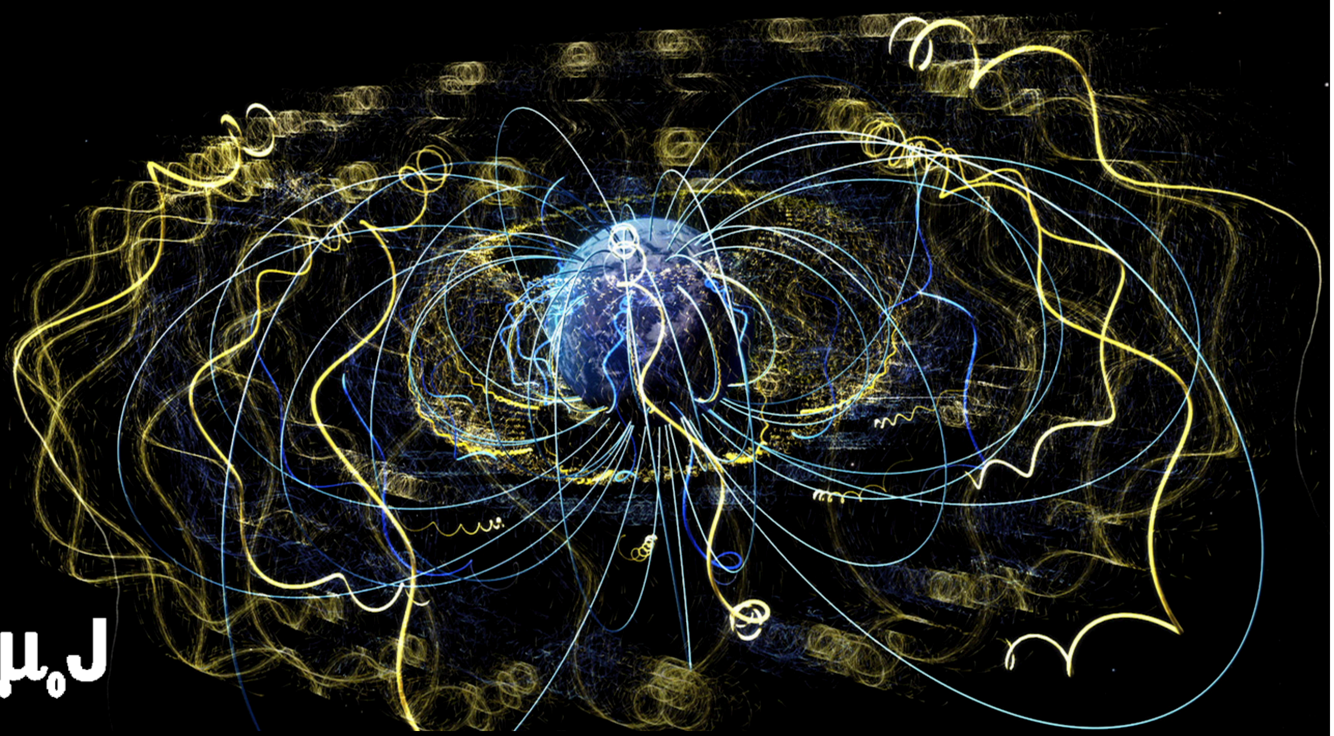
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism II: 3812

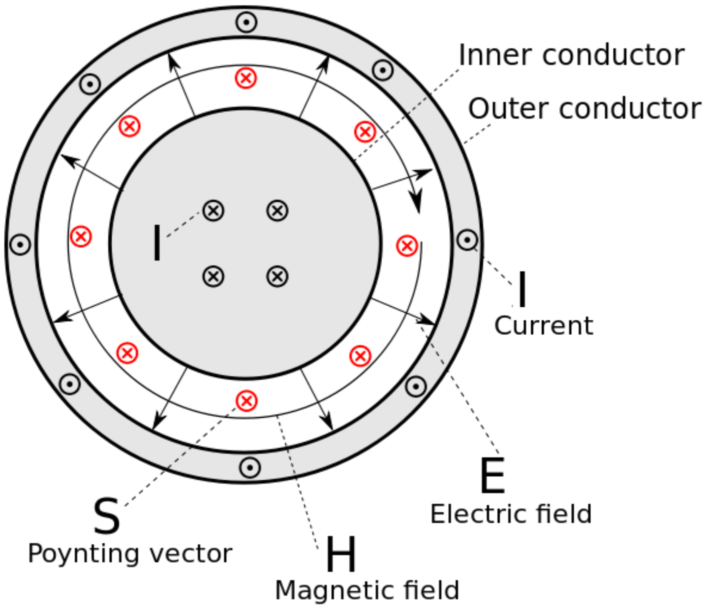
Professor Jasper Halekas
Van Allen 70
MWF 9:30-10:20 Lecture

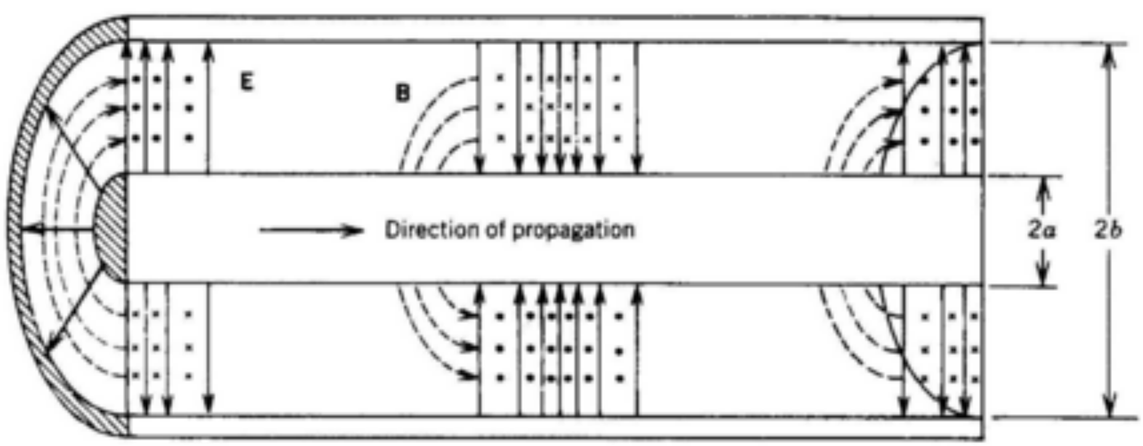
XKCD with the Best Take as Usual

Announcements

- It appears very likely that we will be going to online-only classes, most likely after spring break
  - This class will continue, in a virtual format if need be
    - Lecture notes etc. will be posted as usual
    - Lectures will be either recorded or webcast (TBD)
      - Preferences?
  - Homework will have to be turned in electronically
    - Is this an issue for anyone?
  - Exams will likely have to be take-home, open book, with a constrained duration
    - Is this an issue for anyone?
- Suggestions to ease the transition are welcome!

Coaxial TEM Waveguide





Coaxial Wave Guide

- Coaxial cable supports TEM modes

$$\tilde{\vec{E}}(\vec{r},t) = \tilde{\vec{E}}_0(x,y)e^{i(kz-\omega t)}$$
$$\tilde{\vec{B}}(\vec{r},t) = \tilde{\vec{B}}_0(x,y)e^{i(kz-\omega t)}$$

$$\text{TEM} \Rightarrow \tilde{\vec{E}}_0 = E_{0x}\hat{x} + E_{0y}\hat{y}$$
$$\tilde{\vec{B}}_0 = B_{0x}\hat{x} + B_{0y}\hat{y}$$

$$\nabla \times \vec{E} = -\partial \vec{B}/\partial t$$
$$\Rightarrow \partial E_{0y}/\partial x - \partial E_{0x}/\partial y = 0$$
$$-ikE_{0y} = i\omega B_{0x}$$
$$ikE_{0x} = i\omega B_{0y}$$

$$\nabla \times \vec{B} = \frac{1}{c^2}\partial \vec{E}/\partial t$$
$$\Rightarrow \partial B_{0y}/\partial x - \partial B_{0x}/\partial y = 0$$
$$-ikB_{0y} = -\frac{i\omega}{c^2}E_{0x}$$
$$ikB_{0x} = -\frac{i\omega}{c^2}E_{0y}$$

Combine to find:

$$E_{0x} = c B_{0y} \quad , \quad E_{0y} = -c B_{0x}$$

$$\& \quad k = \omega / c$$

$$\partial E_{0y}/\partial x - \partial E_{0x}/\partial y = 0$$

$$\Rightarrow \partial B_{0x}/\partial x + \partial B_{0y}/\partial y = 0$$

$$\partial B_{0y}/\partial x - \partial B_{0x}/\partial y = 0$$

$$\Rightarrow \partial E_{0x}/\partial x + \partial E_{0y}/\partial y = 0$$

Summary: $B_{0z}, E_{0z} = 0$

$\nabla \cdot \vec{B}_0 = 0 \quad , \quad \nabla \cdot \vec{E}_0 = 0$

$\nabla \times \vec{B}_0 = 0 \quad , \quad \nabla \times \vec{E}_0 = 0$

Solutions: - Cylindrical symmetry
- Vacuum
- Boundary conditions $\vec{E}_{\parallel} = 0$ & $B_{\perp} = 0$ at $r = a, b$

$$\Rightarrow \vec{E}_0(s, \varphi) = \frac{A}{s} \hat{s} \qquad B = A/c$$

$$\vec{B}_0(s, \varphi) = \frac{B}{s} \hat{\varphi}$$

Finally

$$\vec{E}(s,\varphi,z,t) = \frac{A}{s}\cos(kz-\omega t+\delta)\,\hat{s}$$

$$\vec{B}(s,\varphi,z,t) = \frac{A}{cs}\cos(kz-\omega t+\delta)\,\hat{\varphi}$$

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$$

$$= \frac{A^2}{cs^2\mu_0}\cos^2(kz-\omega t+\delta)\,\hat{z}$$

propagates along coax

Check Your Understanding I

- Consider a wave with an electric field amplitude E_o in vacuum, and a wave with the same electric field amplitude E_o in a dielectric material with permittivity $\varepsilon > \varepsilon_o$
  - In which case is the electric field energy density greater?
  - Physically, why is the electric field energy density greater in that case?

Q1: $u_E = \frac{1}{2} \varepsilon E_0^2$

$u_{E vac} = \frac{1}{2} \varepsilon_0 E_0^2$

$u_E > u_{E vac}$

$\longrightarrow \vec{E}_{imposed}$

$\longrightarrow \vec{P} = \varepsilon_0 \chi_e \vec{E}_{imp}$

$\longleftarrow \Delta \vec{E}$

$\longrightarrow \vec{E}_0 = \vec{E}_{imp} - \Delta \vec{E}$

- Field weakened by polarization
- Wave energy goes with imposed field, not weakened field

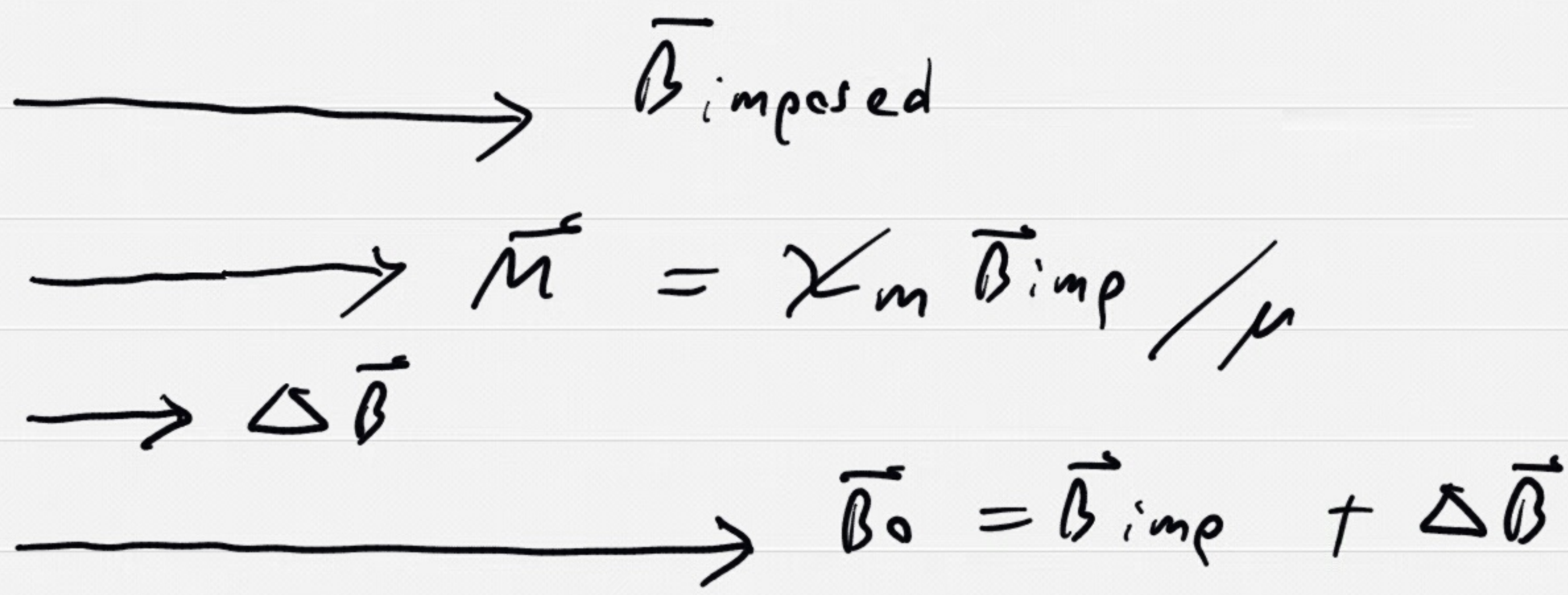
Check Your Understanding II

- Consider a wave with a magnetic field amplitude B_o in vacuum, and a wave with the same magnetic field amplitude B_o in a dielectric material with permeability $\mu > \mu_o$
  - In which case is the magnetic field energy density greater?
  - Physically, why is the magnetic field energy density greater in that case?

Q2: $u_B = B_0^2 / 2\mu$

$u_{B\,vac} = B_0^2 / 2\mu_0$

$u_B < u_{B\,vac}$



- Field amplified by magnetization
- Wave energy goes with imposed field, not amplified field

Note: For wave $B_0 / E_0 = k/\omega$
$= \sqrt{\mu \varepsilon} > 1$

- This factor ensures that $u_B = u_E$ in both vacuum & dielectric

Check Your Understanding III

- Why are electromagnetic waves damped in conducting materials?

A. Free currents are driven by the electric fields

B. Faraday's Law

C. Ampere's Law

D. Ohm's Law

E. All of the above

Q3:

Ohm's Law $\vec{J} = \sigma \vec{E}$

Faraday's Law $\nabla \times \vec{E} = -\partial \vec{B}/\partial t$

Ampere's Law $\nabla \times \vec{B} = \mu \varepsilon \, \partial \vec{E}/\partial t$

Becomes $\nabla \times \vec{B} = \mu \varepsilon \, \partial \vec{E}/\partial t + \mu \vec{J}$

Wave: Changing $B \to E$

Changing $E \to B$

Damped Wave: Changing $B \to E$

$E \to J$

Changing $E, J \to B$

Dissipation $\vec{J} \cdot \vec{E} = \sigma E^2 = dW/dt$

energy loss to particles from fields