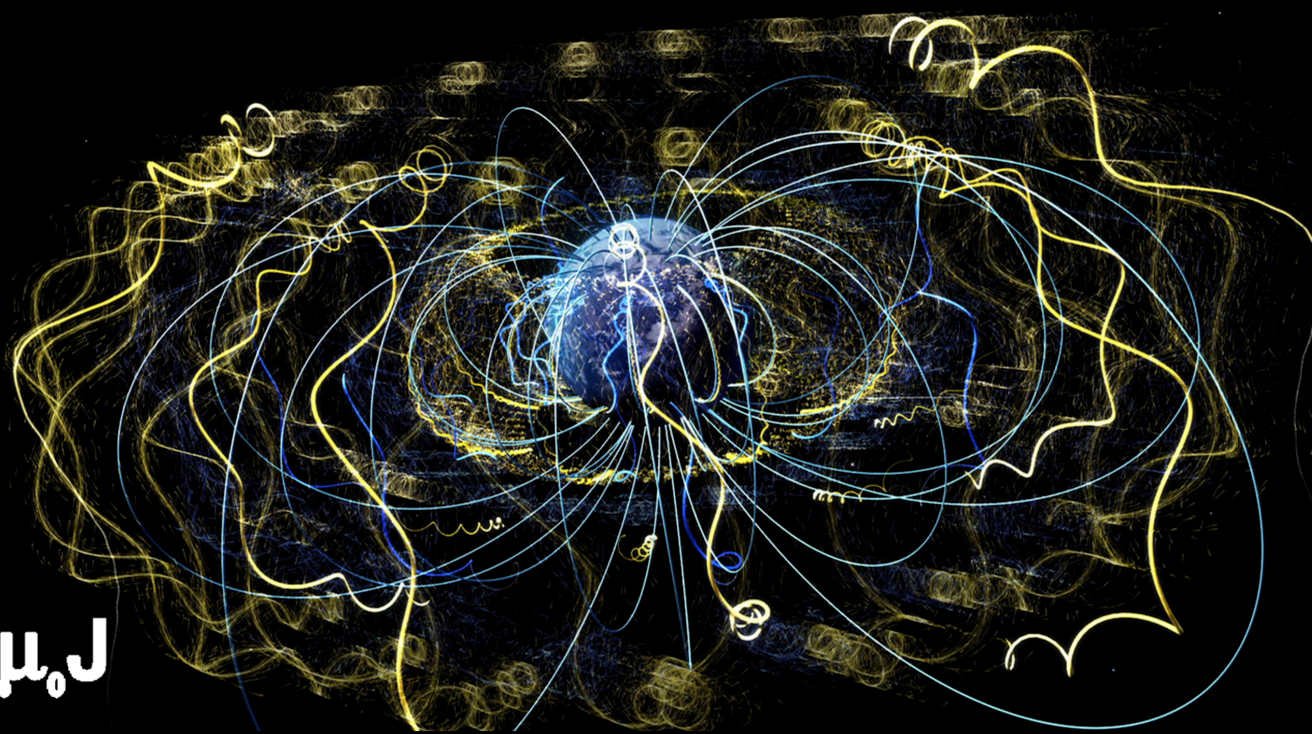
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism II: 3812

Professor Jasper Halekas

Van Allen 70

MWF 9:30-10:20 Lecture

Announcements I

- Final Exam scheduled for Monday, May 11 at 3:00-5:00pm in Van 70 (this room)

Announcements II

- Midterm #1 next Wednesday Feb. 26
  - In this room, during normal class time
  - Covers all sections of Ch. 7-9.2 except:
    - No questions on 8.2.4 (angular momentum) and 8.3 (magnetic forces do no work)
- Equation Sheet posted
  - Review, print out, and annotate as desired
- Practice midterms posted
  - Solutions to follow

Polarization

- For waves of interest to us, the displacement is perpendicular to propagation direction - i.e. "transverse"

- Wave is a vector, not a scalar

$$\tilde{\vec{f}}(x,t) = \tilde{A}\, e^{i(kx-\omega t)}\, \hat{n}$$

$\hat{n}$ is the "polarization vector"

$\vec{k}$ is a vector w/ magnitude $k = 2\pi/\lambda$ and pointing in propagation direction

kx in 1-d $\rightarrow$ $\vec{k}\cdot\vec{r}$ in 3-d

$$\vec{k}\cdot\vec{r} = k_x x + k_y y + k_z z$$

3-d Wave $\boxed{\tilde{\vec{f}}(\vec{r},t) = \tilde{A}\, e^{i(\vec{k}\cdot\vec{r}-\omega t)}\, \hat{n}}$

Transverse $\vec{k}\cdot\hat{n} = 0$

EM Waves

Maxwell's Eq. w/o sources

$$\nabla \cdot \vec{E} = 0 \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\partial \vec{B}/\partial t \qquad \nabla \times \vec{B} = \mu_0 \varepsilon_0 \, \partial \vec{E}/\partial t$$

$$\Rightarrow \boxed{\begin{aligned} \nabla^2 \vec{E} &= \frac{1}{c^2} \partial^2 \vec{E}/\partial t^2 \\ \nabla^2 \vec{B} &= \frac{1}{c^2} \partial^2 \vec{B}/\partial t^2 \end{aligned}}$$

w/ $c = 1/\sqrt{\mu_0 \varepsilon_0}$

3-d Wave Eq. for $\vec{E}, \vec{B}$

Maxwell's Eqs. still couple $\vec{E}, \vec{B}$

Monochromatic Waves

$$\tilde{\vec{E}}(\vec{r}, t) = \tilde{\vec{E}}_0 \, e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

$$\tilde{\vec{B}}(\vec{r}, t) = \tilde{\vec{B}}_0 \, e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

w/ $\omega = ck$

$$\nabla \cdot (\tilde{\vec{E}}) = e^{i(\vec{k}\cdot\vec{r}-\omega t)} \nabla \cdot (\tilde{\vec{E}}_0)$$

$$+ \tilde{\vec{E}}_0 \cdot \nabla e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$= \tilde{\vec{E}}_0 \cdot i\vec{k}\, e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$= 0$$

$$\Rightarrow \tilde{\vec{E}}_0 \cdot \vec{k} = 0$$

similarly $\tilde{\vec{B}}_0 \cdot \vec{k} = 0$

$\Rightarrow$ EM waves are transverse

$$\nabla \times \tilde{\vec{E}} = e^{i(\vec{k}\cdot\vec{r}-\omega t)} \nabla \times (\tilde{\vec{E}}_0)$$

$$- \tilde{\vec{E}}_0 \times \nabla e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$= -\tilde{\vec{E}}_0 \times i\vec{k}\, e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$= -\partial \vec{B}/\partial t$$

$$= -\tilde{\vec{B}}_0 \cdot -i\omega \cdot e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\Rightarrow \boxed{\tilde{\vec{B}}_0 = (\vec{k} \times \tilde{\vec{E}}_0)/\omega}$$

$\Rightarrow \vec{B}, \vec{E}, \vec{k}$ mutually perpendicular

$e^{i(\vec{k}\cdot\vec{r}-\omega t)}$ same for $\tilde{\vec{E}}$ / $\tilde{\vec{B}}$
so $\vec{E}$ & $\vec{B}$ are in phase

$$\tilde{\vec{B}}_0 = (\vec{k}\times\tilde{\vec{E}}_0)/\omega$$

$$\tilde{\vec{E}}_0\cdot\vec{k} = 0 \qquad \tilde{\vec{B}}_0\cdot\vec{k} = 0$$

$$\Rightarrow |\vec{B}_0| = |\vec{k}|\,|\vec{E}_0|/\omega$$

$$= |\vec{E}_0|/c$$

- Polarization Convention

$$\hat{n} = \hat{E}_0$$

transverse $\hat{n}\cdot\vec{k} = 0$

$$\hat{B}_0 = \hat{k}\times\hat{n}$$

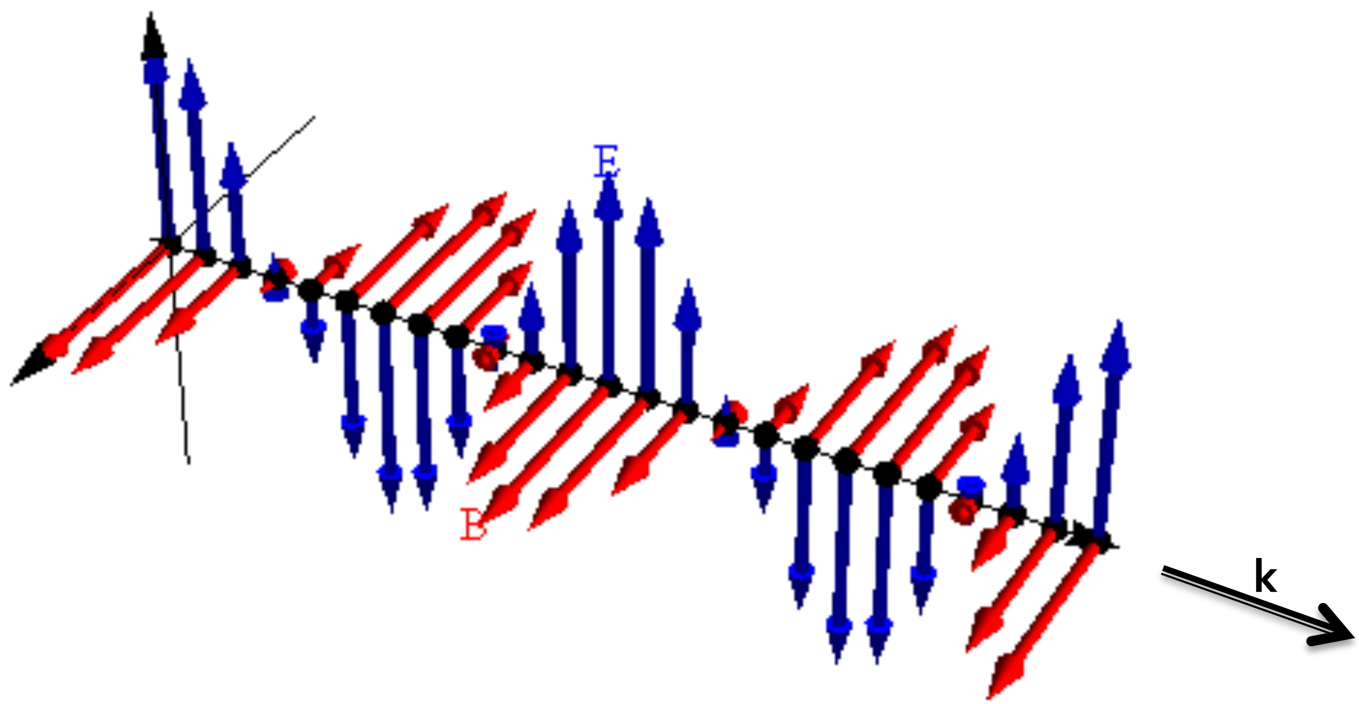
Complex: $\tilde{\vec{E}}(\vec{r},t) = \tilde{E}_0\,\hat{n}\,e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

$\tilde{\vec{B}}(\vec{r},t) = \frac{\tilde{E}_0}{c}(\hat{k}\times\hat{n})\,e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

Real: $\vec{E}(\vec{r},t) = E_0\,\hat{n}\cos(\vec{k}\cdot\vec{r}-\omega t+\delta)$

$\vec{B}(\vec{r},t) = \frac{E_0}{c}(\hat{k}\times\hat{n})\cos(\vec{k}\cdot\vec{r}-\omega t+\delta)$

EM Wave Propagation



Energy & Momentum

$$u_{EM} = \frac{1}{2}\varepsilon_0 E^2 + B^2/2\mu_0$$
$$= \frac{1}{2}\varepsilon_0 E_0^2 \cos^2(\vec{k}\cdot\vec{r} - \omega t + \delta)$$
$$+ \frac{1}{2\mu_0}\frac{E_0^2}{c^2}\cos^2(\vec{k}\cdot\vec{r} - \omega t + \delta)$$

$$= \left(\frac{1}{2}\varepsilon_0 E_0^2 + \frac{1}{2}\frac{E_0^2}{\mu_0}\cdot\mu_0\varepsilon_0\right)\cdot\cos^2(\vec{k}\cdot\vec{r} - \omega t + \delta)$$
$$= \varepsilon_0 E_0^2 \cos^2(\vec{k}\cdot\vec{r} - \omega t + \delta)$$

$$\vec{S} = (\vec{E}\times\vec{B})/\mu_0$$

$$= \frac{E_0^2}{\mu_0 c}\cos^2(\vec{k}\cdot\vec{r} - \omega t + \delta)\ \hat{n}\times(\hat{k}\times\hat{n})$$
$$= c\,\varepsilon_0 E_0^2 \cos^2(\vec{k}\cdot\vec{r} - \omega t + \delta)\ \hat{k}$$

Note $\vec{S} = c\, u_{EM}\,\hat{k}$

$\Rightarrow$ Energy propagates at c in $\hat{k}$ direction