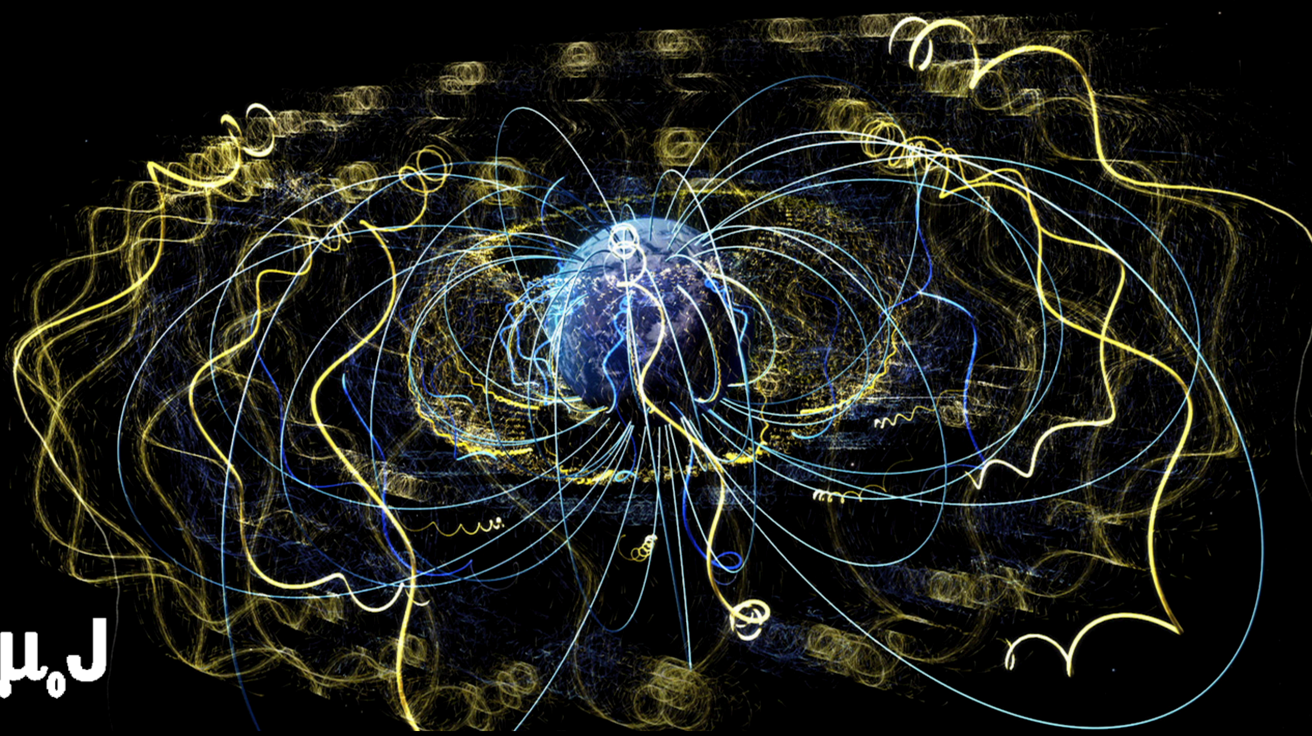
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

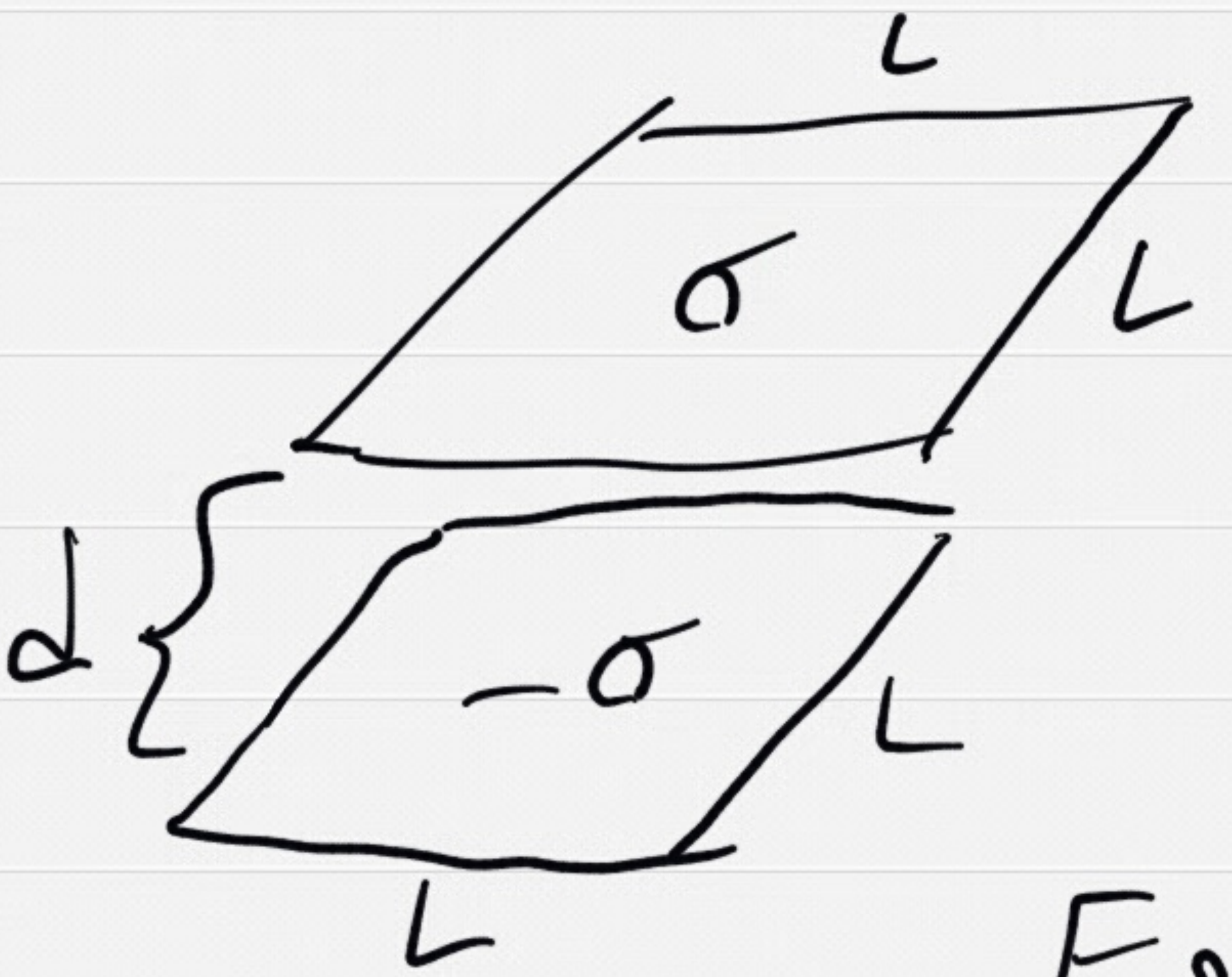
$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism II: 3812

Professor Jasper Halekas
Van Allen 70
MWF 9:30-10:20 Lecture

Example: Capacitor

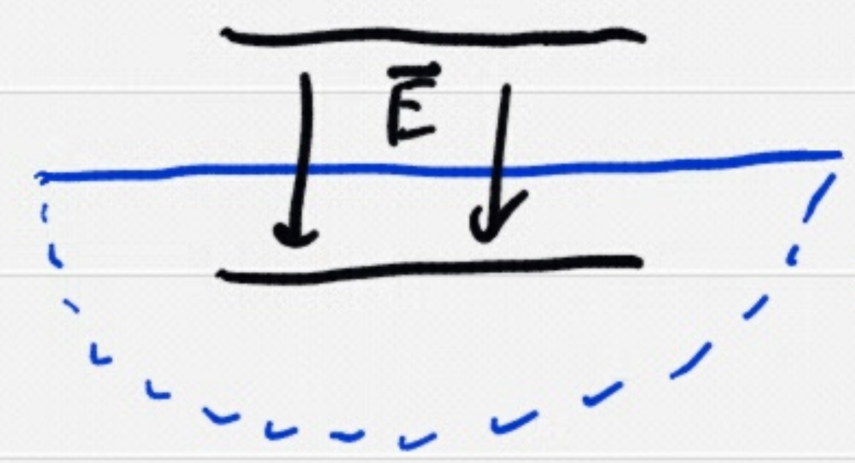


$\vec{E} = -\sigma/\varepsilon_0 \, \hat{z}$

between plates

Force on bottom plate

$$\vec{F} = \oint \overleftrightarrow{T} \cdot d\vec{a}$$



$$F_z = \oint \left(T_{zx} \, da_x + T_{zy} \, da_y + T_{zz} \, da_z \right)$$

$$= \oint T_{zz} \, da_z$$

($da_x = da_y = 0$ for surface between plates)

$$T_{zz} = \varepsilon_0 \left(E_z E_z - \tfrac{1}{2} \delta_{zz} E^2 \right)$$

$$= \frac{\varepsilon_0}{2} \left(E_z^2 - E_x^2 - E_y^2 \right)$$

$$= \frac{\varepsilon_0}{2} \left(\frac{\sigma}{\varepsilon_0} \right)^2$$

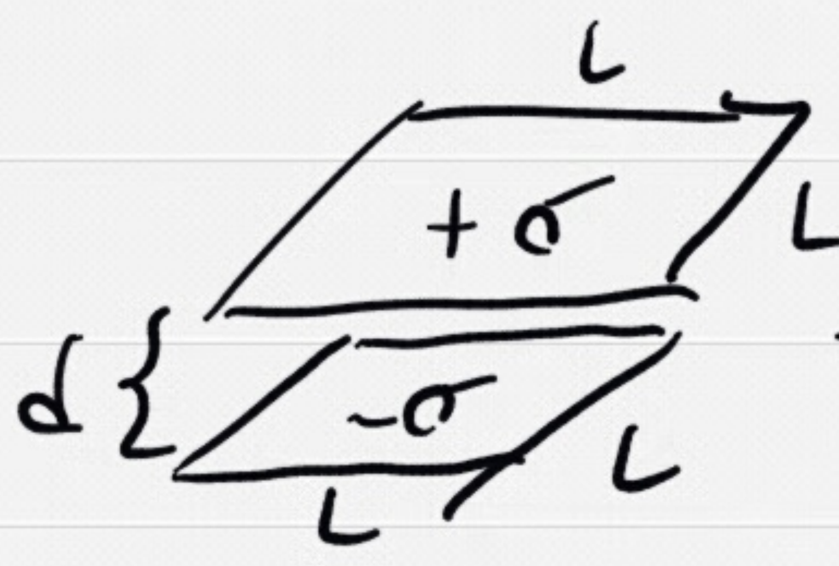
$$= \frac{\sigma^2}{2\varepsilon_0}$$

$$\boxed{F_z = \oint T_{zz} \, da_z = \frac{\sigma^2 L^2}{2\varepsilon_0}}$$

Note $F_z = \sigma L^2 \cdot \frac{\sigma}{2\varepsilon_0}$

$= Q_{bot} \cdot E_{top}$

Example: Moving Capacitor



$\longrightarrow \vec{v} = v\hat{y}$

Between plates: $\vec{E} = -\sigma/\varepsilon_0 \, \hat{z}$

$\vec{B} = -\mu_0 \sigma v \, \hat{x}$

$$\Rightarrow \vec{g}_{EM} = \mu_0 \varepsilon_0 \vec{S}$$
$$= \varepsilon_0 (\vec{E} \times \vec{B})$$
$$= \mu_0 \sigma^2 v \hat{y}$$

$$\vec{P}_{EM} = \vec{g}_{EM} \cdot \text{volume}$$
$$= \mu_0 \sigma^2 v d L^2 \hat{y}$$

Now move top plate down w/ $u_z = -u\hat{z}$

$$F_{yb} = Q_{top}(\hat{u}_z \times \vec{B}_x) \quad \text{on top plate}$$
$$= \sigma L^2 \cdot (-u\hat{z} \times -\tfrac{1}{2}\mu_0 \sigma v \hat{x})$$

↑ average field @ top plate

$$= \tfrac{1}{2}\mu_0 \sigma^2 L^2 u v \hat{y}$$

$$\Delta P_{yb} = \int F_{yb} \, dt = \tfrac{1}{2}\mu_0 \sigma^2 L^2 v \int u \, dt$$
$$= \tfrac{1}{2}\mu_0 \sigma^2 L^2 v d$$

— Also $\vec{E}$ from $\partial \vec{B}/\partial t$ as top plate moves

$$\vec{B}_{above} = 0$$
$$\vec{B}_{below} = -\mu_0 \sigma v \hat{x}$$

$$\oint \vec{E} \cdot d\vec{l} = d/dt \int \vec{B} \cdot d\vec{a}$$
$$(E_{above} - E_{below}) \cdot L = d/dt\,(B_{below} \cdot L \cdot d)$$
$$= B_{below} \cdot L \cdot -u$$
$$\Rightarrow E_{above} - E_{below} = B_{below} \cdot -u$$
$$= \mu_0 \sigma v u$$

$$\Rightarrow \vec{E}_{above} = \tfrac{1}{2} \mu_0 \sigma v u \hat{y}$$
$$\vec{E}_{below} = -\tfrac{1}{2} \mu_0 \sigma v u \hat{y}$$

$F_{ye} = Q_{bottom} \cdot E_{below}$ on bottom plate

$$= -\sigma L^2 \cdot -\tfrac{1}{2} \mu_0 \sigma v u \hat{y}$$
$$= \tfrac{1}{2} \mu_0 \sigma^2 L^2 v u \hat{y}$$

$$\Rightarrow \Delta p_{ye} = \tfrac{1}{2} \mu_0 \sigma^2 L^2 v d \hat{y}$$

$$\Rightarrow \Delta p_y = \mu_0 \sigma^2 L^2 v d \hat{y}$$

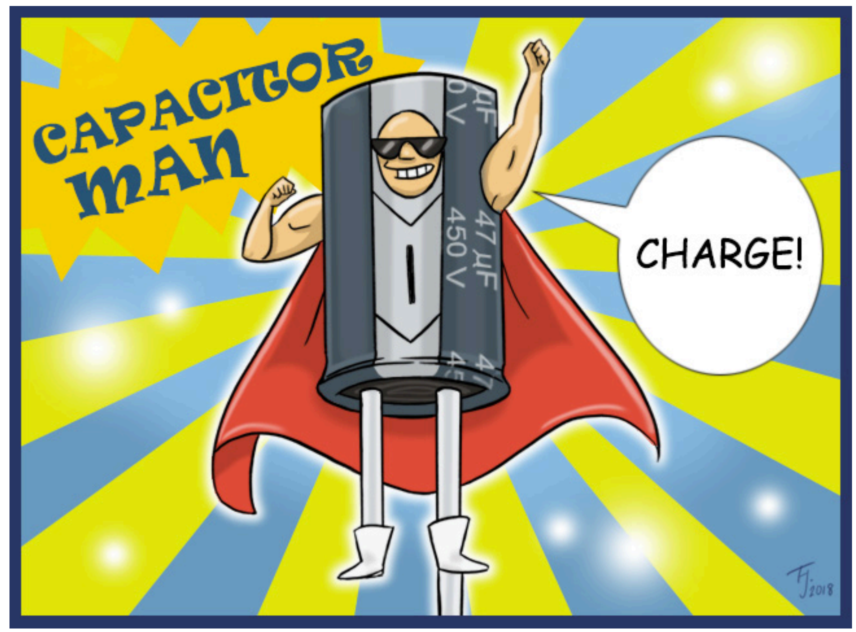
$$= p_{yEM}(0) \quad //$$

Capacitor gains momentum from EM fields!

Check Your Understanding

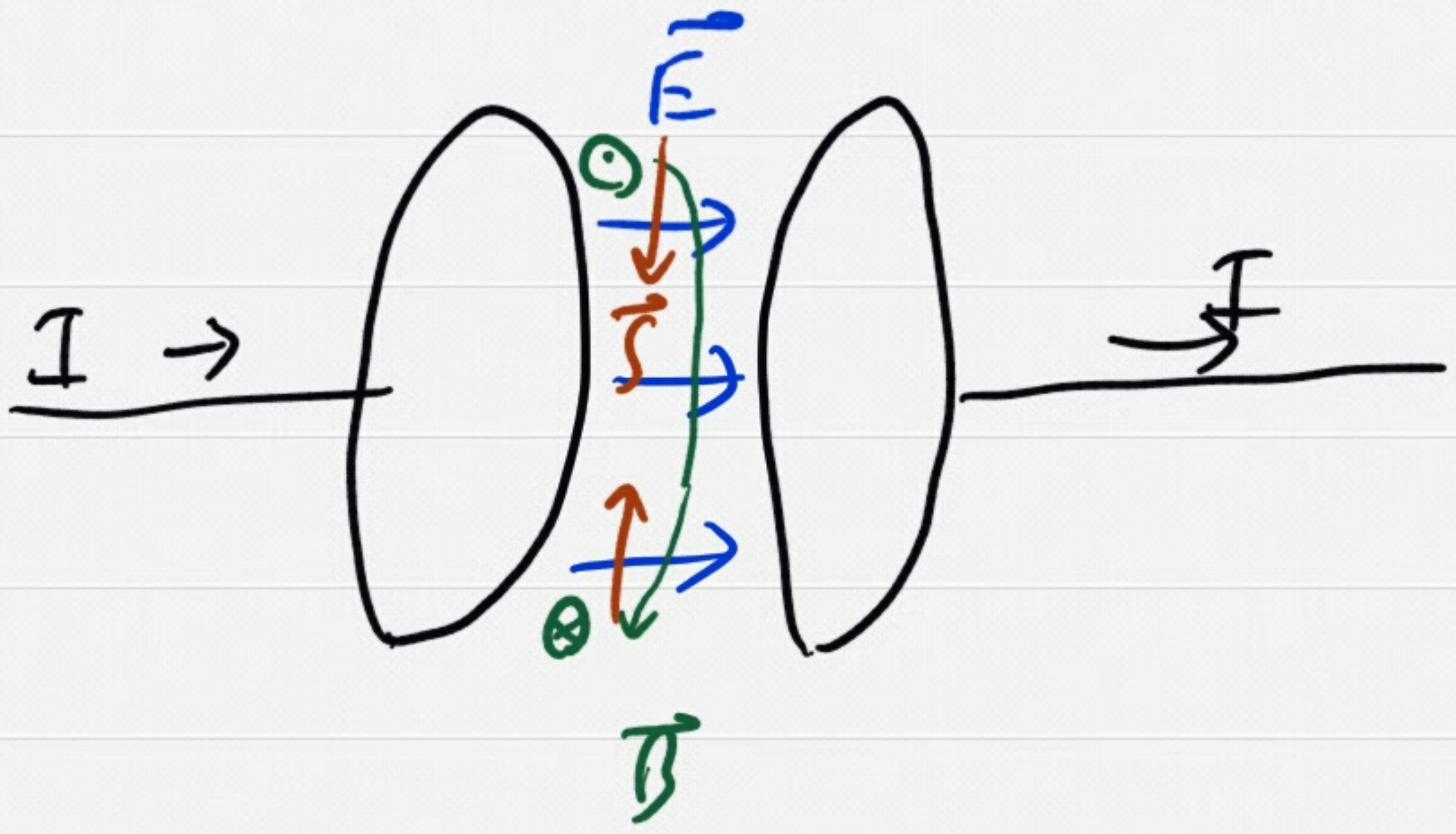
- Which direction do the electric and magnetic fields point between the plates of a charging capacitor?

- Which direction is the Poynting vector?



$$\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$$

Q1:



$$\vec{E} = \sigma/\varepsilon_0 \ \hat{z}$$

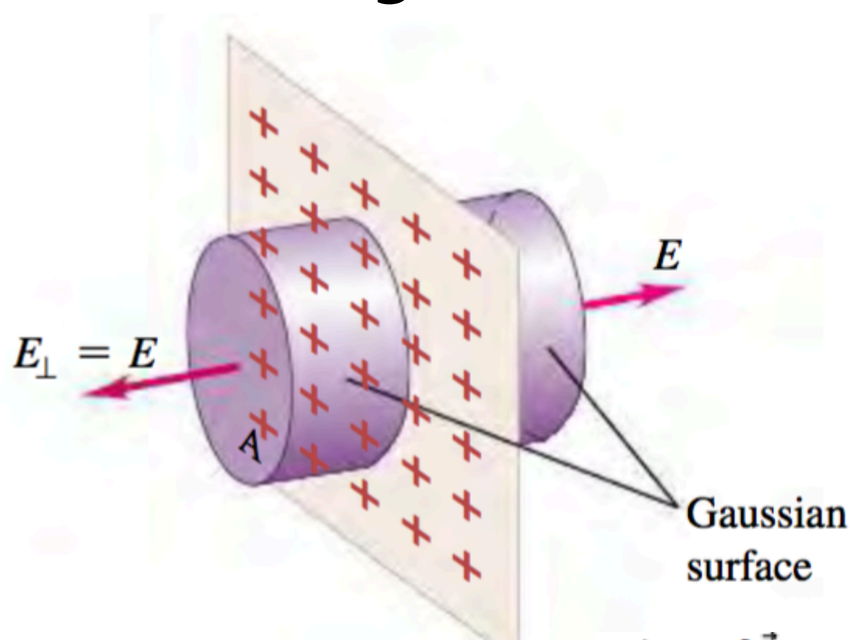
$$\vec{B} = \frac{\mu_0 I r}{2\pi R^2} \hat{\varphi}$$

$$\vec{S} = \frac{1}{\mu_0} (\vec{E} \times \vec{B})$$

$$= \boxed{-\frac{\sigma I r}{2\pi \varepsilon_0 R^2} \ \hat{r}}$$

Check Your Understanding

- Use the Maxwell stress tensor to show that there is no net force on a finite portion of an infinite sheet of charge with area A.



$$T_{ij} = \varepsilon_0\left(E_iE_j - \tfrac{1}{2}\delta_{ij}E^2\right) + \tfrac{1}{\mu_0}\left(B_iB_j - \tfrac{1}{2}\delta_{ij}B^2\right)$$

$$\vec{F} = \frac{d\vec{p}_{mech}}{dt} = \oint \overleftrightarrow{T}\cdot\overrightarrow{da} - \frac{d\vec{p}_{EM}}{dt}$$

Q2: $\vec{E} = \sigma/2\varepsilon_0 \, \hat{n}$ w/ $\hat{n} = \pm \hat{z}$

$$T = \varepsilon_0 \begin{pmatrix} -\frac{1}{2}E^2 & 0 & 0 \\ 0 & -\frac{1}{2}E^2 & 0 \\ 0 & 0 & \frac{1}{2}E^2 \end{pmatrix}$$

$$= \frac{\sigma^2}{8\varepsilon_0} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\int_{top} \overleftrightarrow{T} \cdot d\vec{a} = \int T_{zz} \, da_z = \frac{\sigma^2}{8\varepsilon_0}$$

$$\int_{bottom} \overleftrightarrow{T} \cdot d\vec{a} = \int T_{zz} \cdot -da_z = -\sigma^2/8\varepsilon_0$$

$$\int_{sides} \overleftrightarrow{T} \cdot d\vec{a} = \int T_{xx} \cdot da_x \, \hat{x} + \int T_{yy} \cdot da_y \, \hat{y}$$

$$= 0$$

$$\Rightarrow \int \overleftrightarrow{T} \cdot d\vec{a} = 0$$