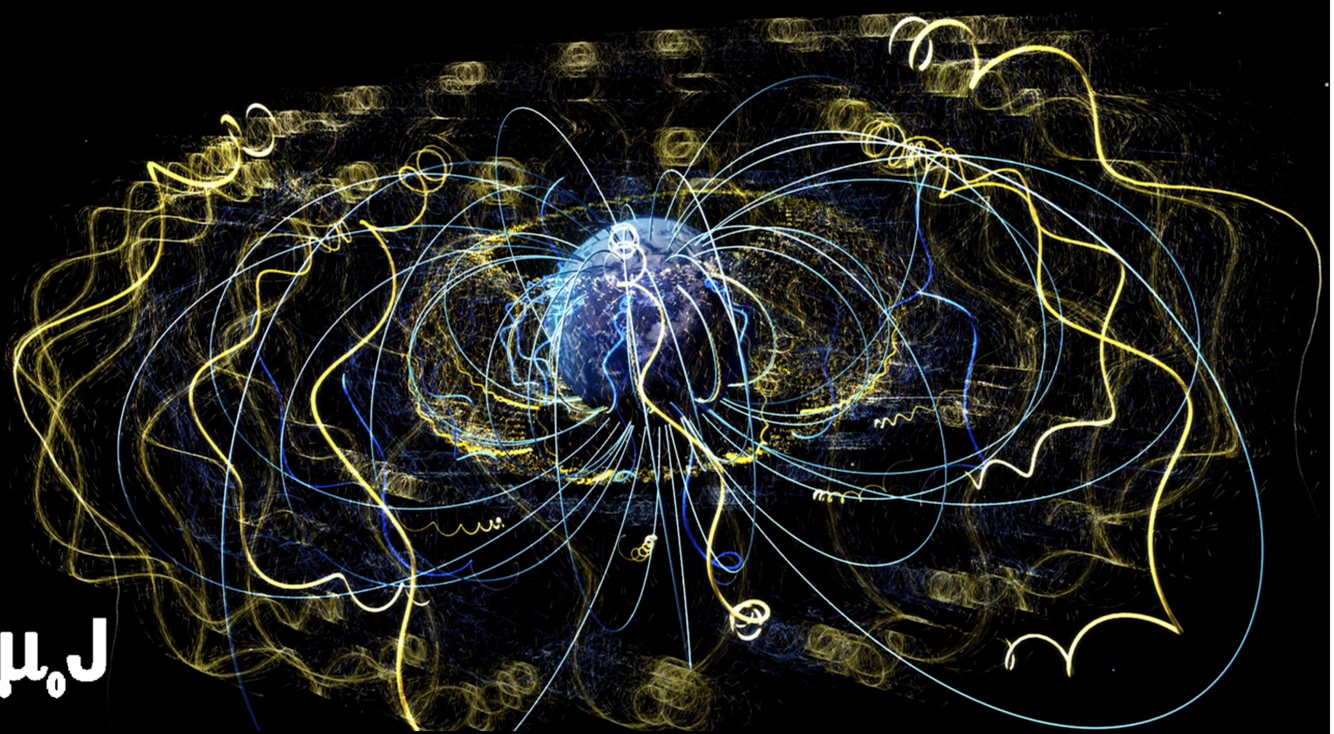
$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot B = 0$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} + \mu_0 J$$



Electricity and Magnetism II: 3812

Professor Jasper Halekas

Virtual by Zoom!

MWF 9:30-10:20 Lecture

Announcements

- Midterm 2 will be next Wednesday 4/22, during normal class hours
  - The midterm will cover Chapters 9.3-11, except for:
    - 9.4.3. The frequency dependence of permittivity
    - 10.1.4. Lorentz force law in potential form
    - 10.2.2. Jefimenko's equations
    - 11.2.2-11.2.3. Radiation reaction
- Equation sheet and sample midterms (with solutions) from last year posted
- Problem solving session Friday
- Review session Monday

Midterm 2 Rules & Directions

- The exam will be posted on the course web page at 9:30am. You must submit your answers to me by e-mail by 11:30am. The exam is intended to take roughly one hour – the extra hour is grace period to check your work, scan it, and submit it.

- This exam is open book and open notes. However, it is not open internet, and it is not open solutions manual. Please do not utilize solutions, online or otherwise, to solve the problems. I trust you all not to abuse this unique situation.

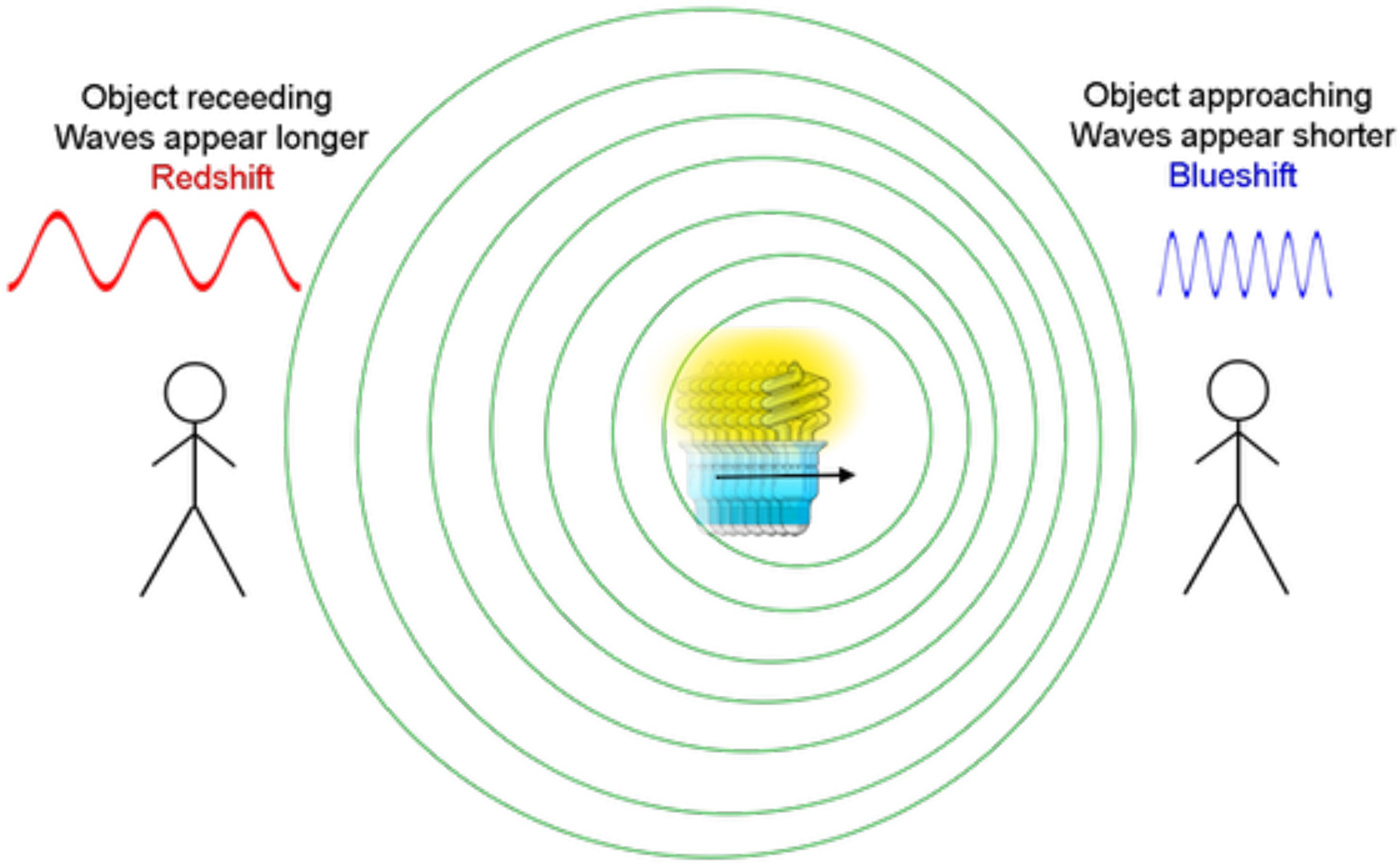
- Read all the questions carefully and answer every part of each question. Show your work on all problems – partial credit may be granted for correct logic or intermediate steps, even if your final answer is incorrect. Make sure to clearly indicate your final answer.

- Unless otherwise instructed, express your answers in terms of fundamental constants like μ_o and ε_o, rather than calculating numerical values.

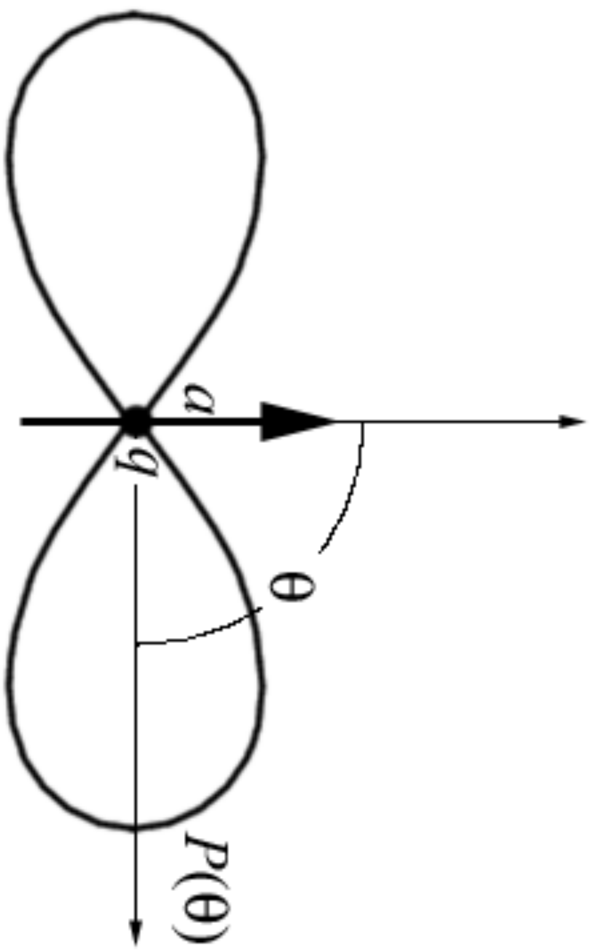
- Please ask if you have any questions, including clarification about the instructions, during the exam. The class Zoom meeting will be open during the exam.

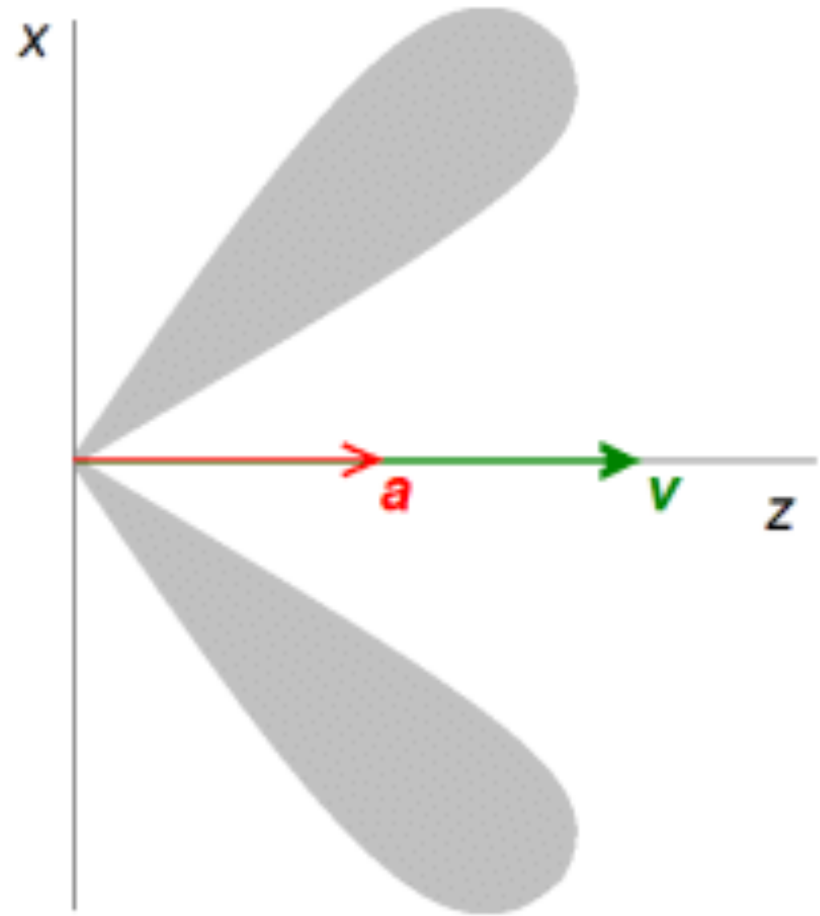
- This test is designed to be gender and race neutral.

Doppler Shift



Radiation from Accelerated Point Charge: Parallel Acceleration





Non-low-velocity Case

- Want power in observer frame
- Must Doppler-shift answer in particle frame

$$S_{rad} = \frac{1}{\mu_0 c} E_{rad}^2 = P_0/\text{area}$$

w/ $E_{rad} = \frac{q}{4\pi\epsilon_0} \frac{\Delta r}{(\vec{\Delta r}\cdot\vec{u})^3}\left[\vec{\Delta r}\times(\vec{u}\times\vec{a})\right]$

in particle frame

$$dP = dP_0\left(1 - \frac{\hat{\Delta r}\cdot\vec{v}}{c}\right)$$

$$= dP_0 \cdot \frac{\vec{\Delta r}\cdot\vec{u}}{c\,\Delta r} \quad \text{since } \vec{u} = c\hat{\Delta r} - \vec{v}$$

but $dP_0 = S_0 \cdot da$

$$\Rightarrow dP = S_0\left(\frac{\vec{\Delta r}\cdot\vec{u}}{c\,\Delta r}\right) da$$

$$= S_0\left(\frac{\vec{\Delta r}\cdot\vec{u}}{c\,\Delta r}\right)\Delta r^2\, d\Omega$$

$$= S_0\left(\frac{\vec{\Delta r}\cdot\vec{u}}{c\,\Delta r}\right)\Delta r^2 \sin\theta\, d\theta\, d\varphi$$

$$= \frac{\vec{\Delta r}\cdot\vec{u}}{c\,\Delta r}\cdot\frac{1}{\mu_0 c}\left(\frac{q}{4\pi\epsilon_0}\right)^2\cdot\frac{\Delta r^2}{(\vec{\Delta r}\cdot\vec{u})^6}\cdot\left[\vec{\Delta r}\times(\vec{u}\times\vec{a})\right]^2 d\Omega$$

$$= \frac{q^2}{16\pi^2\epsilon_0}\frac{\left[\hat{\Delta r}\times(\vec{u}\times\vec{a})\right]}{(\vec{\Delta r}\cdot\vec{u})^5}\, d\Omega$$

↗

$$P = \int dP \Rightarrow$$

$$\boxed{P = \frac{\mu_0 q^2 \gamma^6}{6\pi c}\left(a^2 - \left|\frac{\vec{v} \times \vec{a}}{c}\right|^2\right)}$$

w/ $\gamma = 1/\sqrt{1 - v^2/c^2}$

Note: This integral is so nasty even the book won't do it!

Special Case: $\vec{v} \parallel \vec{a}$

$\vec{v} \times \vec{a} = 0$

$$\Rightarrow P = \frac{\mu_0 q^2 a^2}{6\pi c} \gamma^6$$

- same as Larmor formula, but w/ γ^6 factor

What about angular distribution?

$$dP/d\Omega = \frac{q^2}{16\pi^2 \varepsilon_0} \frac{|\Delta\hat{r} \times (\vec{u} \times \vec{a})|^2}{(\Delta\vec{r} \cdot \vec{u})^5}$$

$$\vec{v} \parallel \vec{a} \Rightarrow \vec{u} \times \vec{a} = c\,\Delta\hat{r} \times \vec{a}$$

$$\Rightarrow |\Delta\hat{r} \times (\vec{u} \times \vec{a})|^2 = |\Delta\hat{r} \times (c\,\Delta\hat{r} \times \vec{a})|^2$$

$$= c^2 (a^2 - (\Delta\hat{r} \cdot \vec{a})^2)$$

$$= c^2 a^2 \sin^2\theta$$

Then $\Delta\vec{r} \cdot \vec{u} = c - \Delta\hat{r} \cdot \vec{v}$

$= c - v\cos\theta$ for $\vec{v} \parallel \vec{a}$

$= c(1 - \frac{v}{c}\cos\theta)$

$= c(1 - \beta\cos\theta)$

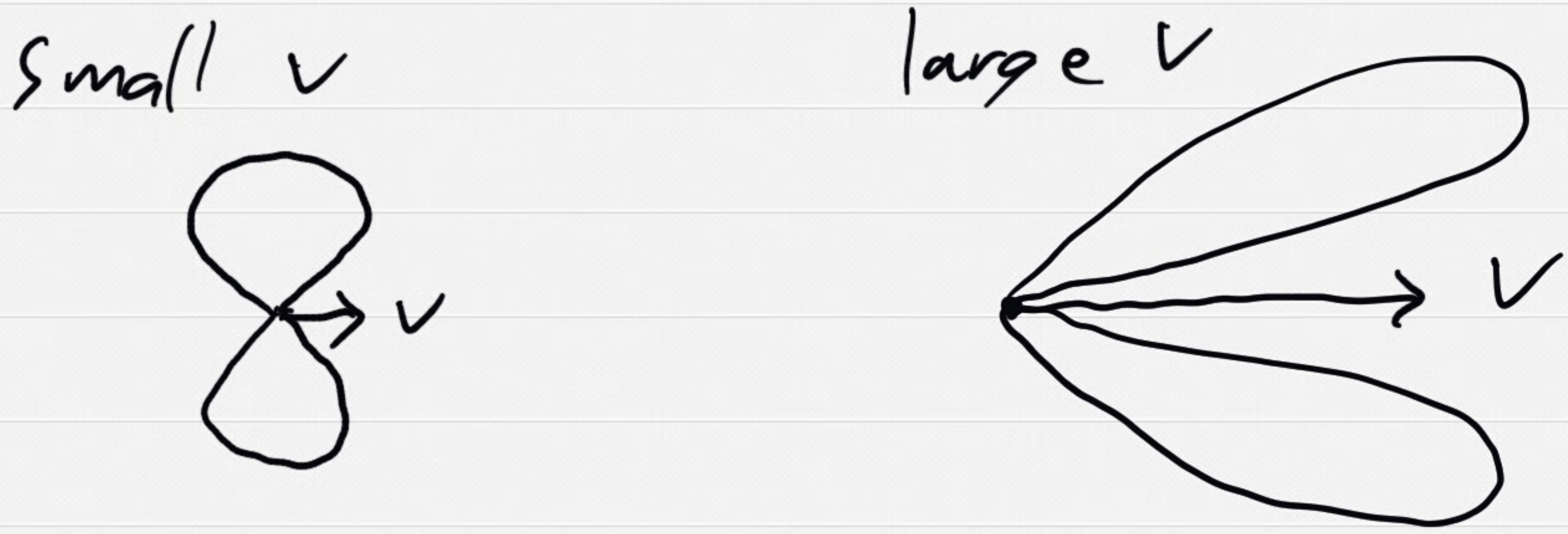
w/ $\beta = v/c$

↪

So $\frac{dP}{d\Omega} = \frac{q^2}{16\pi^2\varepsilon_0} \frac{c^2 a^2 \sin^2\theta}{c^5 (1-\beta\cos\theta)^5}$

$$\Rightarrow \boxed{\frac{dP}{d\Omega} = \frac{\mu_0 q^2 a^2 \sin^2\theta}{16\pi^2 c (1-\beta\cos\theta)^5}}$$

- Power highly concentrated near $\theta = 0$, but 0 at $\theta = 0$, for large v



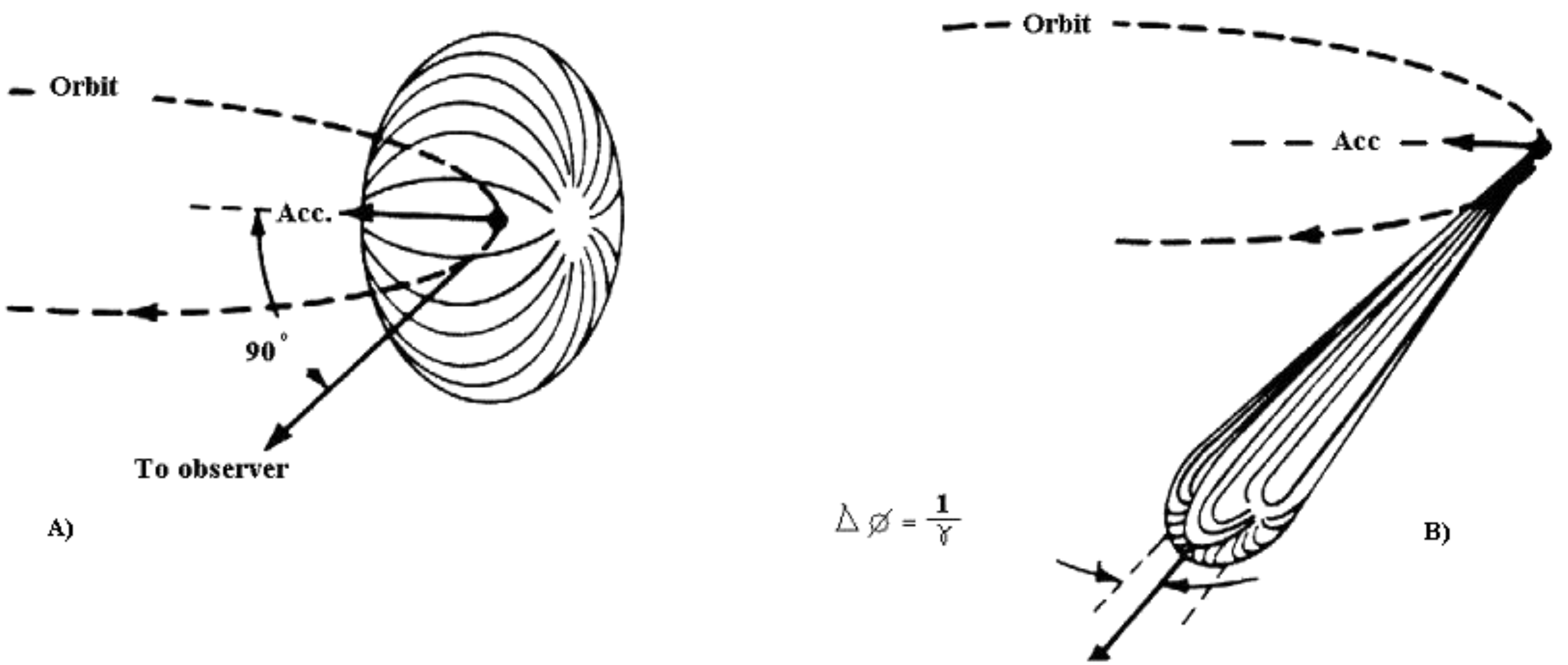
$$\beta = 0 \Rightarrow \frac{dP}{d\Omega} = \frac{\mu_0 q^2 a^2 \sin^2\theta}{16\pi^2 c}$$

$$S = \frac{dP}{da} = \frac{dP}{d\Omega}\frac{d\Omega}{da}$$

$$= \frac{dP}{d\Omega} \cdot \frac{1}{\Delta r^2}$$

$$= \frac{\mu_0 q^2 a^2 \sin^2\theta}{16\pi^2 c \, \Delta r^2} \quad //$$

Radiation from Accelerated Point Charge: Perpendicular Acceleration



Synchrotron Radiation

Radiation Reaction

Larmor Formula

$$P_{rad} = dW_{rad}/dt = \frac{\mu_0 q^2 a^2}{6\pi c}$$

$$-dW_{rad}/dt = \vec{F}_{rad} \cdot \vec{v}$$

$$\int_{t_1}^{t_2} \vec{F}_{rad} \cdot \vec{v}\, dt = -\int_{t_1}^{t_2} \frac{\mu_0 q^2}{6\pi c} a^2\, dt$$

but $\int a^2\, dt$

$$= \int_{t_1}^{t_2} d\vec{v}/dt \cdot d\vec{v}/dt\, dt$$

$$= \vec{v} \cdot d\vec{v}/dt \Big|_{t_1}^{t_2} - \int_{t_1}^{t_2} d^2\vec{v}/dt^2 \cdot \vec{v}\, dt$$

$$= -\int_{t_1}^{t_2} \dot{\vec{a}} \cdot \vec{v}\, dt \quad \text{if } \vec{v}(t_1) = \vec{v}(t_2)$$

$$\Rightarrow \boxed{\vec{F}_{rad} = \frac{\mu_0 q^2}{6\pi c} \dot{\vec{a}}}$$

Ultimately due to force of charge on itself!