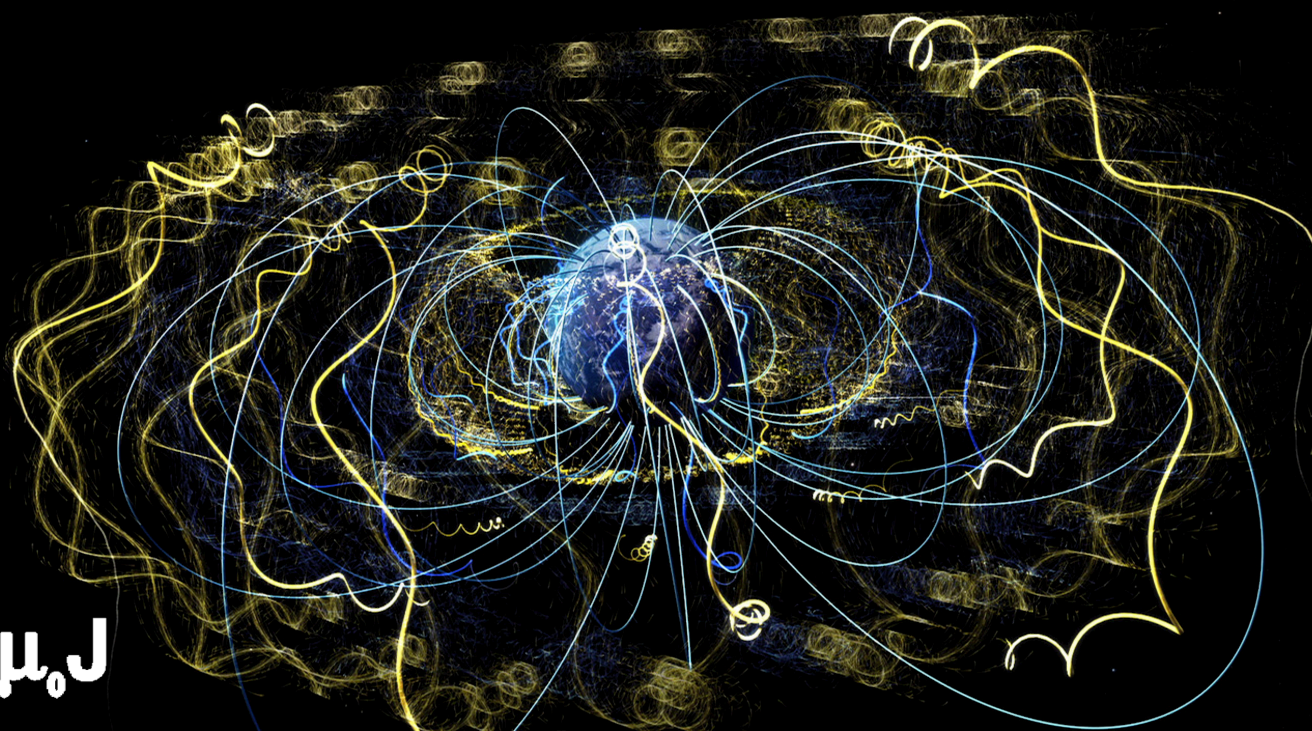
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism II: 3812

Professor Jasper Halekas
Van Allen 70
MWF 9:30-10:20 Lecture

Ch. 9	Electromagnetic Waves
9.1	Waves in 1-D

Wave = coherent propagating structure

Wave Eq. in 1-D

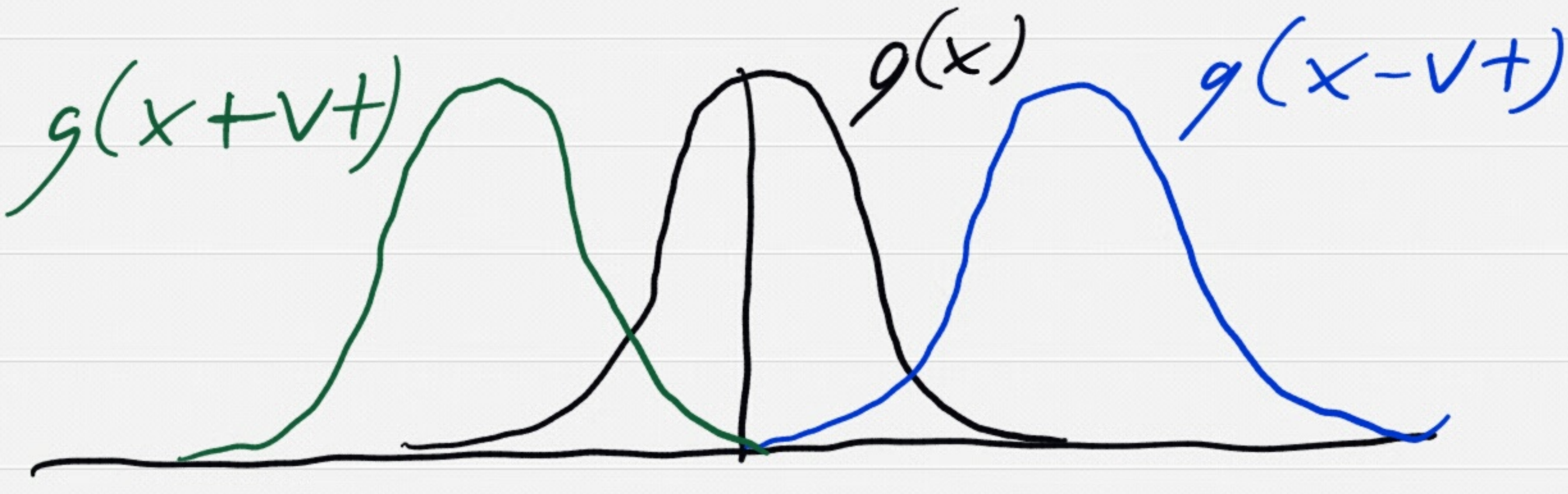
$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

solutions $f(x,t) = g(x \pm vt)$
$= g(u)$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 g}{\partial u^2}\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 g}{\partial u^2}$$

$$\frac{\partial^2 f}{\partial t^2} = \frac{\partial^2 g}{\partial u^2}\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 g}{\partial u^2}$$

$$\Rightarrow \frac{\partial^2 g}{\partial u^2} = \frac{1}{v^2}\cdot v^2 \frac{\partial^2 g}{\partial u^2} \quad //$$



Sinusoidal Waves

$$f(x,t) = A\cos(k(x-vt)+\delta)$$

A = Amplitude
δ = phase constant
k = wave number
v = phase velocity

ν (or f) = Frequency
ω = Angular Frequency
T = Period
λ = Wavelength

$$k = 2\pi/\lambda \quad , \quad \omega = 2\pi\nu = kv$$

$$\nu = 1/T = \frac{kv}{2\pi} = v/\lambda$$

Can write

$$f(x,t) = A\cos(kx-\omega t+\delta)$$

$\omega < 0$ or $k < 0$
is a leftward-travelling wave

Complex Notation

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\Rightarrow A\cos(kx - \omega t + \delta)$$
$$= Re\left[Ae^{i(kx-\omega t+\delta)}\right]$$
$$= Re\left[Ae^{i\delta}e^{i(kx-\omega t)}\right]$$

Define $\tilde{f}(x,t) = \tilde{A}e^{i(kx-\omega t)}$

w/ $\tilde{A} = Ae^{i\delta}$

Then $f(x,t) = Re[\tilde{f}(x,t)]$

Combining Waves

$$A_1\cos(kx - \omega t + \delta_1) + A_2\cos(kx - \omega t + \delta_2)$$

$$= Re\left[A_1e^{i\delta_1}e^{i(kx-\omega t)} + A_2e^{i\delta_2}e^{i(kx-\omega t)}\right]$$

$$= Re\left[(A_1e^{i\delta_1} + A_2e^{i\delta_2})e^{i(kx-\omega t)}\right]$$

$$= Re\left[A_3e^{i\delta_3}e^{i(kx-\omega t)}\right]$$

$$\Rightarrow A_3e^{i\delta_3} = A_1e^{i\delta_1} + A_2e^{i\delta_2}$$

$$A_3 = |\tilde{A}_3| = \sqrt{\tilde{A}_3\tilde{A}_3^*}$$

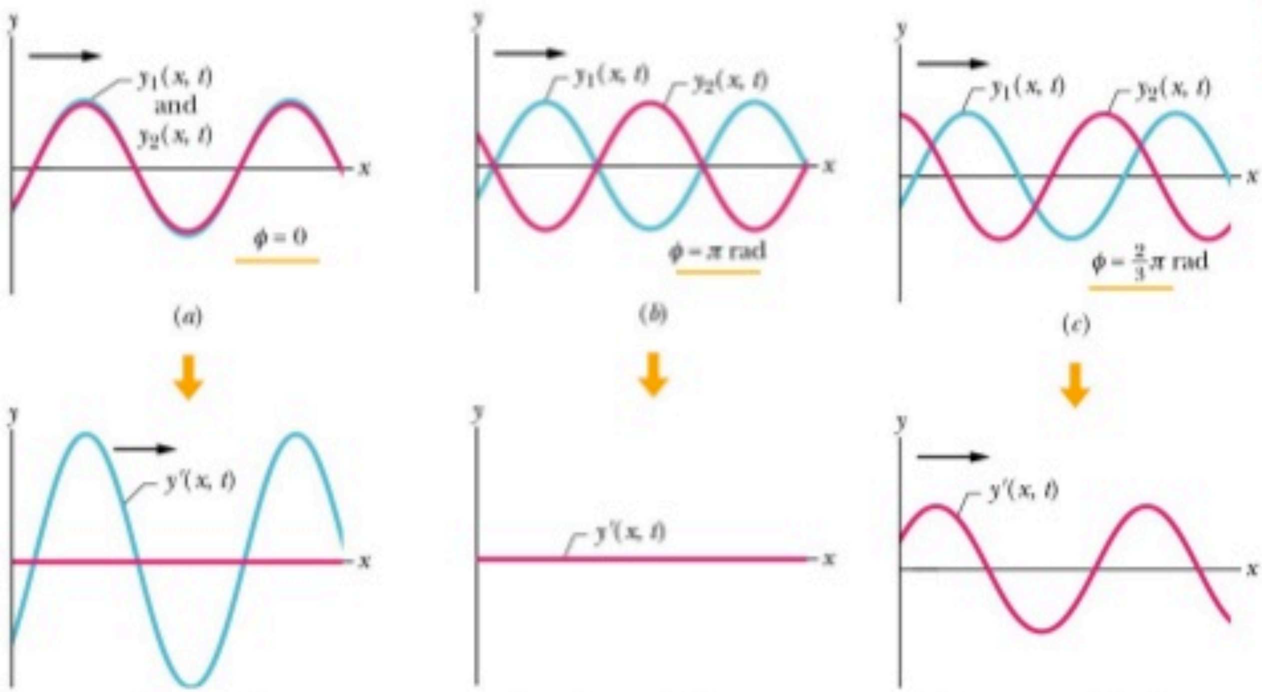
$$\delta_3 = \tan^{-1}\left[\frac{Im(A_3e^{i\delta_3})}{Re(A_3e^{i\delta_3})}\right]$$

Combining Waves

Two identical waves out of phase:

$$y_1(x,t) = A\cos(kx - \omega t) \qquad y_2(x,t) = A\cos(kx - \omega t + \boxed{\phi})$$

Wave 2 is little ahead or behind wave 1



constructive destructive intermediate

Reflection and Transmission

Reflection & Transmission

$$\tilde{f}_I = \tilde{A}_I e^{i(k_1 x - \omega t)} \qquad x < 0$$

$$\tilde{f}_T = \tilde{A}_T e^{i(k_2 x - \omega t)} \qquad x > 0$$

$$\tilde{f}_R = \tilde{A}_R e^{i(-k_1 x - \omega t)} \qquad x < 0$$

$$\tilde{f}_I(0) + \tilde{f}_R(0) = \tilde{f}_T(0)$$

$$\partial \tilde{f}_I / \partial x \big|_0 + \partial \tilde{f}_R / \partial x \big|_0 = \partial \tilde{f}_T / \partial x \big|_0$$

$$\Rightarrow \quad \tilde{A}_I + \tilde{A}_R = \tilde{A}_T$$

$$k_1 (\tilde{A}_I - \tilde{A}_R) = k_2 \tilde{A}_T$$

$$\Rightarrow \quad \tilde{A}_R = \frac{k_1 - k_2}{k_1 + k_2} \tilde{A}_I$$

$$\tilde{A}_T = \frac{2 k_1}{k_1 + k_2} \tilde{A}_I$$

- Can write in terms of

$$v_1 = \omega / k_1 , \quad v_2 = \omega / k_2$$

$$\tilde{A}_R = \frac{v_2 - v_1}{v_2 + v_1} \tilde{A}_I$$

$$\tilde{A}_T = \frac{2 v_2}{v_2 + v_1} \tilde{A}_I$$

$$\Rightarrow A_R e^{i\delta_R} = \frac{v_2 - v_1}{v_2 + v_1} A_I e^{i\delta_I}$$

$$A_T e^{i\delta_T} = \frac{2v_2}{v_2 + v_1} A_I e^{i\delta_I}$$

$v_2 > v_1$: $\delta_R = \delta_I = \delta_T$

$A_R, A_I, A_T > 0$

$$A_R = \frac{v_2 - v_1}{v_2 + v_1} A_I$$

$$A_T = \frac{2v_2}{v_2 + v_1} A_I$$

$v_2 < v_1$: to get $A_R, A_I, A_T > 0$

need

$$e^{i\delta_R} = -1 \cdot e^{i\delta_I}$$

or $\delta_R = \delta_I + \pi$

(reflected wave inverted)

$$\Rightarrow A_R = \frac{v_1 - v_2}{v_1 + v_2} A_I$$

$$A_T = \frac{2v_2}{v_1 + v_2} A_I$$

Special Cases:

$v_1 = v_2 \Rightarrow A_R = 0$

$A_T = A_I$

$v_2 = 0 \Rightarrow A_R = A_I,$

$A_T = 0$

(reflected wave inverted)