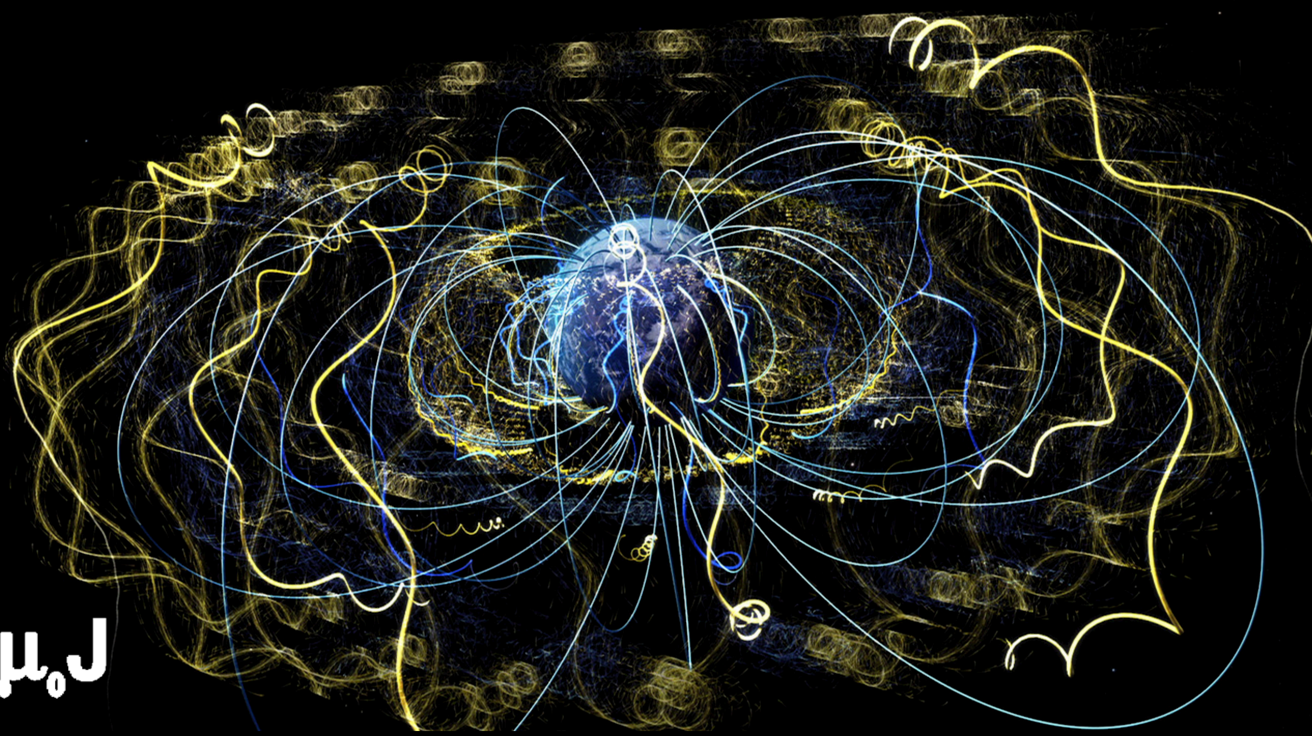
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

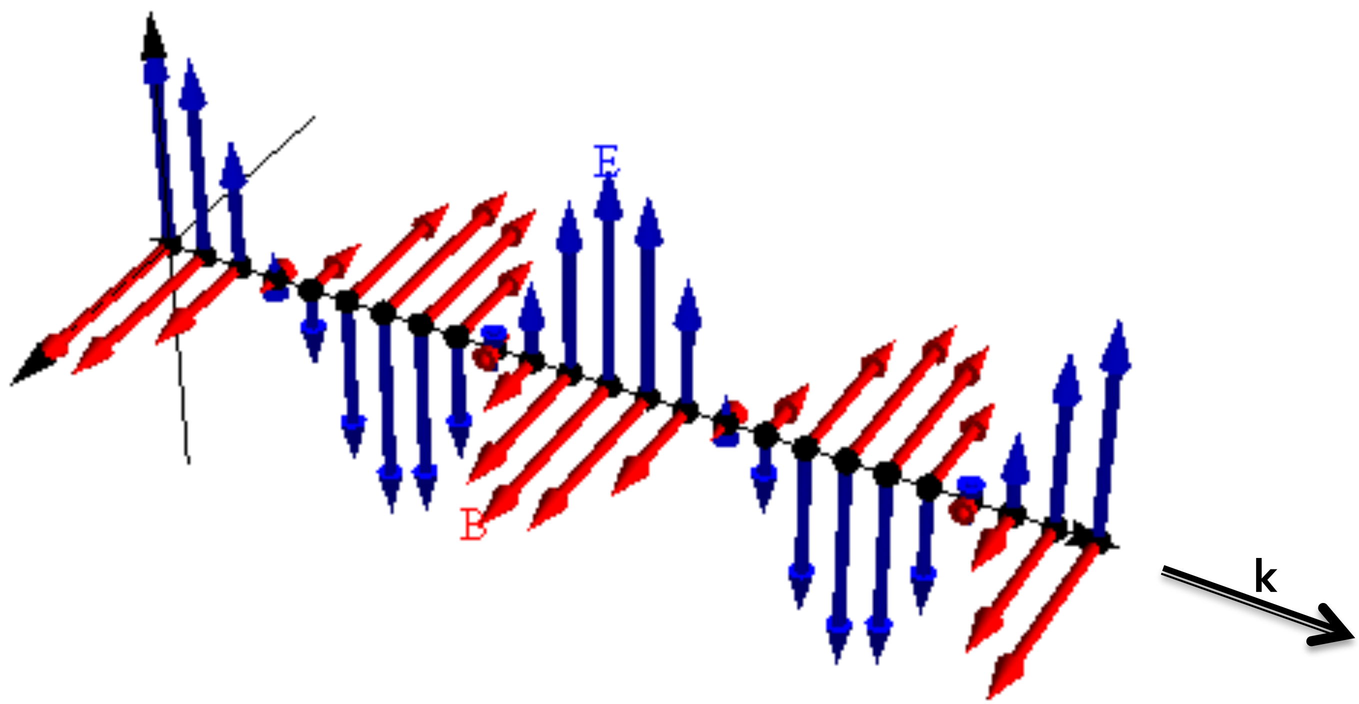
$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism II: 3812

Professor Jasper Halekas
Van Allen 70
MWF 9:30-10:20 Lecture

EM Wave Propagation



Energy & Momentum

$$u_{EM} = \frac{1}{2}\varepsilon_0 E^2 + B^2/2\mu_0$$
$$= \frac{1}{2}\varepsilon_0 E_0^2 \cos^2(\vec{k}\cdot\vec{r} - \omega t + \delta)$$
$$+ \frac{1}{2\mu_0}\frac{E_0^2}{c^2}\cos^2(\vec{k}\cdot\vec{r} - \omega t + \delta)$$

$$= \left(\frac{1}{2}\varepsilon_0 E_0^2 + \frac{1}{2}\frac{E_0^2}{\mu_0}\cdot\mu_0\varepsilon_0\right)\cdot\cos^2(\vec{k}\cdot\vec{r} - \omega t + \delta)$$
$$= \varepsilon_0 E_0^2 \cos^2(\vec{k}\cdot\vec{r} - \omega t + \delta)$$

$$\vec{S} = (\vec{E}\times\vec{B})/\mu_0$$

$$= \frac{E_0^2}{\mu_0 c}\cos^2(\vec{k}\cdot\vec{r} - \omega t + \delta)\ \hat{n}\times(\hat{k}\times\hat{n})$$
$$= c\,\varepsilon_0 E_0^2 \cos^2(\vec{k}\cdot\vec{r} - \omega t + \delta)\ \hat{k}$$

Note $\vec{S} = c\, u_{EM}\,\hat{k}$

$\Rightarrow$ Energy propagates at c in $\hat{k}$ direction

$$\vec{g}_{EM} = \vec{S}/c^2 = \frac{u_{EM}}{c}\hat{k}$$

$\Rightarrow$ momentum = energy / c
as expected for massless quantities

Average over wave:

$$\langle\cos^2\rangle = 1/2$$

$$\Rightarrow \langle u_{EM}\rangle = \frac{1}{2}\varepsilon_0 E_0^2 \quad \langle\vec{S}\rangle = \frac{1}{2}c\,\varepsilon_0 E_0^2\,\hat{k}$$
$$\langle\vec{g}\rangle = \frac{\varepsilon_0}{2c}E_0^2\,\hat{k}$$

Light Intensity

$$I = \langle S \rangle = \frac{1}{2} c \varepsilon_0 E_0^2$$

Radiation Pressure

volume of light impacting area
A in time Δt

$$= A \cdot \Delta r = A \cdot c \cdot \Delta t$$

So Momentum impacting A
is $\Delta p = \langle g \rangle \cdot A \cdot c \cdot \Delta t$
if absorbed

Pressure $P = F/A$

$$= \frac{\Delta p}{\Delta t} \cdot \frac{1}{A}$$

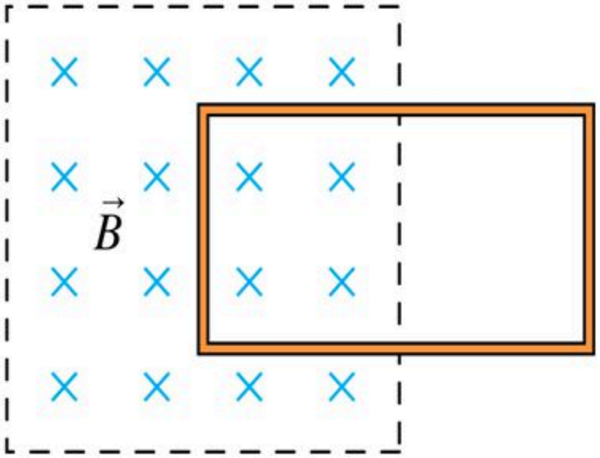
$$= \langle g \rangle \cdot c$$

$$= \frac{1}{2} \varepsilon_0 E_0^2 = I/c$$

If light reflected
Δp doubled and $P = 2I/c$

Check Your Understanding

A conducting loop is halfway into a magnetic field. Suppose the magnetic field begins to increase rapidly in strength. What happens to the loop?



$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{E} = \rho/\varepsilon_0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\oint \vec{E} \cdot \overrightarrow{dl} = -\frac{d}{dt} \int \vec{B} \cdot \overrightarrow{da}$$

$$\oint \vec{E} \cdot \overrightarrow{da} = Q_{enc}/\varepsilon_0$$

$$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot \overrightarrow{da}$$

$$\oint \vec{B} \cdot \overrightarrow{da} = 0$$

Q_1:

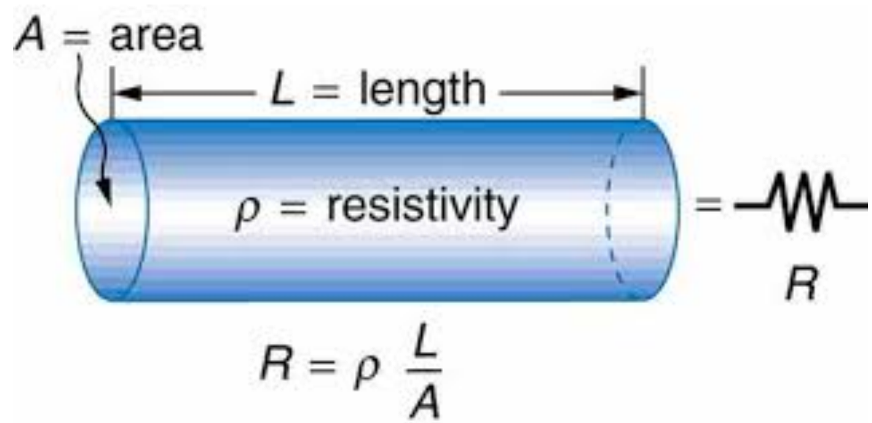
$$\mathcal{E} = -d\Phi_B/dt$$
$$= -d/dt\,(BA)$$
$$= -A\,dB/dt$$

$I = \mathcal{E}/R$ opposite dB/dt
so CCW

$$\vec{F} = \vec{I}L \times \vec{B}$$
$$= ILB\,\hat{x}$$ to right
$$= \frac{\mathcal{E}LB}{R}\,\hat{x}$$

Check Your Understanding

Consider a cylindrical resistor with a potential difference applied between the two ends. Assume the potential difference varies with time exponentially: $V(t) = V_0 \exp(\alpha t)$



A. Compute the magnetic field at the resistor's surface due to conduction current

B. Compute the magnetic field at the resistor's surface due to displacement current

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \oint \vec{E} \cdot \overrightarrow{dl} = -\frac{d}{dt} \int \vec{B} \cdot \overrightarrow{da}$$

$$\nabla \cdot \vec{E} = \rho / \varepsilon_0 \qquad \oint \vec{E} \cdot \overrightarrow{da} = Q_{enc} / \varepsilon_0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad \oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot \overrightarrow{da}$$

$$\nabla \cdot \vec{B} = 0 \qquad \oint \vec{B} \cdot \overrightarrow{da} = 0$$

Q2: $\vec{E} = -\nabla V$

$$= \Delta V / L \ \hat{z}$$

$$= V_0 \ e^{\alpha t} / L \ \hat{z}$$

$$I = \Delta V / R = \frac{V_0 e^{\alpha t}}{R}$$

$$\oint \vec{B} \, d\vec{l} = \mu_0 I_{enc} = \mu_0 I \quad \text{at } s = r$$

$$B \cdot 2\pi r = \mu_0 I$$

$$\Rightarrow \boxed{\vec{B}_1 = \frac{\mu_0 V_0 \ e^{\alpha t}}{2\pi r R} \hat{\varphi}}$$

$$= \boxed{\frac{\mu_0 V_0 \ e^{\alpha t}}{\rho L} \frac{r}{2} \hat{\varphi}}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \, d\Phi_E / dt$$

$$= \mu_0 \varepsilon_0 \, d/dt \, (E \cdot \pi r^2)$$

$$B \cdot 2\pi r = \mu_0 \varepsilon_0 \cdot \pi r^2 \cdot dE/dt$$

$$\Rightarrow \boxed{\vec{B}_2 = \frac{\mu_0 \varepsilon_0 \alpha V_0 \ e^{\alpha t}}{L} \cdot \frac{r}{2} \hat{\varphi}}$$

Check Your Understanding

- Given the following electromagnetic wave:
  - $E_y(x, t) = E_o \cos(kx - \omega t)$ $B_z(x, t) = E_o/c \cos(kx - \omega t)$

1. What is the polarization vector?
2. What is the propagation direction?
3. What are the elements of the Maxwell Stress Tensor?

$$T_{ij} = \varepsilon_0\left(E_i E_j - \tfrac{1}{2}\delta_{ij}E^2\right) + \tfrac{1}{\mu_0}\left(B_i B_j - \tfrac{1}{2}\delta_{ij}B^2\right)$$

Q3 : $\hat{n}$ along $\vec{E}$

so $\boxed{\hat{n} = \hat{y}}$

$\vec{K}$ along $\vec{E} \times \vec{B}$

so $\hat{K} = \hat{y} \times \hat{z}$

or $\boxed{\hat{K} = \hat{x}}$

All off-diagonal elements $= 0$

$$T_{xx} = \varepsilon_0\left(-\tfrac{1}{2}E^2\right) + \frac{1}{\mu_0}\left(-\tfrac{1}{2}B^2\right)$$

$$= -\tfrac{1}{2}\left(\varepsilon_0 E_0^2 + \frac{E_0^2}{\mu_0 c^2}\right)\cos^2(kx - \omega t)$$

$$= -\tfrac{1}{2}\varepsilon_0 E_0^2\left(1 + \frac{\varepsilon_0\mu_0}{\varepsilon_0\mu_0}\right)\cos^2(kx - \omega t)$$

$$= \boxed{-\varepsilon_0 E_0^2 \cos^2(kx - \omega t)}$$

$$T_{yy} = \varepsilon_0\left(\tfrac{1}{2}E^2\right) + \frac{1}{\mu_0}\left(-\tfrac{1}{2}B^2\right)$$

$$= \boxed{0}$$

$$T_{zz} = \varepsilon_0\left(-\tfrac{1}{2}E^2\right) + \frac{1}{\mu_0}\left(\tfrac{1}{2}B^2\right)$$

$$= \boxed{0}$$