Electricity and Magnetism II [3812] Midterm 1
Wednesday February 26, 2020

Directions:

This exam is closed book. You are allowed a copy of the latest equation sheet posted on the course website. You may annotate your equation sheet.

Read all the questions carefully and answer every part of each question. Show your work on all problems – partial credit may be granted for correct logic or intermediate steps, even if your final answer is incorrect.

Unless otherwise instructed, express your answers in terms of fundamental constants like μ_0 and ε_0, rather than calculating numerical values.

If the question asks for an explanation, write at least one full sentence explaining your reasoning.

Please ask if you have any questions, including clarification about the instructions, during the exam.

<u>This test is designed to be gender and race neutral.</u>

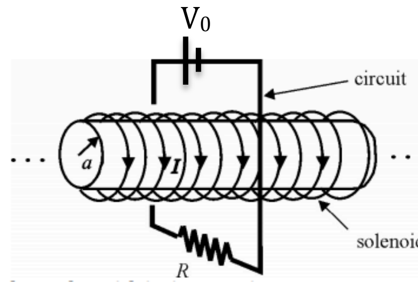
Good luck!

Honor Pledge: I understand that sharing information with anyone during this exam by talking, looking at someone else's test, or any other form of communication, will be interpreted as evidence of cheating. I also understand that if I am caught cheating, the result will be no credit (0 points) for this test, and disciplinary action may result.

Sign Your Name____________________________________

Print Your Name____________________________________

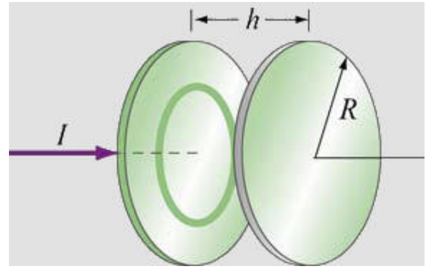
Question 1 (25 points): A circuit consists of a battery with an EMF V_0, and a resistance *R*. A solenoid of radius *a*, with *n* turns per length, is introduced into the middle of the circuit, as shown. The initial current in the circuit $I_c(0) = V_0/R$, and the solenoid current $I_s(t)$, are both clockwise as shown.



1a (15 points): If the solenoid current increases linearly with time as $I_s(t) = kt$, what is the induced EMF in the circuit? *Hint: The magnetic field magnitude in the solenoid is* $B = \mu_0 n I_s$.

1b (10 points): What is the current $I_c(t)$ in the circuit, as the solenoid current $I_s(t)$ is increasing?

Question 2 (40 points): A parallel plate capacitor has plates of radius *R*, separated by a distance *h*. The capacitor is charged by a voltage source that produces a voltage drop between the plates of $\Delta V = V_0 \sin(\omega t)$.



2a (5 points): Find the electric field $\vec{E}(t)$ between the plates from this voltage source, assuming the plates are close enough that the electric field is uniform between them.

2b (15 points): Find the magnetic field $\vec{B}(s,t)$ induced by the changing electric field from 2a, as a function of radius *s* and time *t*, between the plates.

2c (5 points): Find the Poynting vector $\vec{S}(s,t)$ between the plates due to the fields from 2a and 2b.

2d (5 points): Find the electromagnetic energy density $u(s, t)$ between the plates corresponding to the fields from 2a and 2b.

2e (10 points): Show that the continuity equation $\frac{\partial u}{\partial t} = -\nabla \cdot \vec{S}$ is satisfied everywhere between the plates, if you drop all terms of order $(\omega/c)^2$ or higher in the energy density. *Note: This approximation is necessary because we haven't accounted for second-order electric fields induced by the changing magnetic field between the plates*.

Question 3 (35 points): An electromagnetic wave with wave number *k* and angular frequency ω carries electromagnetic momentum density $\vec{g} = g_0 cos^2(kz - \omega t)\hat{z}$.

3a (10 points): If the polarization vector of the wave is $\hat{n} = \hat{y}$, find the complete real forms of the vector electric and magnetic fields, as a function of position and time.

3b (10 points): Find the T_{zz} component of the Maxwell stress tensor.

3c (5 points): Using your result from 3b, find the magnitude and direction of the total force exerted by the wave, if it is absorbed on a surface of area A lying in the x-y plane.

3d (10 points): Show that your result from 3c gives the correct time-averaged radiation pressure on the surface.