside

Cylindrical Symmetry



$$\oint \vec{E} \cdot d\vec{a} = \int E \cdot s \, d\varphi \, dz$$

(neglecting ends)

$$= 2\pi s L \cdot E$$

$$= Q_{enc}/\varepsilon_0 \quad \text{w/ } Q = \int_V \rho \, d\tau$$

$$\Rightarrow \vec{E}(s) = \frac{Q}{2\pi s L \varepsilon_0} \hat{s}$$

$$= \frac{\lambda L}{2\pi s L \varepsilon_0} \hat{s} \quad \text{w/ } \lambda = Q/L$$

$$= \boxed{\frac{\lambda \hat{s}}{2\pi \varepsilon_0 s}}$$

— Any cylindrically symmetric distribution of charge looks like a wire from outside