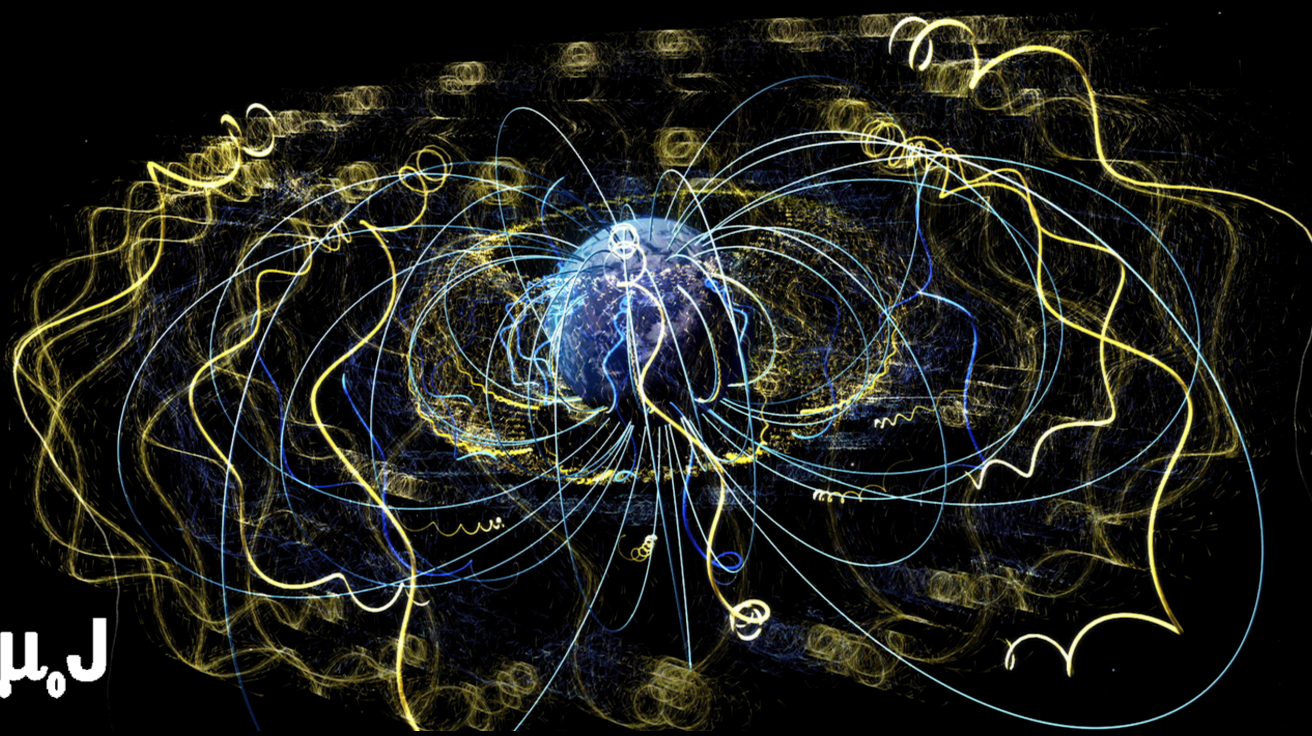
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism II: 3812

Professor Jasper Halekas

Virtual by Zoom!

MWF 9:30-10:20 Lecture

Sample Question

- What are the relative amplitudes and phases of the electric and magnetic fields for:
  - Plane waves in vacuum?
  - Plane waves in a linear dielectric?
  - Plane waves in a conductor?

Q1: $\tilde{\vec{E}} = \tilde{E}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)} \hat{n}$

Vacuum: $\nabla \times \vec{E} = -\partial \vec{B}/\partial t$

$$\Rightarrow \tilde{\vec{B}} = \frac{k}{\omega} \tilde{E}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)} (\hat{k} \times \hat{n})$$

$$|\vec{E}|/|\vec{B}| = \omega/k = c = 1/\sqrt{\mu_0 \varepsilon_0}$$

$$\tilde{E}_0 = E_0 e^{i\delta} \quad w/ \quad \delta_E = \delta_B$$

Dielectric: $\nabla \times \vec{E} = -\partial \vec{B}/\partial t$

$$\Rightarrow \tilde{\vec{B}} = \frac{k}{\omega} \tilde{E}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)} (\hat{k} \times \hat{n})$$

$$|\vec{E}|/|\vec{B}| = \omega/k = v = 1/\sqrt{\mu \varepsilon}$$

$$\delta_E = \delta_B$$

Conductor: $\tilde{\vec{E}} = \tilde{E}_0 e^{i(\tilde{\vec{k}}\cdot\vec{r} - \omega t)} \hat{n}$

$$\nabla \times \vec{E} = -\partial \vec{B}/\partial t$$

$$\Rightarrow \tilde{\vec{B}} = \frac{\tilde{k}}{\omega} \tilde{E}_0 e^{i(\tilde{\vec{k}}\cdot\vec{r} - \omega t)} (\hat{k} \times \hat{n})$$

$$|\vec{E}|/|\vec{B}| = \omega/|\tilde{k}| = \omega/\sqrt{k_r^2 + k_i^2}$$

$$\tilde{k} \tilde{E}_0 = |\tilde{k}| E_0 e^{i\delta} e^{i\varphi}$$

$$\Rightarrow \delta_B = \delta_E + \varphi \quad w/ \quad \varphi = \tan^{-1}\left(\frac{k_i}{k_r}\right)$$

Sample Question

- For the following potentials:
  - $V(r,t) = t^2/(2y^2)$
  - $A(r,t) = t/(c^2y)\ \hat{y}$
- Compute the electric and magnetic fields
- What gauge are these potentials in?

Q2: $\vec{E} = -\nabla V - \partial \vec{A}/\partial t$

$$= \frac{t^2}{y^3}\hat{y} - \frac{1}{c^2 y}\hat{y}$$

$$\vec{B} = \nabla \times \vec{A} = 0$$

$$\nabla \cdot \vec{A} = \partial A_y / \partial y = -\frac{t}{c^2 y^2} \neq 0$$

Not Coulomb

$$\mu_0 \varepsilon_0 \, \partial V/\partial t = \frac{1}{c^2} \cdot t/y^2 = -\nabla \cdot \vec{A}$$

$\Rightarrow$ Lorenz

Sample Question

- Consider a point charge q traveling in a circle around the origin with radius R and angular frequency ω
- What are the Liénard-Wiechert potentials at the origin?

Q 3: $\vec{w}(t) = R\cos(\omega t)\hat{x} + R\sin(\omega t)\hat{y}$

$\vec{v}(t) = -\omega R\sin(\omega t)\hat{x} + \omega R\cos(\omega t)\hat{y}$

$$\Delta\vec{r} = \vec{r} - \vec{w}(t_r)$$

$$= -\vec{w}(t_r) \quad @ \text{ origin}$$

$$= -R\cos(\omega t_r)\hat{x} - R\sin(\omega t_r)\hat{y}$$

$$\Delta r = |\Delta\vec{r}| = R$$

$\Delta\vec{r}\cdot\vec{v} = 0$ for circular motion

so $V(0,t) = \frac{1}{4\pi\varepsilon_0}\frac{qc}{Rc}$

$$= \boxed{\frac{q}{4\pi\varepsilon_0 R}}$$

$$\vec{A}(0,t) = \frac{\vec{v}}{c^2}V$$

$$= \boxed{\frac{q\omega}{4\pi\varepsilon_0 c^2}\left(-\sin(\omega t_r)\hat{x} + \cos(\omega t_r)\hat{y}\right)}$$

Sample Question

- Consider a point charge q traveling in a circle around the origin with radius R and angular frequency ω (same as previous question)
- Use the multipole approximation to find a first-order approximation for the power radiated by this charge

Q4: $P_{rad} \cong \frac{\mu_0}{6\pi c}(\ddot{p}(t_0))^2$

w/ $t_0 = t - r/c$

$$\vec{p} = q\vec{w}$$

$$\ddot{\vec{p}} = q\ddot{\vec{w}}$$

$$= -q\omega^2 R(\cos(\omega t_0)\hat{x} + \sin(\omega t_0)\hat{\phi})$$

$$|\ddot{\vec{p}}| = q\omega^2 R$$

$$\Rightarrow \boxed{P_{rad} \cong \frac{\mu_0 q^2 \omega^4 R^2}{6\pi c}}$$

Could also get from Larmor

$$P_{rad} = \frac{\mu_0 q^2 a^2}{6\pi c}$$

$$\& \quad a = \omega^2 R$$

$$\Rightarrow P_{rad} = \frac{\mu_0 q^2 \omega^4 R^2}{6\pi c}$$