

## Electricity and Magnetism II [3812] Practice Midterm 2

### Directions:

This exam is closed book. You are allowed a copy of the latest equation sheet posted on the course website. You may annotate your equation sheet.

Read all the questions carefully and answer every part of each question. Show your work on all problems – partial credit may be granted for correct logic or intermediate steps, even if your final answer is incorrect.

Unless otherwise instructed, express your answers in terms of fundamental constants like  $\mu_0$  and  $\epsilon_0$ , rather than calculating numerical values.

If the question asks for an explanation, write at least one full sentence explaining your reasoning.

Please ask if you have any questions, including clarification about the instructions, during the exam.

This test is designed to be gender and race neutral.

## Good luck!

**Honor Pledge:** I understand that sharing information with anyone during this exam by talking, looking at someone else's test, or any other form of communication, will be interpreted as evidence of cheating. I also understand that if I am caught cheating, the result will be no credit (0 points) for this test, and disciplinary action may result.

**Sign Your Name** \_\_\_\_\_

**Print Your Name** \_\_\_\_\_

**Question 1 (30 points):**

**1a (15 points).** Use the complex forms for the electric and magnetic fields of an electromagnetic plane wave in a linear dielectric material with permittivity  $\epsilon$  and permeability  $\mu$  to find the ratio of the energy density stored in the oscillating magnetic field to the energy density stored in the oscillating electric field.

**1b (15 points).** If the plane wave travels from the dielectric material into vacuum, at normal incidence, what fraction of the electromagnetic energy is reflected, and what fraction is transmitted? You may express this in terms of  $\beta$ , but if you do, please explicitly show what  $\beta$  is in this case.

**Problem 2 (35 points):** This problem addresses the following scalar and vector potentials, valid on the x-axis, for  $x > vt$  ( $B$  and  $v$  are constants,  $c$  is the speed of light):

$$V(x, t) = \frac{B}{x-vt} \quad \vec{A}(x, t) = \frac{Bv}{c^2(x-vt)} \hat{x}$$

**2a (10 points):** Find the electric and magnetic fields corresponding to these potentials, along the x-axis, for  $x > vt$ .

**2b (10 points):** Are these potentials in the Coulomb or Lorenz gauge, or both, or neither?

**2c (15 points):** Show that for an appropriate choice of  $B$ , these are in fact the Liénard-Wiechert potentials for a particle traveling at constant velocity along the x-axis, evaluated along the x-axis, for  $x > vt$ .

**Problem 3 (35 points):** Imagine the following model for electric dipole radiation. A single point charge  $q$  executes harmonic motion along the z-axis (centered at the origin), so its position can be expressed as  $z(t) = d \cos(\omega t)$ , with  $d$  a constant.

**3a (20 points).** Find the corresponding dipole moment  $\vec{p} = \sum q_i \vec{r}_i$  as a function of time, and then use the multipole approximation to find a first order approximation for the Poynting flux radiated from this configuration as a function of position and time.

**3b (15 points).** Assuming  $v \ll c$  and  $d \ll r$ , calculate the Poynting flux radiated from an accelerating point charge in this configuration, as a function of position and time, and show that it matches the answer given above.