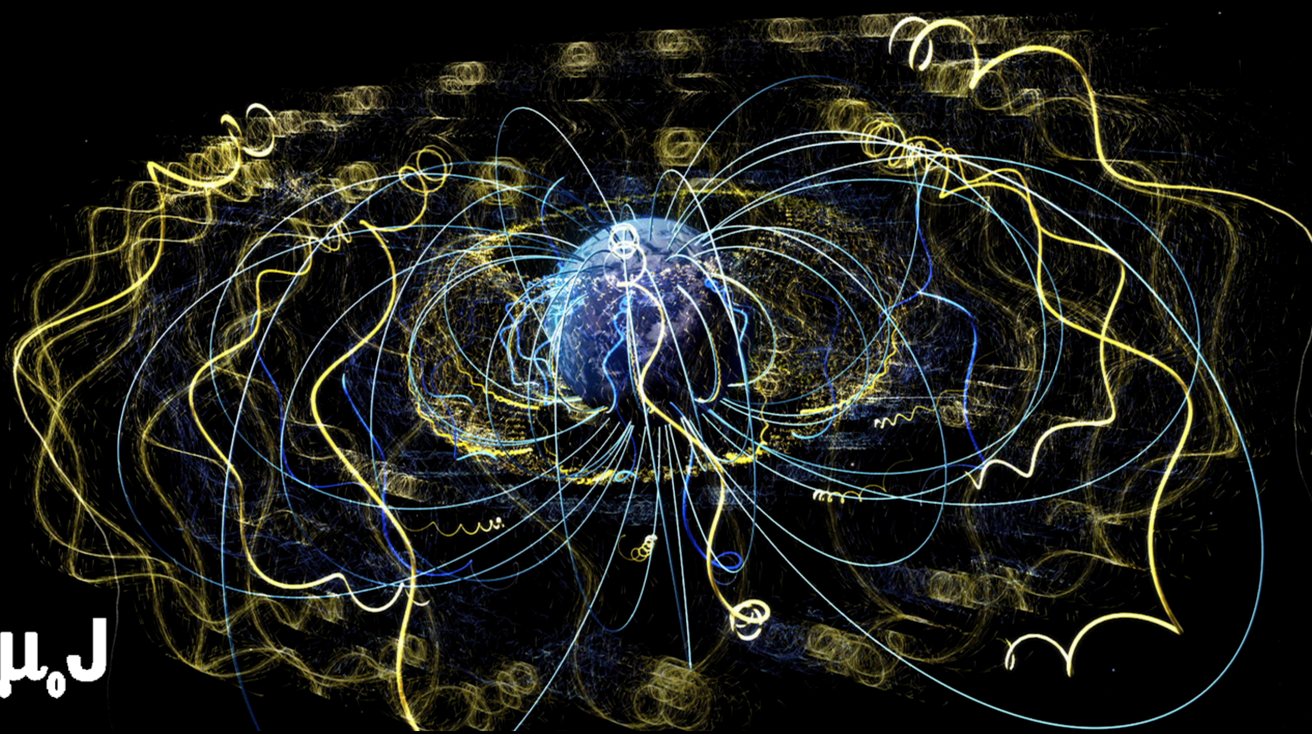
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism II: 3812

Professor Jasper Halekas

Van Allen 70

MWF 9:30-10:20 Lecture

Announcements

- Our university is transitioning to online-only instruction, starting March 23 (first Monday following spring break), and continuing until at least April 3 (two weeks)
  - This class will continue in a virtual format
    - Lectures will be webcast via Zoom (at least on a trial basis)
    - Lecture notes will be posted as usual
    - Office hours will be virtual via Zoom
  - Homework #7 and #8 will have to be turned in electronically
    - Due dates/times remain the same
  - There are no exams in this class during the online-only period
    - If the online-only period is eventually extended, we will have take-home exams. If that occurs, more information will be forthcoming.

Zoom Information

- I plan to utilize Zoom for lectures and office hours
  - Zoom is an online videoconferencing tool which allows me to share audio, video, and anything on my computer screen
    - Lectures will be recorded and posted on ICON for those who can't make the lecture, or want to review the lecture
    - Office hours will not be recorded, and will utilize a "waiting room" to allow private discussion if necessary
- Zoom meeting # for lecture: 372-343-761
  - Join URL: https://uiowa.zoom.us/j/372343761
- Zoom meeting # for office hours: 146-384-269
  - Join URL: https://uiowa.zoom.us/j/146384269
- You will be able to access these meetings directly through these join URLs, or through the Zoom links on the course ICON page

Ch. 10	Potentials & Fields
10.1	Potential Formulation

Maxwells Eqs.

$$\nabla \cdot \vec{E} = \rho/\varepsilon_0 \qquad \nabla \times \vec{E} = -\partial \vec{B}/\partial t$$
$$\nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{B} = \mu_0 \varepsilon_0 \partial \vec{E}/\partial t + \mu_0 \vec{J}$$

Magnetostatics: $\nabla \cdot \vec{B} = 0$

$$\Rightarrow \boxed{\vec{B} = \nabla \times \vec{A}}$$

still valid!

Electrostatics: $\nabla \times \vec{E} = 0$

$$\Rightarrow \vec{E} = -\nabla V$$

no longer true!

Fix:
$$\nabla \times \vec{E} = -\partial \vec{B}/\partial t$$
$$= -\partial/\partial t (\nabla \times \vec{A})$$
$$= \nabla \times (-\partial \vec{A}/\partial t)$$
$$\Rightarrow \nabla \times (\vec{E} + \partial \vec{A}/\partial t) = 0$$
$$\Rightarrow \vec{E} + \partial \vec{A}/\partial t = -\nabla V$$
$$\Rightarrow \boxed{\vec{E} = -\nabla V - \partial \vec{A}/\partial t}$$

What about Gauss's Law & Ampere's Law?

Gauss: $\nabla\cdot\vec{E} = \nabla\cdot(-\nabla V) + \nabla\cdot(-\partial\vec{A}/\partial t)$

$= \rho/\varepsilon_0$

$$\Rightarrow \boxed{\nabla^2 V + \partial/\partial t(\nabla\cdot\vec{A}) = -\rho/\varepsilon_0}$$

Poisson's Eq. w/ extra term

Ampere: $\nabla\times\vec{B} = \nabla\times(\nabla\times\vec{A})$

$= \nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A}$

$= \mu_0\vec{J} + \mu_0\varepsilon_0\,\partial/\partial t(-\nabla V - \partial\vec{A}/\partial t)$

$$\Rightarrow \boxed{\nabla^2\vec{A} - \mu_0\varepsilon_0\,\partial^2\vec{A}/\partial t^2 - \nabla(\nabla\cdot\vec{A} + \mu_0\varepsilon_0\,\partial V/\partial t) = -\mu_0\vec{J}}$$

Uggh!

Gauge Transformations

- would like to simplify ....
- Anything that leaves $\vec{E}, \vec{B}$ unchanged is okay
  - E.g. adding a constant to V

Try $\vec{A}' = \vec{A} + \vec{\alpha}$ w/ $\vec{\alpha}, \beta$ functions
$V' = V + \beta$

Must have $\nabla \times \vec{A}' = \nabla \times \vec{A}$
so $\nabla \times \vec{\alpha} = 0$
$\Rightarrow \vec{\alpha} = \nabla \lambda$ w/ λ a scalar function

Must have $-\nabla V' - \partial \vec{A}'/\partial t = -\nabla V - \partial \vec{A}/\partial t$
so $\nabla \beta + \partial \vec{\alpha}/\partial t = 0$
$\Rightarrow \nabla \beta + \partial/\partial t (\nabla \lambda) = 0$
$\Rightarrow \nabla(\beta + \partial \lambda/\partial t) = 0$
$\Rightarrow \beta + \partial \lambda/\partial t = f(t)$ ind. of position
We'll pick $f(t) = 0$
$\Rightarrow \beta = -\partial \lambda/\partial t$

So any
$$\boxed{\begin{aligned} \vec{A}' &= \vec{A} + \nabla \lambda \\ V' &= V - \partial \lambda/\partial t \end{aligned}}$$
Gives same $\vec{E}$ and $\vec{B}$

$$\boxed{\text{Coulomb Gauge}}$$

Pick $\nabla \cdot \vec{A} = 0$

$$\Rightarrow \nabla^2 V = -\rho/\varepsilon_0 \quad \text{Poisson's Eq.}$$

But $\nabla^2 \vec{A} - \mu_0 \varepsilon_0 \partial^2 \vec{A}/\partial t^2 - \nabla \mu_0 \varepsilon_0 \partial V/\partial t = -\mu_0 \vec{J}$

yuck!

Note: V in Coulomb gauge only depends on instantaneous charge distribution. But $\vec{A}$ (& $\vec{B}$ and $\vec{E}$) have propagation delays since causal influences propagate at speed of light

$\boxed{\text{Lorenz Gauge}}$

Pick $\nabla \cdot \vec{A} = -\mu_0 \varepsilon_0 \partial V / \partial t$

$$\Rightarrow \nabla^2 V - \mu_0 \varepsilon_0 \partial^2 V / \partial t^2 = -\rho / \varepsilon_0$$
$$\nabla^2 \vec{A} - \mu_0 \varepsilon_0 \partial^2 \vec{A} / \partial t^2 = -\mu_0 \vec{J}$$

Symmetric in $V, \vec{A}$ and $\rho, \vec{J}$

Define d'Alembertian operator

$$\Box^2 = \nabla^2 - \mu_0 \varepsilon_0 \partial^2 / \partial t^2$$

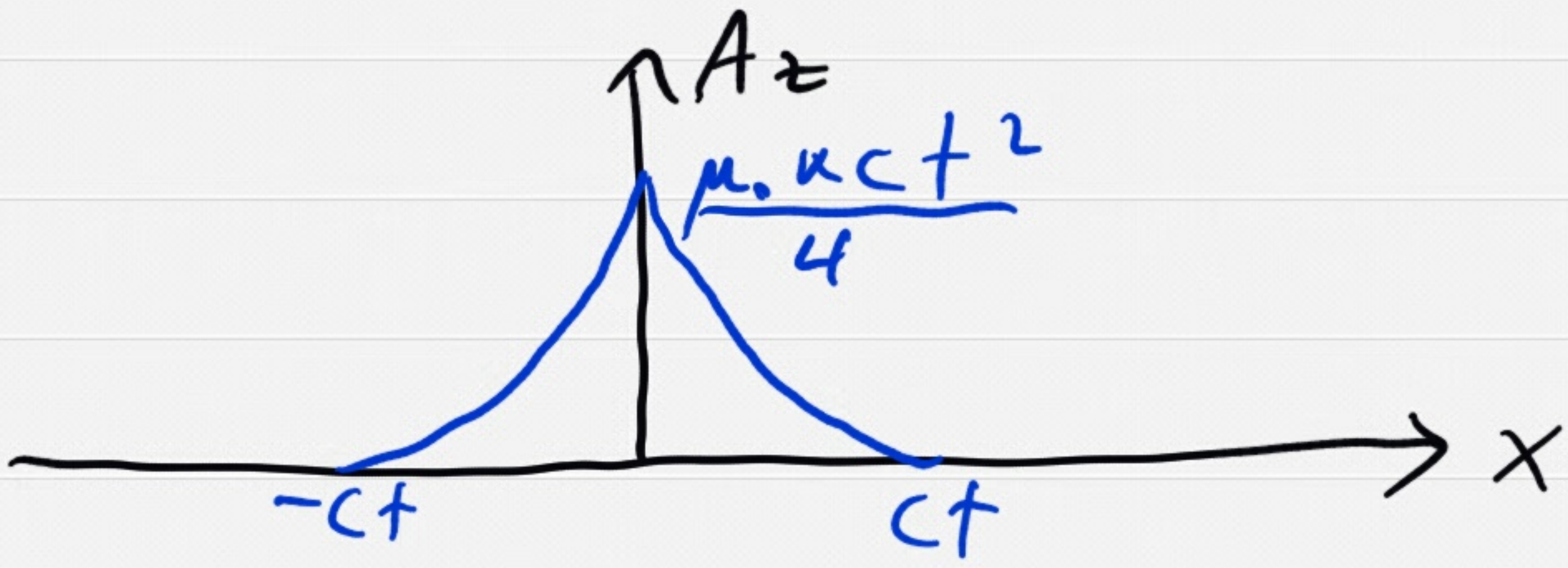
$$\boxed{\begin{aligned} \Box^2 V &= -\rho / \varepsilon_0 \\ \Box^2 \vec{A} &= -\mu_0 \vec{J} \end{aligned}}$$

w/ $\rho, \vec{J} = 0 \Rightarrow$ Wave Equation

w/ $\rho, \vec{J} \neq 0 \Rightarrow$ Inhomogeneous Wave Equation

Example:

$$V = 0 \ , \quad \vec{A} = \frac{\mu_0 \kappa}{4c}\left(ct - |x|\right)^2 \hat{z} \quad |x| < ct$$
$$= 0 \quad |x| > ct$$



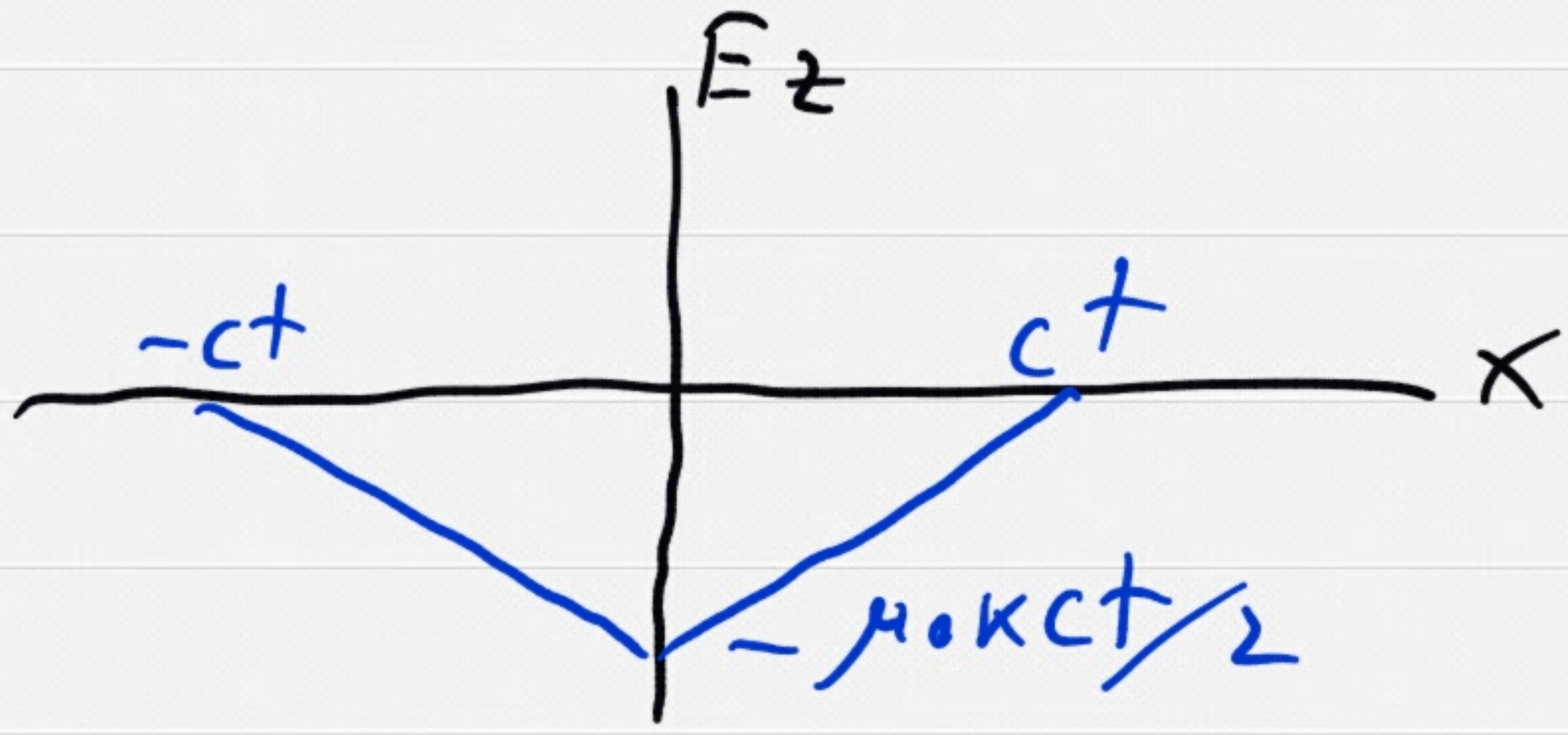
$$\nabla \cdot \vec{A} = \partial A_z / \partial z = 0$$
$$\partial V / \partial t = 0$$

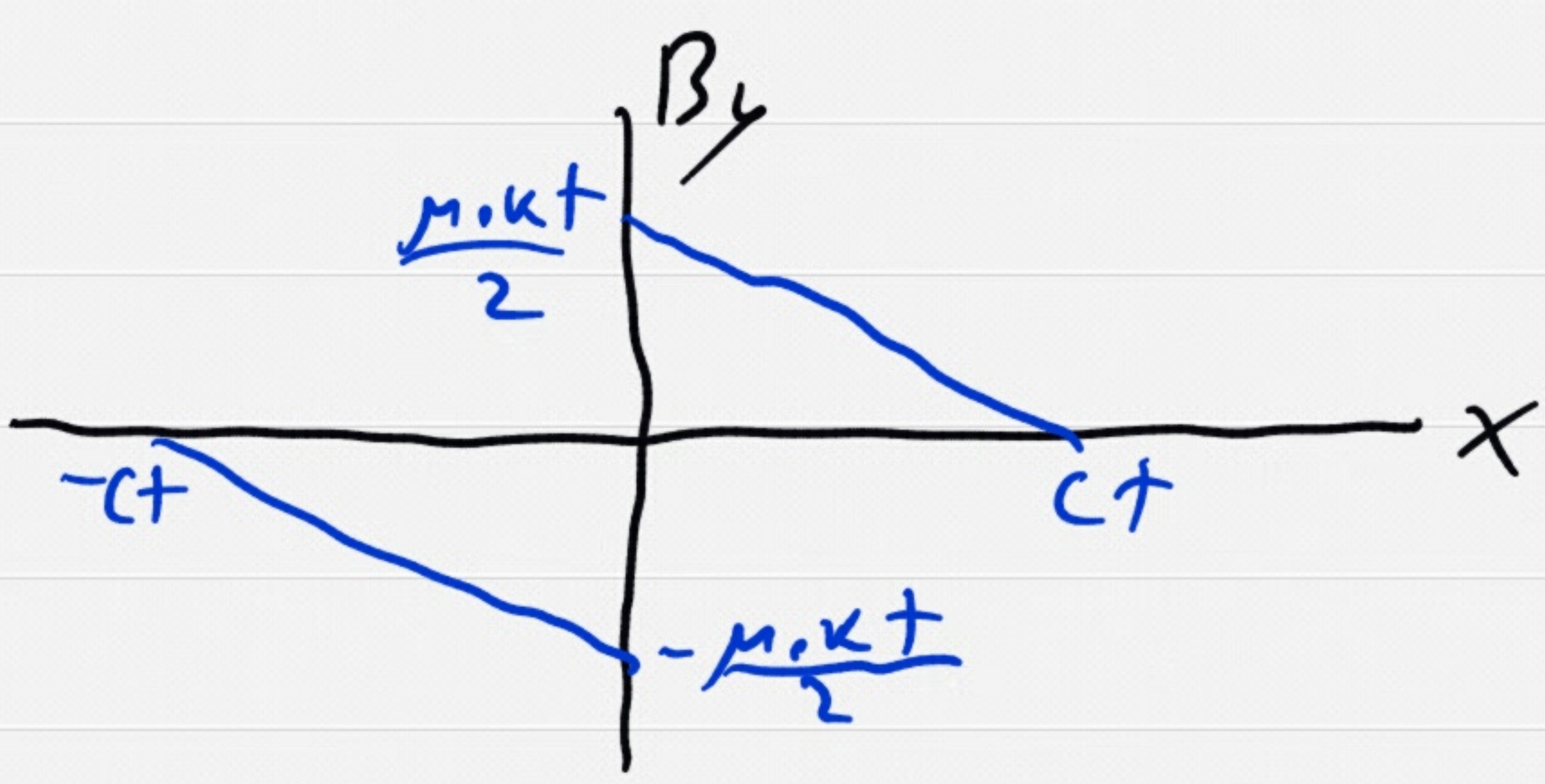
$\Rightarrow$ satisfies both Lorenz & Coulomb

$$\vec{E} = -\partial \vec{A}/\partial t = -\frac{\mu_0 \kappa}{2}\left(ct - |x|\right)\hat{z} \quad |x| < ct$$

$$\vec{B} = \nabla \times \vec{A} = -\partial A_z/\partial x \ \hat{y}$$

$$= \frac{\mu_0 \kappa}{2c}\left(ct - |x|\right) \cdot \mathrm{sign}(x)\ \hat{y} \quad |x| < ct$$





$\vec{B}$ has a discontinuity at $x = 0$

$$\Delta \vec{B}_{\parallel} = \mu_0 \vec{K}_f \times \hat{n} = \mu_0 \vec{K}_f \times \hat{x}$$
$$= \mu_0 k t \hat{y}$$

$$\Rightarrow \boxed{\vec{K}_f = k t \hat{z}}$$

surface current increasing w/ time

$\nabla \cdot \vec{E} = 0 \Rightarrow \rho_f = 0$

$\vec{E}$ continuous $\Rightarrow \sigma_f = 0$

$\vec{E}$ completely induced

- Note that current starts flowing at $t = 0$
- Fields don't respond until $t = |x|/c$
- Information propagates @ c

Happy Spring Break – And Wash Your Hands!