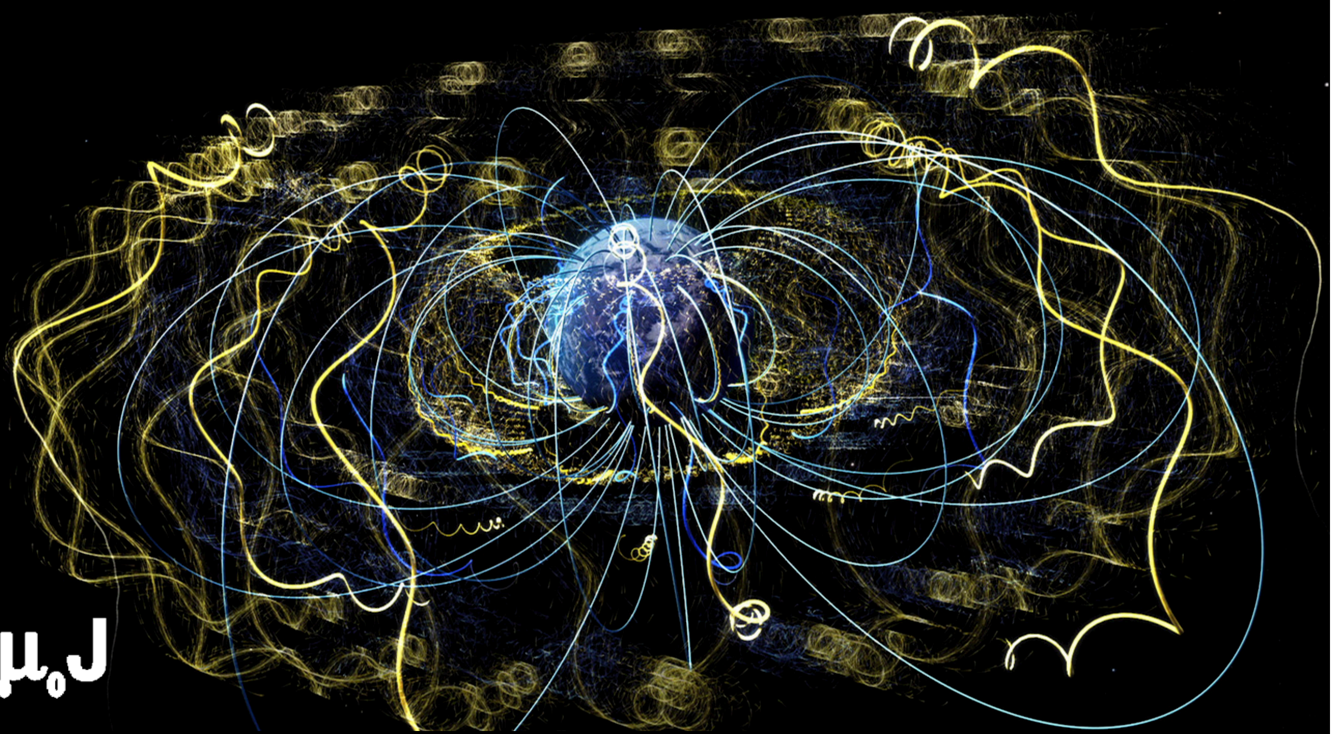
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism II: 3812

Professor Jasper Halekas
Van Allen 70
MWF 9:30-10:20 Lecture

Announcements

- We will be in an online format for the rest of the semester
  - We will hold both lectures and office hours virtually using Zoom
  - Homework assignments should be turned in at the usual time/date in electronic form by e-mail
  - The remaining exams will be take-home open-book format
  - The course schedule and syllabus have been modified to reflect these and other changes

Key Syllabus/Schedule Changes

- Lectures and Office Hours
  - Moved to virtual format
- Material Covered
  - Coverage of Ch. 12 reduced to “selected topics” due to extended spring break
- Homework
  - Electronic turn-in rather than physical
  - Number of assignments reduced (10 instead of 11)
    - Delayed by one week due to extended spring break
- Exams
  - Midterm 2 delayed by one week due to extended spring break
  - Midterm 2 and Final both changed to take-home open-book format with two-hour duration

Revised Schedule

REVISED VERSION 3/18/2020

Physics 3812 Electricity and Magnetism II 2020 Schedule

Dates	Week	Reading (Due Monday unless noted)	HW Due Friday	Notes
Jan. 20-24	Week 1	Ch. 7.1	No HW	*Holiday Monday 1/20*
Jan. 27-31	Week 2	Ch. 7.2	HW 1	
Feb. 3-7	Week 3	Ch. 7.3	HW 2	
Feb. 10-14	Week 4	Ch. 8	HW 3	
Feb. 17-21	Week 5	Ch. 9.1-9.2	HW 4	
Feb. 24-28	Week 6	No Reading	No HW	Midterm 1 Ch. 7-9.2 Wednesday Feb. 26
Mar. 2-6	Week 7	Ch. 9.3-9.4	HW 5	
Mar. 9-13	Week 8	Ch. 9.5-10.1	HW 6	
Mar. 16-20	Spring Break	No Reading	No HW	Spring Break, Woohoo!
Mar. 23-27	Extended Spring Break	No Reading	No HW	COVID-19, isn't it fun?
Mar. 30-Apr. 3	Week 9	Ch. 10.2-10.3	HW 7	
Apr. 6-10	Week 10	Ch. 11.1	HW 8	
Apr. 13-17	Week 11	Ch. 11.2	HW 9	
Apr. 20-24	Week 12	No Reading	No HW	Midterm 2 Ch. 9.3-11 Wednesday Apr 22
Apr. 27-May 1	Week 13	Ch. 12.1-12.3 (Selected Topics)	HW 10	
May 4-8	Week 14	No Reading	No HW	
May 11-15	Finals Week	No Reading	No HW	Final Exam Ch. 7-12 Monday May 11

Discussion of Values

- The next few weeks (or months) are going to be a learning experience for us all
- Communication will be even more important during this time period
  - Please continue to feel free to ask questions during lecture
    - You can virtually "raise your hand" if you have a question
    - You can use the "chat" feature to ask a question by text
    - You can also just un-mute and speak as needed
  - Please raise any other issues or concerns you have, either during virtual office hours or by e-mail

10.2 | Continuous Distributions

Retarded Potentials

Recall Eqs. for $V, \vec{A}$
in Lorenz Gauge

$$\Box^2 \vec{A} = \nabla^2 \vec{A} - \frac{1}{c^2} \partial^2 \vec{A} / \partial t^2 = -\mu_0 \vec{J}$$

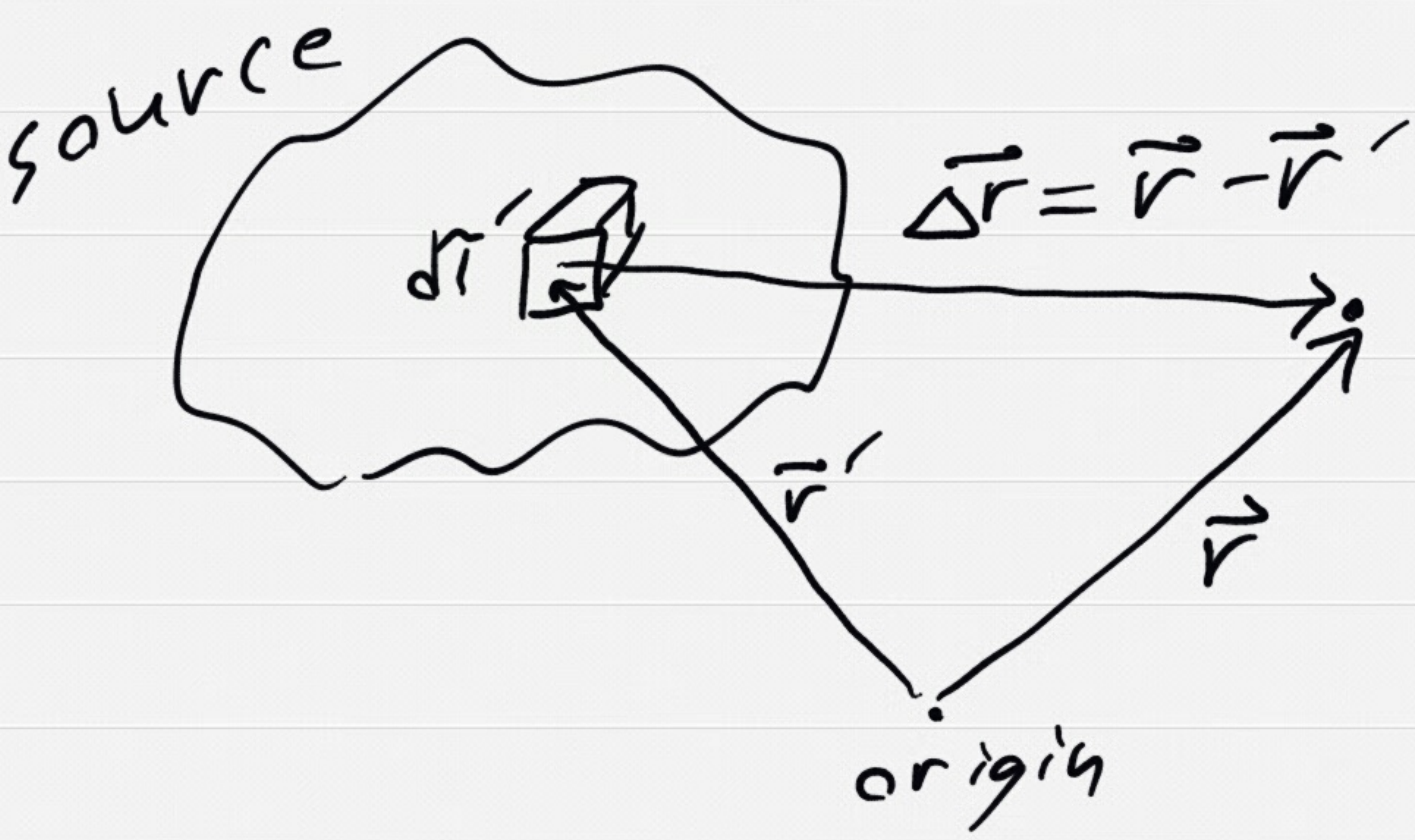
$$\Box^2 V = \nabla^2 V - \frac{1}{c^2} \partial^2 V / \partial t^2 = -\rho / \varepsilon_0$$

- static case

$$\nabla^2 \vec{A} = -\mu_0 \vec{J} \quad , \quad \nabla^2 V = -\rho / \varepsilon_0$$

$$\Rightarrow \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,')}{\Delta r} d\tau'$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{V(\vec{r}\,')}{\Delta r} d\tau'$$



But what if $\vec{r}\,'$ changes??

Retarded Potentials

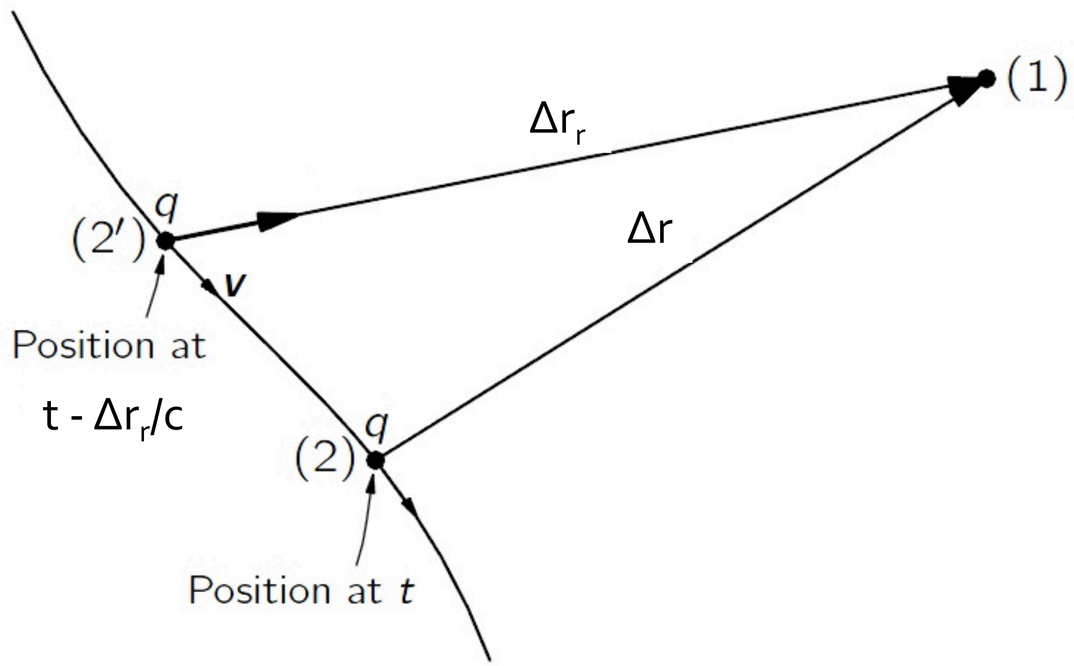


Fig. 21-1. The fields at (1) at the time t depend on the position (2′) occupied by the charge q at the time $t - \Delta r_r/c$

- Information about source takes time to propagate at the speed of light

$$\Delta t = \Delta r / c$$

- Effect seen at time t at $\vec{r}$ happened at $t_r = t - \Delta r / c$ at $\vec{r}'$
  - t_r = "retarded time"

Guess: $$V(\vec{r}, t) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\vec{r}', t_r)}{\Delta r} d\tau'$$
$$\vec{A}(\vec{r}, t) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}', t_r)}{\Delta r} d\tau'$$

In other words, potentials depend on "old" distribution of sources

We'll prove it for V

$$\nabla V = \nabla \left(\frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\vec{r}', t_r)}{\Delta r} d\tau' \right)$$
$$= \frac{1}{4\pi\varepsilon_0} \int \nabla \left(\frac{\rho(\vec{r}', t_r)}{\Delta r} \right) d\tau'$$

$$= \frac{1}{4\pi\varepsilon_0} \int \left[\frac{\nabla \rho(\vec{r}', t_r)}{\Delta r} + \rho(\vec{r}', t_r) \nabla \frac{1}{\Delta r} \right] d\tau'$$

ρ does not depend on $\vec{r}$
but it does depend on $t_r = t - \frac{\Delta r}{c}$

$$\nabla \rho = \partial \rho / \partial t_r \, \nabla t_r$$

$$\partial \rho / \partial t_r = \partial \rho / \partial t = \dot{\rho}$$
$$\nabla t_r = -\frac{1}{c} \nabla (\Delta r)$$

$$\Rightarrow \nabla \rho = -\frac{1}{c} \dot{\rho} \, \nabla (\Delta r)$$

Identities : $\nabla (\Delta r) = \hat{\Delta r}$
$\nabla \left(\frac{1}{\Delta r} \right) = - \hat{\Delta r} / \Delta r^2$

$$\Rightarrow \nabla V = \frac{1}{4\pi\varepsilon_0} \int \left[-\frac{\dot{\rho}}{c} \frac{\hat{\Delta r}}{\Delta r} - \rho \frac{\hat{\Delta r}}{\Delta r^2} \right] d\tau'$$

Next $\nabla^2 V = \frac{1}{4\pi\varepsilon_0} \int \Big[-\frac{1}{c} \nabla \dot{\rho} \cdot \frac{\hat{\Delta r}}{\Delta r} - \frac{\dot{\rho}}{c} \nabla \cdot \left(\frac{\hat{\Delta r}}{\Delta r} \right)$
$- \nabla \rho \cdot \frac{\hat{\Delta r}}{\Delta r^2} - \rho \nabla \cdot \left(\frac{\hat{\Delta r}}{\Delta r^2} \right) \Big] d\tau'$

$$\nabla \dot{\rho} = -\frac{1}{c} \ddot{\rho} \, \nabla (\Delta r) = -\frac{\ddot{\rho}}{c} \hat{\Delta r}$$

Identities : $\nabla \cdot \left(\frac{\hat{\Delta r}}{\Delta r} \right) = \frac{1}{\Delta r^2}$
$\nabla \cdot \left(\frac{\hat{\Delta r}}{\Delta r^2} \right) = 4\pi \delta^3 (\vec{\Delta r})$

$$\Rightarrow \nabla^2 V = \frac{1}{4\pi\varepsilon_0} \int \Big[\frac{\ddot{\rho}}{c^2} \hat{\Delta r} \cdot \frac{\hat{\Delta r}}{\Delta r} - \frac{\dot{\rho}}{c} \cdot \frac{1}{\Delta r^2}$$
$$+ \frac{\dot{\rho}}{c} \hat{\Delta r} \cdot \frac{\hat{\Delta r}}{\Delta r^2} - \rho \cdot 4\pi \delta^3 (\vec{\Delta r}) \Big] d\tau'$$

$$= \frac{1}{4\pi\varepsilon_0} \int \left[\frac{1}{c^2} \frac{\ddot{\rho}}{\Delta r} - \rho \cdot 4\pi \delta^3 (\vec{\Delta r}) \right] d\tau'$$

$$\Rightarrow \nabla^2 V = \frac{1}{c^2} \partial^2/\partial t^2 \left(\frac{1}{4\pi\varepsilon_0} \int \frac{\rho}{\Delta r} d\tau' \right) - \rho/\varepsilon_0$$

$$\Rightarrow \nabla^2 V = \frac{1}{c^2} \partial^2 V/\partial t^2 - \rho/\varepsilon_0$$

$$\Rightarrow \Box^2 V = -\rho/\varepsilon_0 \quad //$$

- So V & $\vec{A}$ just depend on retarded distribution of charge & current

- Beware!!!

The same is not true for $\vec{E}$ and $\vec{B}$

You can't just put the retarded charge & current distribution into Coulomb's Law, Biot-Savart Law

Jefimenko's Equations

$$\vec{E}(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\int\left[\frac{\rho(\vec{r}',t_r)}{\Delta r^2}\hat{\Delta r} + \frac{\dot{\rho}(\vec{r}',t_r)}{c\,\Delta r}\hat{\Delta r} - \frac{\dot{\vec{J}}(\vec{r}',t_r)}{c^2\,\Delta r}\right]d\tau'$$

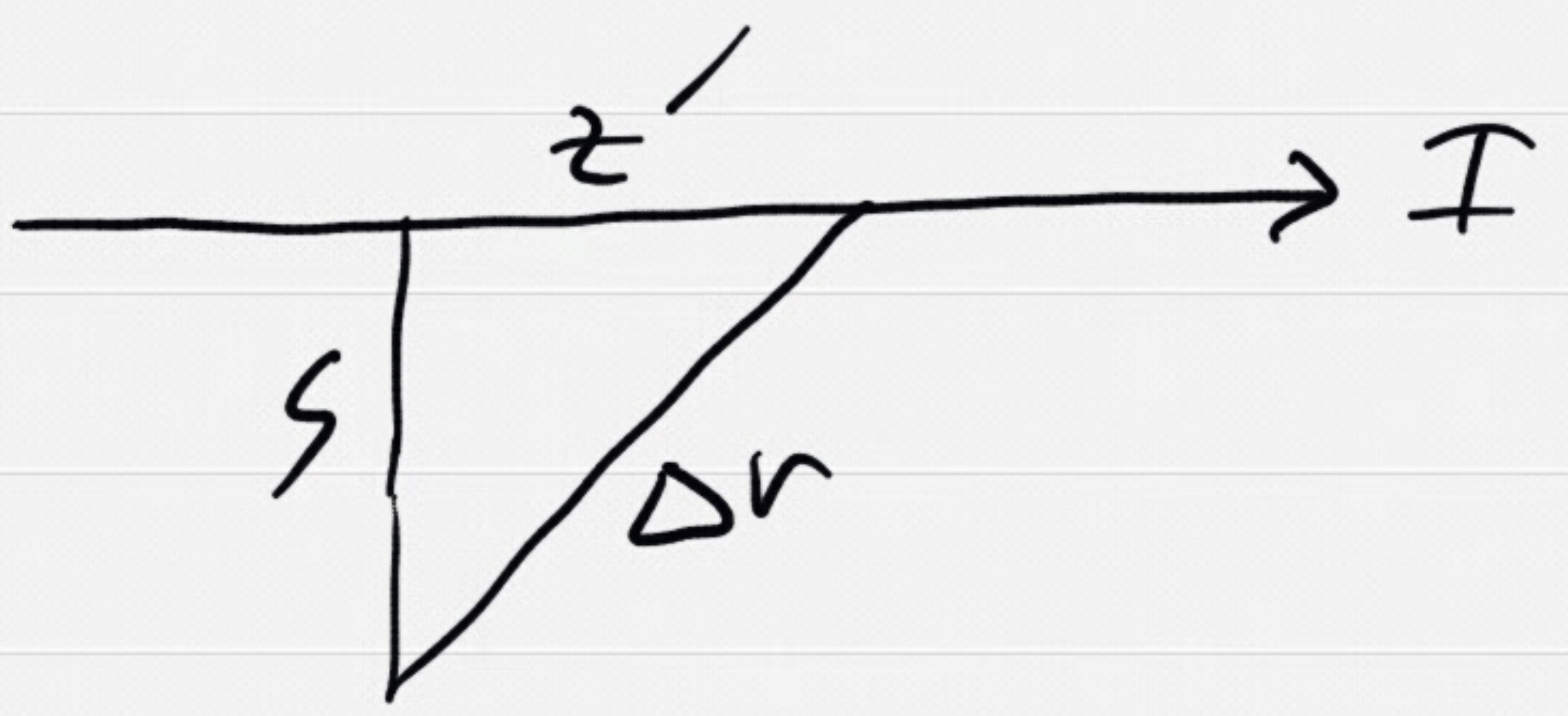
$$\vec{B}(\vec{r},t) = \frac{\mu_0}{4\pi}\int\left(\left[\frac{\vec{J}(\vec{r}',t_r)}{\Delta r^2} + \frac{\dot{\vec{J}}(\vec{r}',t_r)}{c\,\Delta r}\right] \times \hat{\Delta r}\right)d\tau'$$

- Leading order terms are Coulomb & Biot-Savart
- These equations are so ugly no one uses them!

Example

Line current $I = I_0$, turns on everywhere at $t = 0$

$$I(t) = \begin{cases} 0 & t \le 0 \\ I_0 & t > 0 \end{cases}$$



$\rho = 0 \Rightarrow V = 0$

$$\vec{A}(\vec{r}, t) = \frac{\mu_0}{4\pi} \int_{-\infty}^{\infty} \frac{I(t_r)\,\hat{z}}{\Delta r}\, dz'$$

$$= \frac{\mu_0}{2\pi} \int_{0}^{\infty} \frac{I(t_r)\,\hat{z}}{\Delta r}\, dz'$$

$$t_r = t - \Delta r / c = t - \frac{1}{c}\sqrt{s^2 + z'^2}$$

$$I(t_r) = \begin{cases} 0 & t_r \le 0 \\ I_0 & t_r > 0 \end{cases}$$

$$t_r > 0 \Rightarrow t > \frac{1}{c}\sqrt{s^2 + z'^2}$$

$$\Rightarrow z'^2 < (ct)^2 - s^2$$

So $\vec{A}(\vec{r},t) = \frac{\mu_0 I_0}{2\pi}\hat{z}\int_0^{\sqrt{(ct)^2-s^2}} \frac{1}{\sqrt{s^2+z'^2}}\,dz'$

$$= \frac{\mu_0 I_0}{2\pi}\hat{z}\,\ln\left(\sqrt{s^2+z'^2}+z'\right)\Big|_0^{\sqrt{(ct)^2-s^2}}$$

$$= \frac{\mu_0 I_0}{2\pi}\hat{z}\,\ln\left(\frac{ct+\sqrt{(ct)^2-s^2}}{s}\right)$$

$\vec{E}(\vec{r},t) = -\partial\vec{A}/\partial t$

$$= -\frac{\mu_0 I_0}{2\pi}\hat{z}\cdot\frac{s\cdot\frac{1}{s}\left(c+\frac{1/2}{\sqrt{(ct)^2-s^2}}\cdot 2c^2t\right)}{ct+\sqrt{(ct)^2-s^2}}$$

$$= -\frac{\mu_0 I_0}{2\pi}\hat{z}\,\frac{c\left(1+ct/\sqrt{(ct)^2-s^2}\right)}{ct+\sqrt{(ct)^2-s^2}}$$

$$= \boxed{-\frac{\mu_0 I_0}{2\pi}\frac{c\,\hat{z}}{\sqrt{(ct)^2-s^2}}}$$

$\vec{B}(\vec{r},t) = \nabla\times\vec{A} = -\partial A_z/\partial s\,\hat{\varphi}$

$$= -\frac{\mu_0 I_0}{2\pi}\hat{\varphi}\,\frac{s}{ct+\sqrt{(ct)^2-s^2}}\cdot\left(-ct/s^2+\frac{1}{2\sqrt{(ct/s)^2-1}}\cdot 2\frac{ct}{s}\cdot -ct/s^2\right)$$

$$= -\frac{\mu_0 I_0}{2\pi}\hat{\varphi}\cdot\frac{-ct/s}{ct+\sqrt{(ct)^2-s^2}}\left(1+\frac{ct/s}{\sqrt{(ct/s)^2-1}}\right)$$

$$= \boxed{\frac{\mu_0 I_0}{2\pi s}\frac{ct\,\hat{\varphi}}{\sqrt{(ct)^2-s^2}}}$$

$t\to\infty \Rightarrow \vec{E}\to 0,\ \vec{B}\to\frac{\mu_0 I_0}{2\pi s}\hat{\varphi}$

static solutions