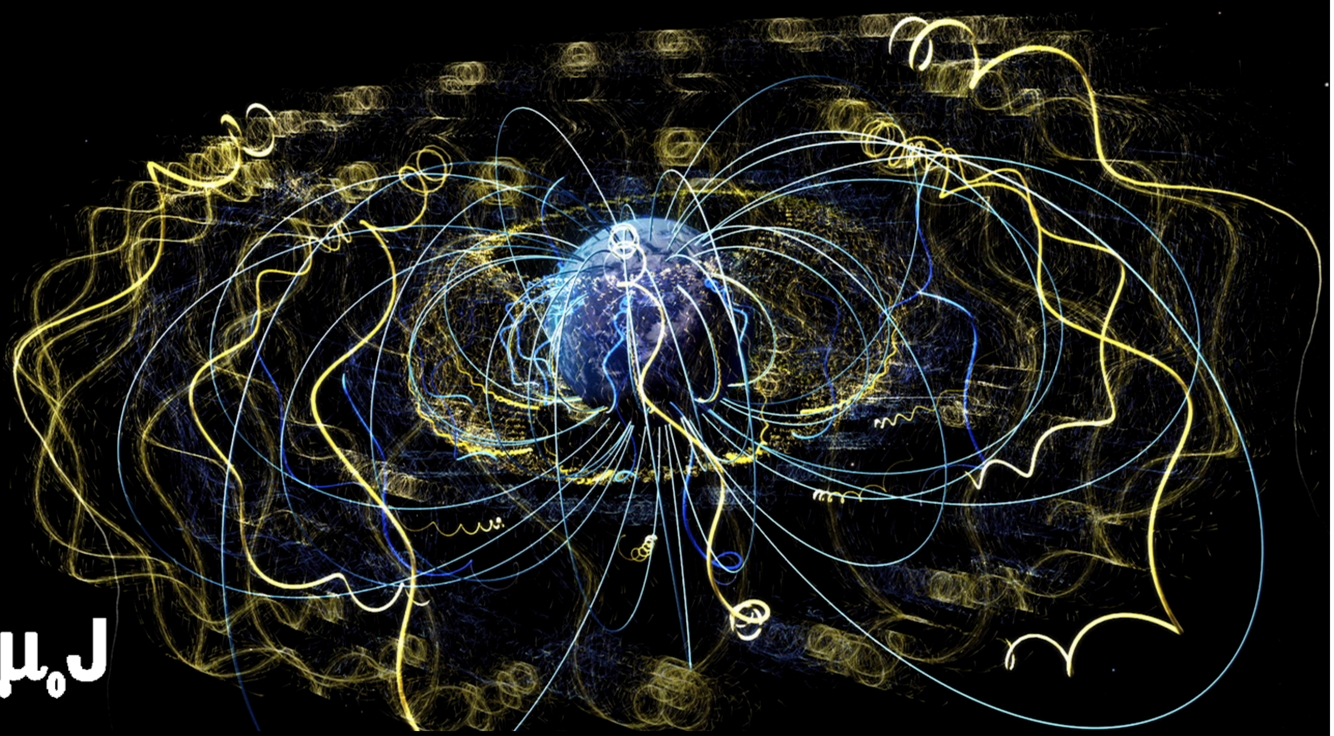
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

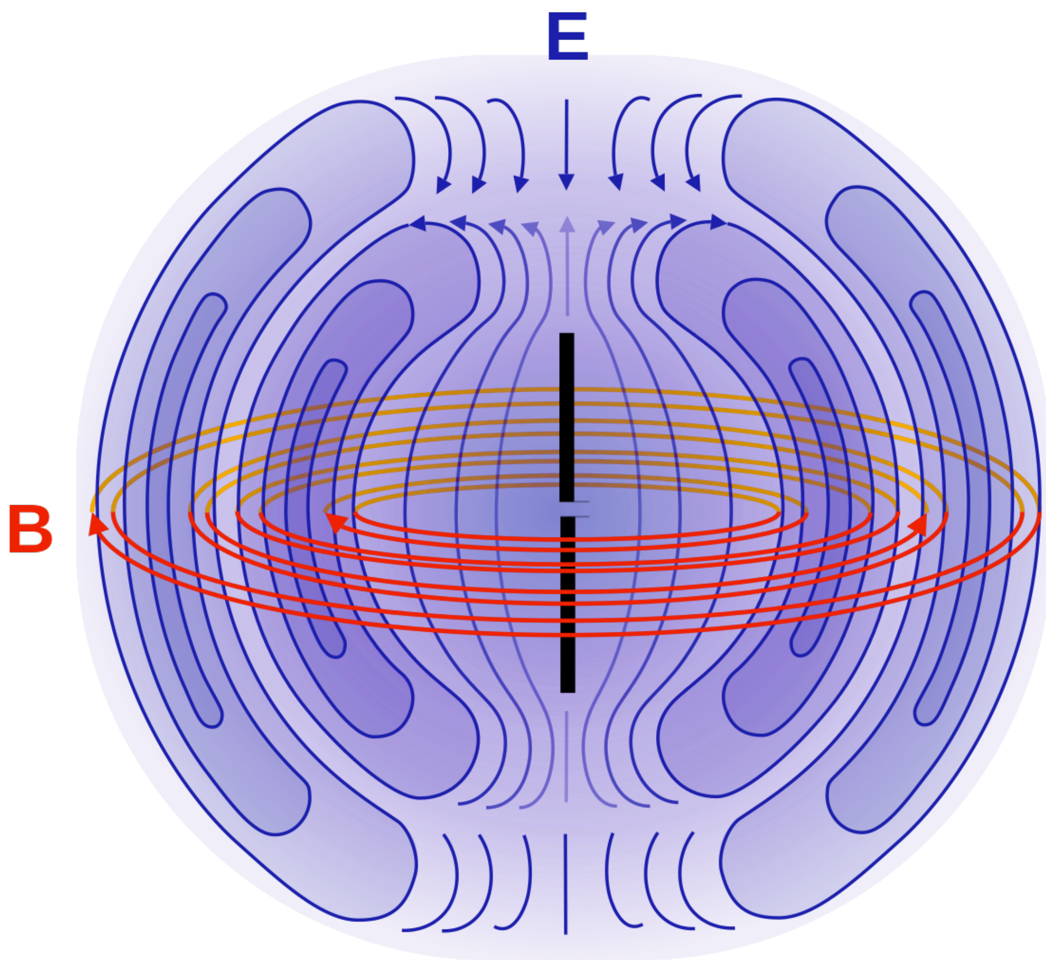
$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism II: 3812

Professor Jasper Halekas
Virtual by Zoom!
MWF 9:30-10:20 Lecture

Electric Dipole Radiation



Dipole Fields: Summary

$$\vec{E} \cong -\frac{\mu_0 p_0 \omega^2}{4\pi} \frac{\sin\theta}{r} \cos(\omega(t - r/c))\,\hat{\theta}$$

$$\vec{B} \cong -\frac{\mu_0 p_0 \omega^2}{4\pi c} \frac{\sin\theta}{r} \cos(\omega(t - r/c))\,\hat{\varphi}$$

$$\vec{S}(\vec{r},t) = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$$

$$\boxed{\vec{S} = \frac{\mu_0}{c}\left\{\frac{p_0\omega^2}{4\pi} \frac{\sin\theta}{r} \cos(\omega(t - r/c))\right\}^2 \hat{r}}$$

$$\boxed{\langle \vec{S} \rangle = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \frac{\sin^2\theta}{r^2}\,\hat{r}}$$

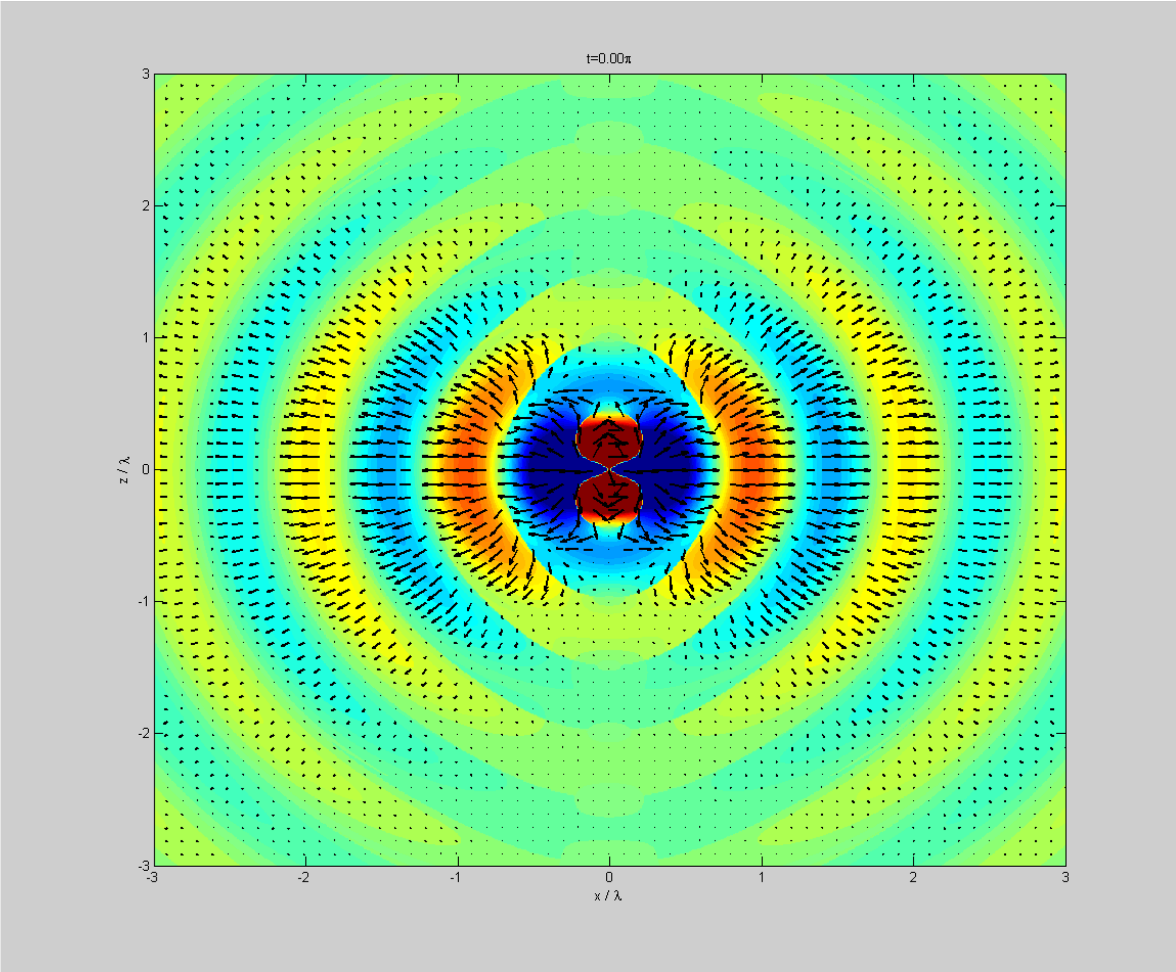
$$\langle P \rangle = \oint \langle \vec{S} \rangle \cdot d\vec{a}$$

$$= \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \int_0^{2\pi}\int_0^{\pi} \frac{\sin^2\theta}{r^2} \cdot r^2 \sin\theta \, d\theta \, d\varphi$$

$$= \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \cdot 2\pi \cdot (2 - 2/3)$$

$$\boxed{\langle P \rangle = \frac{\mu_0 p_0^2 \omega^4}{12\pi c}}$$

Electric Dipole Radiation



Arrows = Poynting Flux

Rayleigh Scattering

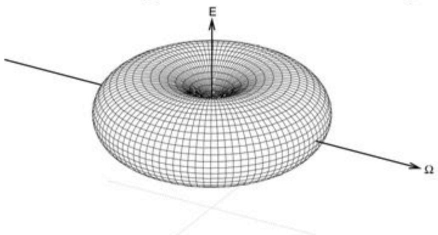
ω^4 term $\Rightarrow$ blue light more efficiently radiated by dipole molecules

- inverse process means blue light also more efficiently absorbed by dipole molecules

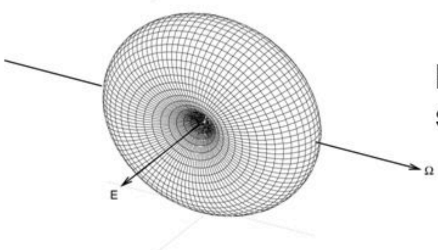
$\Rightarrow$ more efficient scattering of blue light

$\Rightarrow$ blue sky & red sunsets

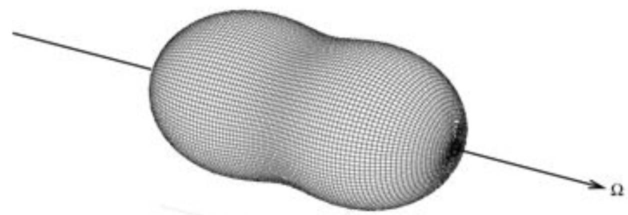
Rayleigh Scattering



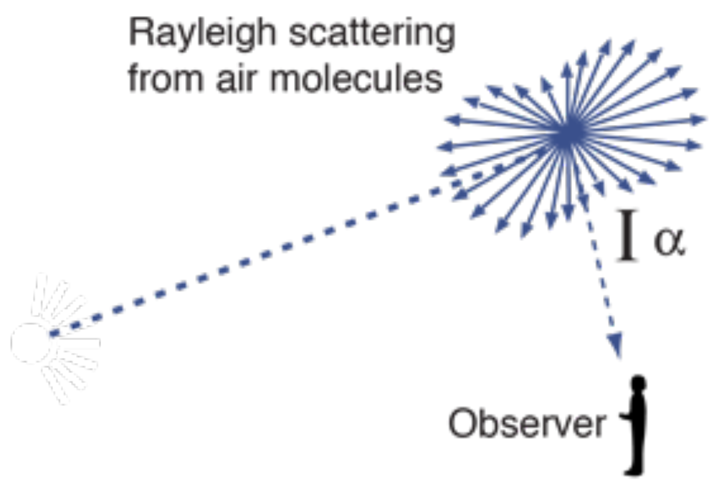
vertical polarization state



horizontal polarization state



Average Radiation Intensity

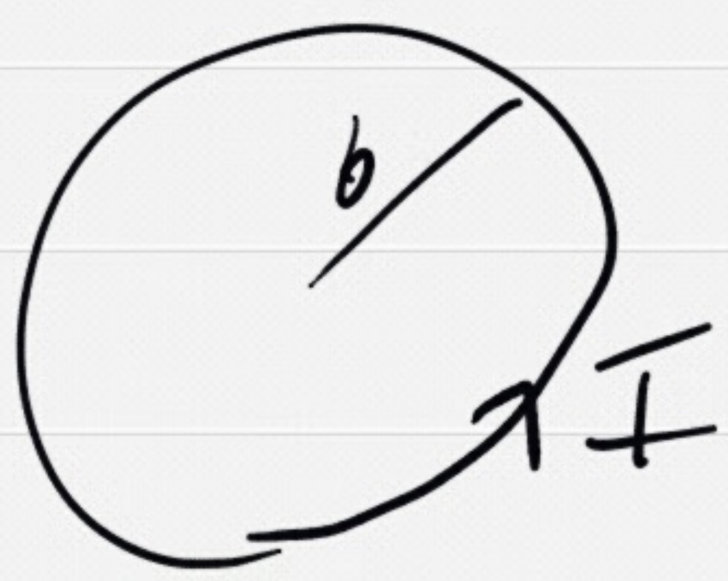


$$I \,\alpha\, \frac{1}{\lambda^4}$$

The strong wavelength dependence of Rayleigh scattering enhances the short wavelengths, giving us the blue sky.

The scattering at 400 nm is 9.4 times as great as that at 700 nm for equal incident intensity.

Magnetic Dipole Radiation



$$I(t) = I_0 \cos(\omega t)$$

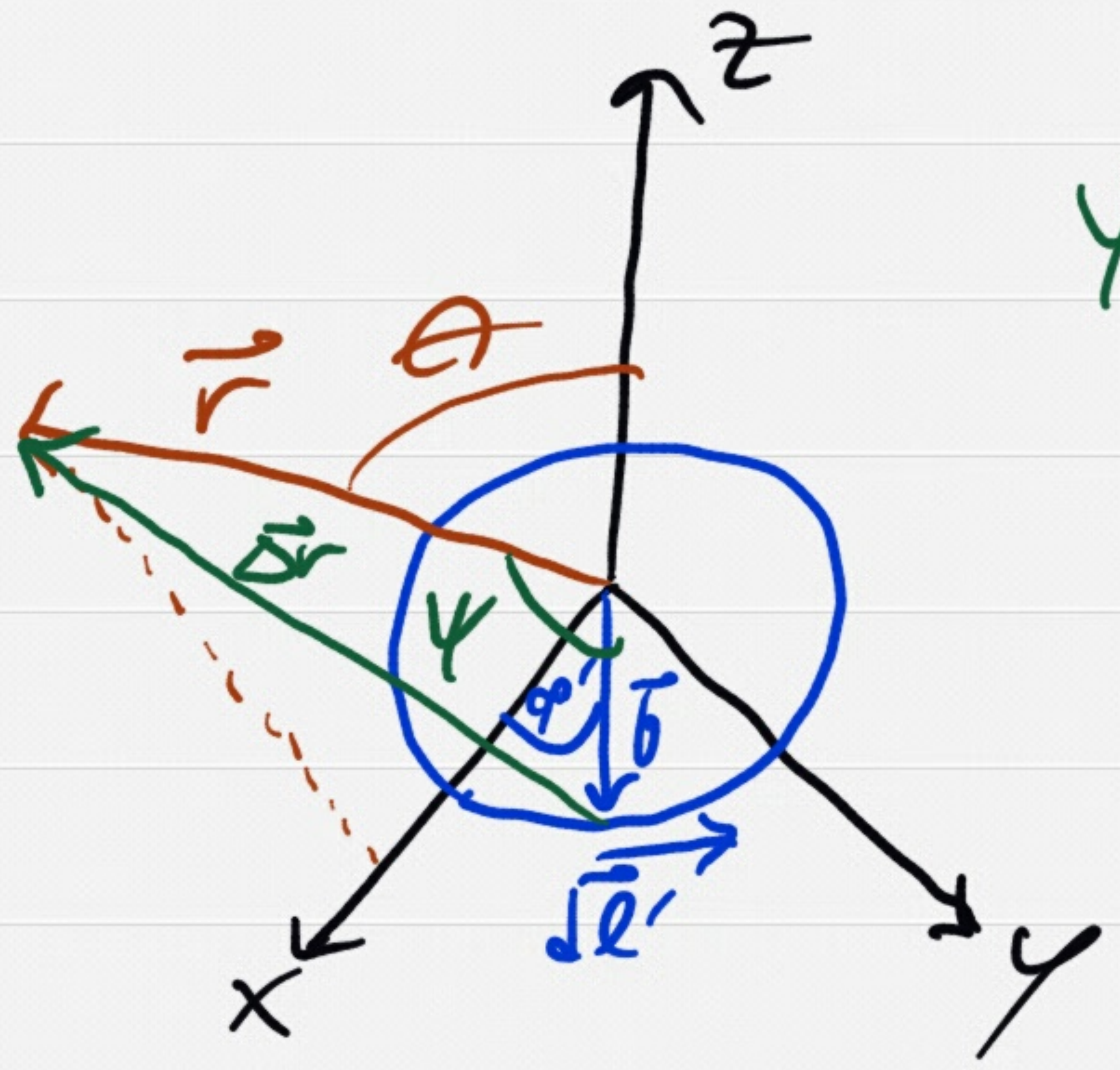
$$\begin{aligned}\vec{m} &= I\vec{a} \\ &= \pi b^2 I_0 \cos(\omega t)\,\hat{z} \\ &= m_0 \cos(\omega t)\,\hat{z} \\ &= \vec{m}_0 \cos(\omega t)\end{aligned}$$

$V(\vec{r},t) = 0$ since net neutral

$$\begin{aligned}\vec{A}(\vec{r},t) &= \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,',t_r)\,d\tau'}{\Delta r} \\ &= \frac{\mu_0}{4\pi}\int \frac{\vec{I}(\vec{r}\,',t_r)\,dl'}{\Delta r} \\ &= \frac{\mu_0}{4\pi}\int \frac{I_0\cos\left(\omega\left(t-\frac{\Delta r}{c}\right)\right)d\vec{l}\,'}{\Delta r}\end{aligned}$$

$\vec{A}$ must be symmetric in φ

- Compute for point in x-z plane for simplicity



ψ = angle between $\vec{b}$ and $\vec{r}$

In x-z plane, $\vec{A} = A\hat{y}$

$$dl'_y = b\cos\varphi'$$

$$\Rightarrow \vec{A}(\vec{r},t) = \frac{\mu_0 I_0 b}{4\pi}\hat{y}\int_0^{2\pi}\frac{\cos(\omega(t-\Delta r/c))\cos\varphi'\,d\varphi'}{\Delta r}$$

$$\Delta r = \sqrt{r^2+b^2-2rb\cos\psi}$$

For $y=0$, $\vec{r} = r\sin\theta\,\hat{x} + r\cos\theta\,\hat{z}$

Meanwhile $\vec{b} = b\cos\varphi'\,\hat{x} + b\sin\varphi'\,\hat{y}$

so $\vec{r}\cdot\vec{b} = rb\cos\psi = rb\sin\theta\cos\varphi'$

$$\Rightarrow \Delta r = \sqrt{r^2+b^2-2rb\sin\theta\cos\varphi'}$$

Assume $b \ll r$

$$\Rightarrow \Delta r \cong r\left(1 - \frac{b}{r}\sin\theta\cos\varphi'\right)$$

$$1/\Delta r \cong \frac{1}{r}\left(1 + \frac{b}{r}\sin\theta\cos\varphi'\right)$$

and $\Rightarrow$

$$\cos\left(w\left(t - \frac{\Delta r}{c}\right)\right)$$

$$\cong \cos\left(w\left(t - r/c\right)\right)\cos\left(\frac{wb}{c}\sin\theta\cos\varphi'\right)$$

$$- \sin\left(w\left(t - r/c\right)\right)\sin\left(\frac{wb}{c}\sin\theta\cos\varphi'\right)$$

Assume $b \ll \lambda = c/w$

$$\Rightarrow \cos\left(w\left(t - \frac{\Delta r}{c}\right)\right)$$

$$\cong \cos\left(w\left(t - r/c\right)\right)$$

$$- \frac{wb}{c}\sin\theta\sin\varphi'\,\sin\left(w\left(t - r/c\right)\right)$$

So
$$\vec{A}(\vec{r},t) \cong \frac{\mu_0 I_0 b}{4\pi r}\hat{\varphi}\int_0^{2\pi}\Big\{\cos\left(w\left(t - r/c\right)\right)$$

$$+ b\sin\theta\cos\varphi' \cdot \left(\frac{\cos\left(w\left(t - r/c\right)\right)}{r} - \frac{w}{c}\sin\left(w\left(t - r/c\right)\right)\right)\Big\}\cos\varphi'\,d\varphi'$$

$$\int_0^{2\pi}\cos\varphi'\,d\varphi' = 0, \quad \int_0^{2\pi}\cos^2\varphi'\,d\varphi' = \pi$$

$$\Rightarrow \vec{A}(\vec{r},t) \cong \frac{\mu_0 I_0 b}{4\pi r}\cdot\pi\hat{\varphi}\, b\sin\theta \cdot \Big\{\frac{1}{r}\cos\left(w\left(t - r/c\right)\right)$$
$$- w/c\,\sin\left(w\left(t - r/c\right)\right)\Big\}$$

in x-z plane

For general position

$$\boxed{\vec{A}(\vec{r},t) \cong \frac{\mu_0 m_0}{4\pi} \frac{\sin\theta}{r} \left\{ \frac{\cos(\omega(t - r/c))}{r} - \frac{\omega}{c} \sin(\omega(t - \frac{r}{c})) \right\} \hat{\varphi}}$$

Assume $r \gg \lambda = c/\omega$

($r \gg \lambda \gg d$)

$$\Rightarrow \boxed{\vec{A}(\vec{r},t) \cong -\frac{\mu_0 m_0 \omega}{4\pi c} \frac{\sin\theta}{r} \sin(\omega(t - r/c))\, \hat{\varphi}}$$

Note: $\omega \to 0$

$$\Rightarrow \vec{A}(\vec{r},t) \to \frac{\mu_0 m_0 \sin\theta}{4\pi r^2} \hat{\varphi}$$

static magnetic dipole potential

Magnetic Dipole Radiation Fields

$$\vec{E} = -\partial \vec{A}/\partial t$$

$$\boxed{\vec{E} = \frac{\mu_0 m_0 \omega^2}{4\pi c} \frac{\sin\theta}{r} \cos(\omega(t - r/c))\,\hat{\varphi}}$$

$$\vec{B} = \nabla \times \vec{A} = \frac{1}{r\sin\theta} \partial/\partial\theta \,(\sin\theta\, A_\varphi)\,\hat{r}$$

$$- \frac{1}{r} \partial/\partial r\,(r A_\varphi)\,\hat{\theta}$$

$$\sim \frac{1}{r} \partial/\partial r\,(r A_\varphi)\,\hat{\theta}$$

$$\cong -1/r\, \partial/\partial r \left(-\frac{\mu_0 m_0 \omega}{4\pi c} \sin\theta \sin(\omega(t - r/c)) \right)\hat{\theta}$$

$$\boxed{\vec{B} \cong -\frac{\mu_0 m_0 \omega^2}{4\pi c^2} \frac{\sin\theta}{r} \cos(\omega(t - r/c))\,\hat{\theta}}$$

Similar to electric case, but $\vec{E}$ azimuthal & $\vec{B}$ polar

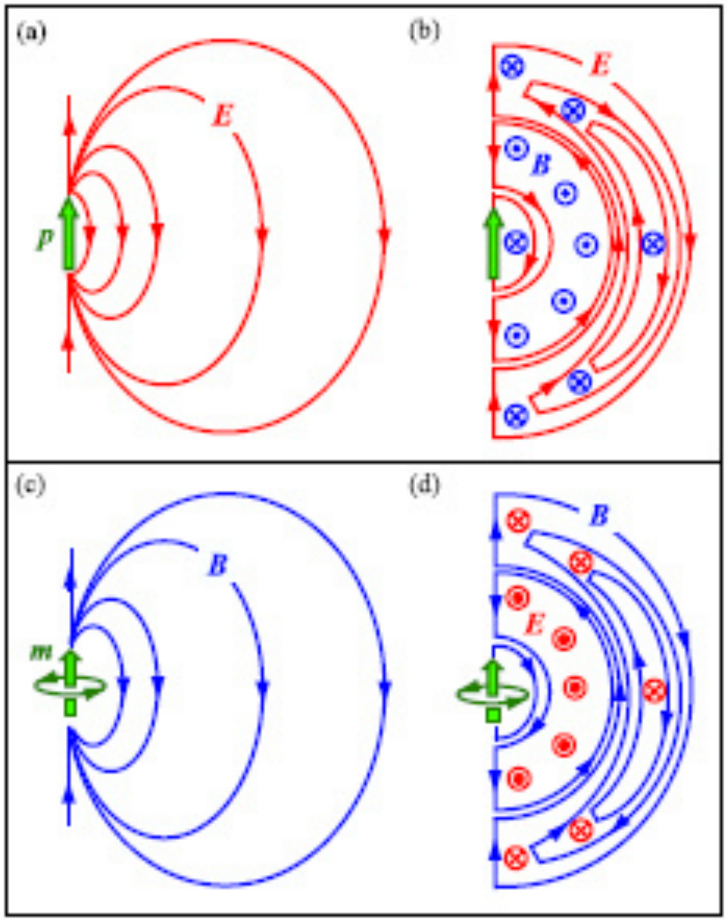
$$\vec{S}(\vec{r}, t) = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$$

$$\boxed{\vec{S} = \frac{\mu_0}{c}\left\{\frac{m_0 \omega^2}{4\pi c} \frac{\sin\theta}{r} \cos(\omega(t - r/c))\right\}^2 \hat{r}}$$

$$\boxed{\langle \vec{S} \rangle = \frac{\mu_0 m_0^2 \omega^4}{32\pi^2 c^3} \frac{\sin^2\theta}{r^2}\,\hat{r}}$$

$$\boxed{P = \frac{\mu_0 m_0^2 \omega^4}{12\pi c^3}}$$

Electric Vs. Magnetic Dipole Radiation



$$\frac{P_{magnetic}}{P_{electric}} = \frac{m_0^2}{p_0^2 c^2}$$

$$= \frac{(\pi b^2 I_0)^2}{(q_0 d)^2 c^2}$$

$$= \frac{(\pi b^2 q_0 \omega)^2}{(q_0 d)^2 c^2}$$

$$\sim \frac{\omega^2 b^2}{c^2} \quad \text{if } b \sim d$$

But, we assumed

$$d \ll c/\omega \ll r$$

So $\frac{\omega b}{c} \ll 1$

and $P_{magnetic} \ll P_{electric}$

unless we somehow set up scenario that eliminates electric dipole component