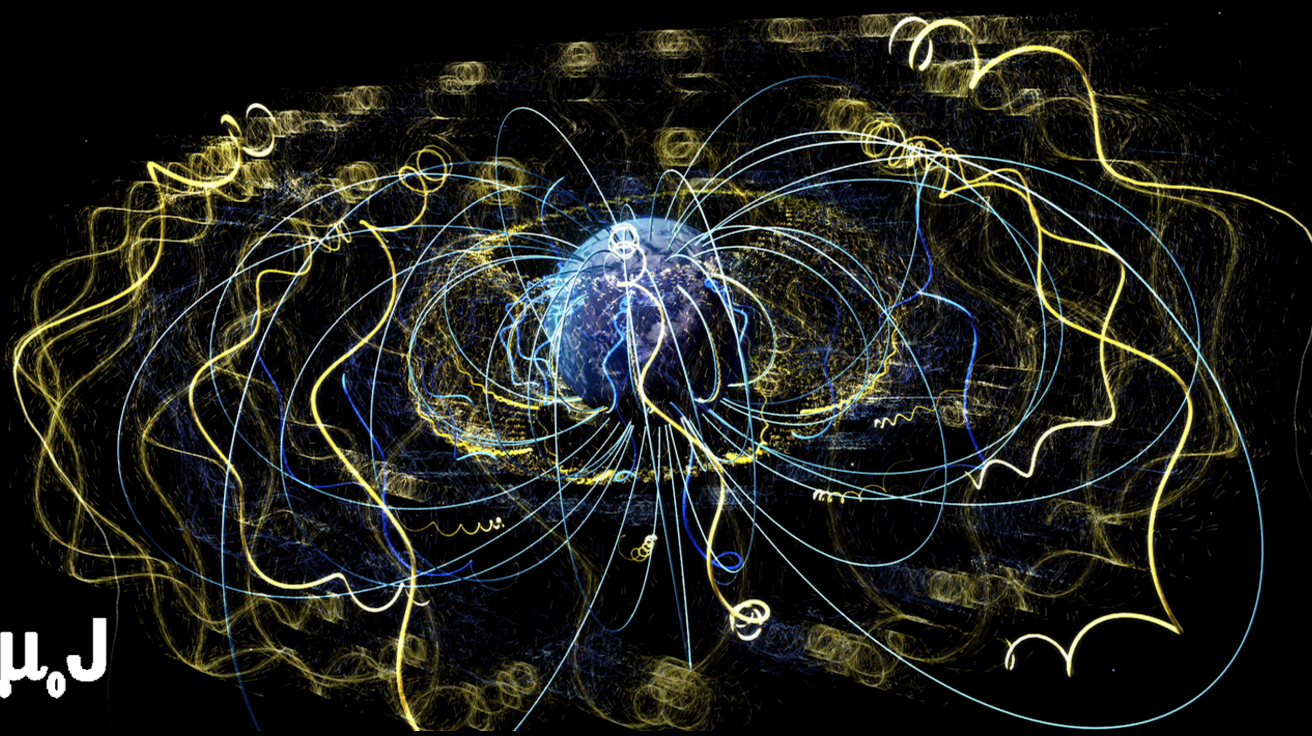
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism II: 3812

Professor Jasper Halekas
Van Allen 70
MWF 9:30-10:20 Lecture

Check Your Understanding: Generator

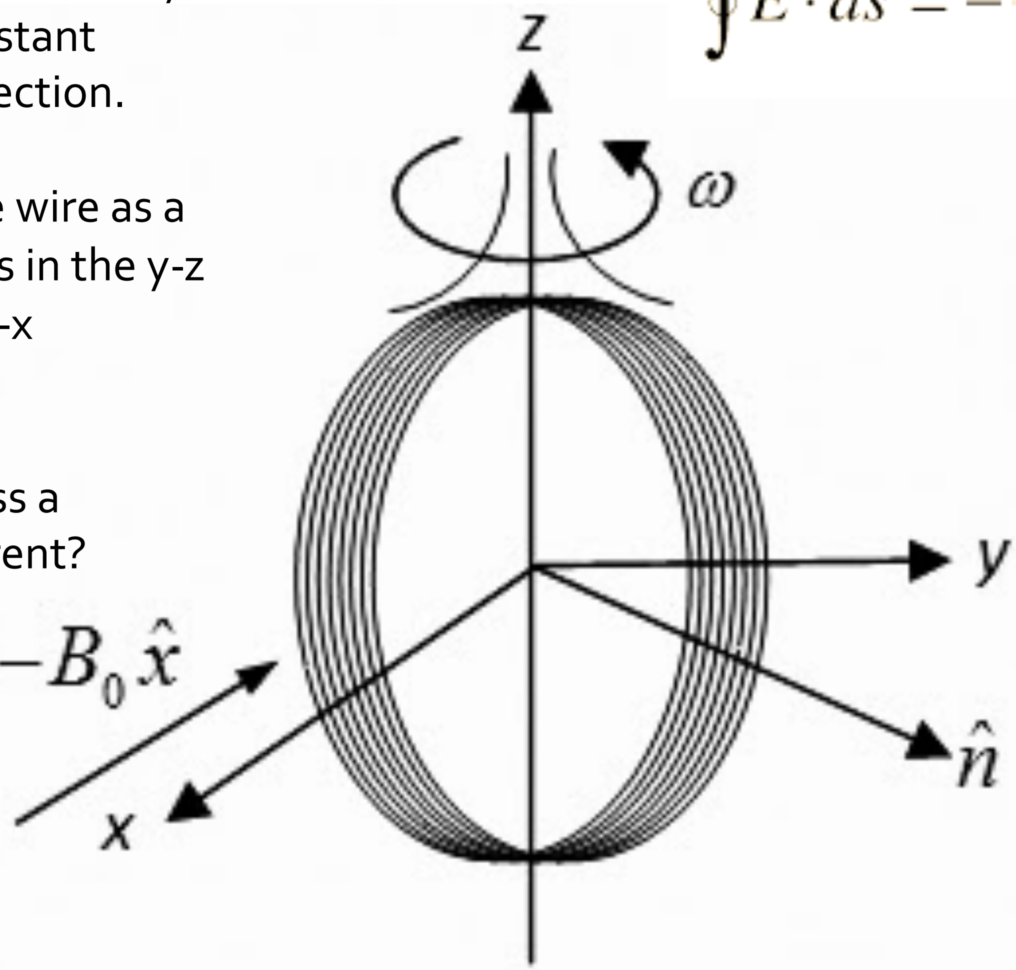
Imagine a wire wrapped in N loops w/ radius r, rotating with angular velocity ω around the z-axis, in a constant magnetic field **B** in the -x direction.

What is the EMF through the wire as a function of time, if the loop is in the y-z plane (with its normal in the -x direction) at time t = 0?

If the wire is connected across a resistance R, what is the current?

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\overline{B} = -B_0 \hat{x}$$



Q1 : $\Phi_B = N B A \cos\theta_{AB}$

$$= N B_0 \cdot \pi r^2 \cdot \cos(\omega t)$$

$$d\Phi_B/dt = -\omega N B_0 \cdot \pi r^2 \cdot \sin(\omega t)$$

$$\mathcal{E} = -d\Phi_B/dt = \boxed{\omega N B_0 \cdot \pi r^2 \cdot \sin(\omega t)}$$

$$I = \mathcal{E}/R = \boxed{\frac{\omega N B_0}{R} \cdot \pi r^2 \sin(\omega t)}$$

Note: $\vec{\tau} = \vec{\mu} \times \vec{B}$

$$= N \cdot I A B \cdot \sin\theta_{AB}$$

$$= N \cdot I \cdot \pi r^2 \cdot B_0 \cdot \sin(\omega t)$$

$$= \frac{\omega N^2 B_0^2}{R} \cdot (\pi r^2)^2 \sin^2(\omega t)$$

$$P_{EM} = I\mathcal{E} = \frac{\omega^2 N^2 B_0^2}{R} (\pi r^2)^2 \sin^2(\omega t)$$

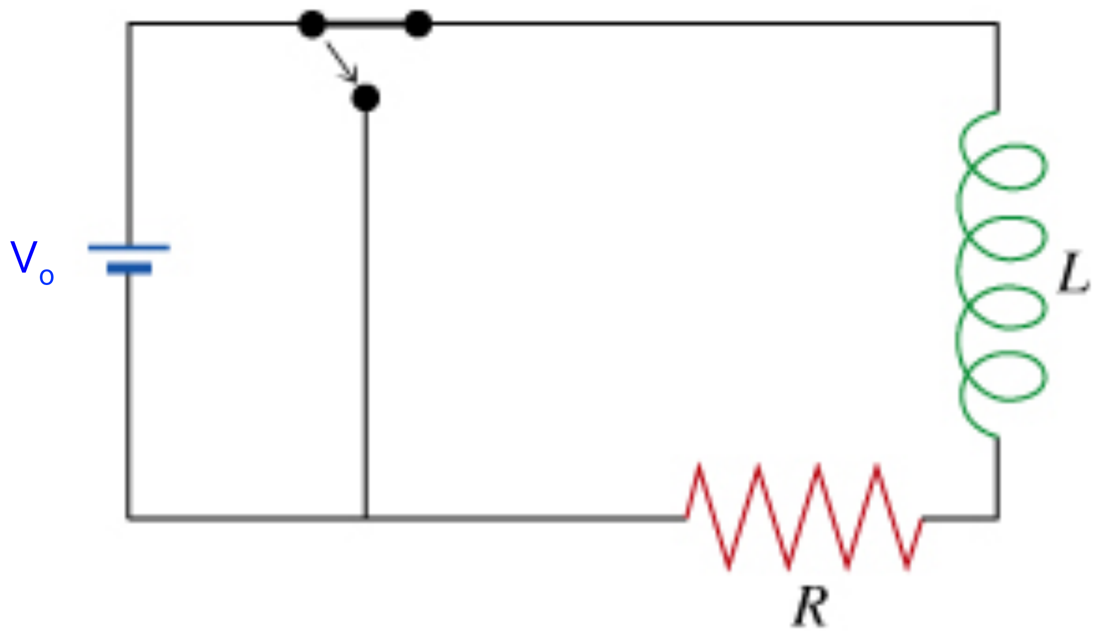
$$P_\tau = \vec{\tau} \cdot \vec{\omega} = \frac{\omega^2 N^2 B_0^2}{R} (\pi r^2) \sin^2(\omega t)$$

Torque provides all power

Check Your Understanding: LR Circuit I

In the following circuit, the battery (w/ EMF V_o) is originally in the circuit. At t = 0, the switch is flipped, taking the battery out of the circuit. What is the current through the resistor as a function of time?

$$\varepsilon_L = -L\frac{dI}{dt}$$



Q2:

$$\mathcal{E} = \mathcal{E}_L = -L\, dI/dt$$

$$\mathcal{E} = IR$$

$$\Rightarrow \quad dI/dt = -\frac{R}{L} I$$

$$\Rightarrow \quad I(t) = I_0 \, e^{-R/L\, t}$$

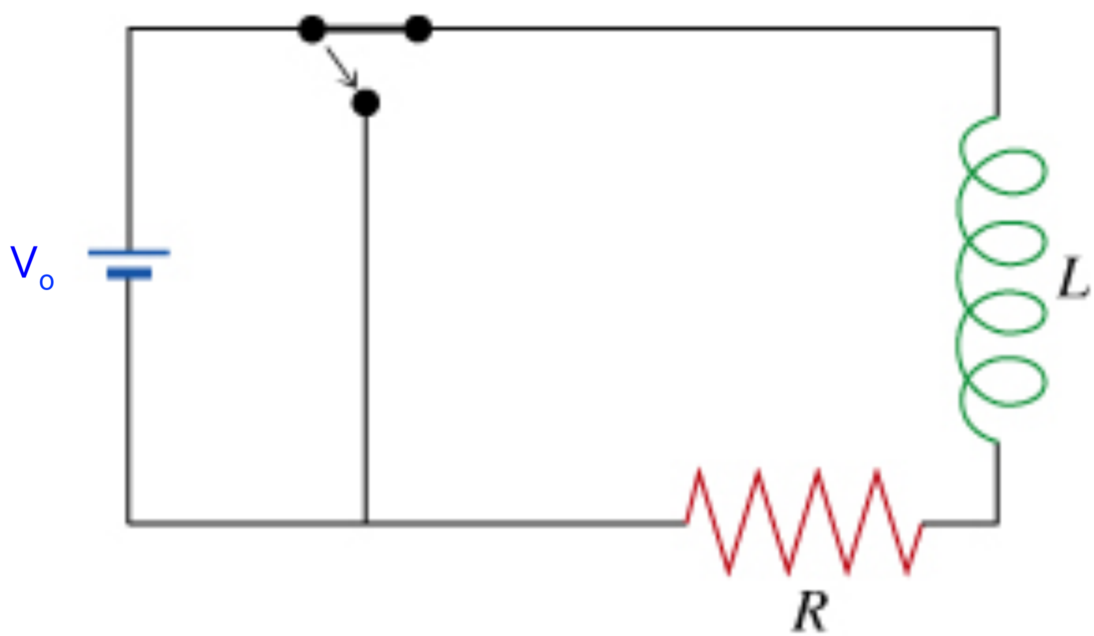
$$= \boxed{V_0/R \; e^{-R/L\, t}}$$

Check Your Understanding: LR Circuit II

Given your answer to part I:

$$U_L = \tfrac{1}{2} L I^2$$

A. What is the initial energy stored in the inductor?
B. What is the total energy dissipated in the resistor as the circuit discharges?



Q3: $U_L = \frac{1}{2} L I^2$

$$= \frac{1}{2} L \cdot \left(\frac{V_0}{R} e^{-R/L\, t}\right)^2$$

$$= \frac{L V_0^2}{2R^2} e^{-\frac{2R}{L} t}$$

$$U_R = \int_0^t P_R$$

$$= \int_0^t I^2 R \, dt'$$

$$= \int_0^t R \frac{V_0^2}{R^2} e^{-\frac{2R}{L} t'} dt'$$

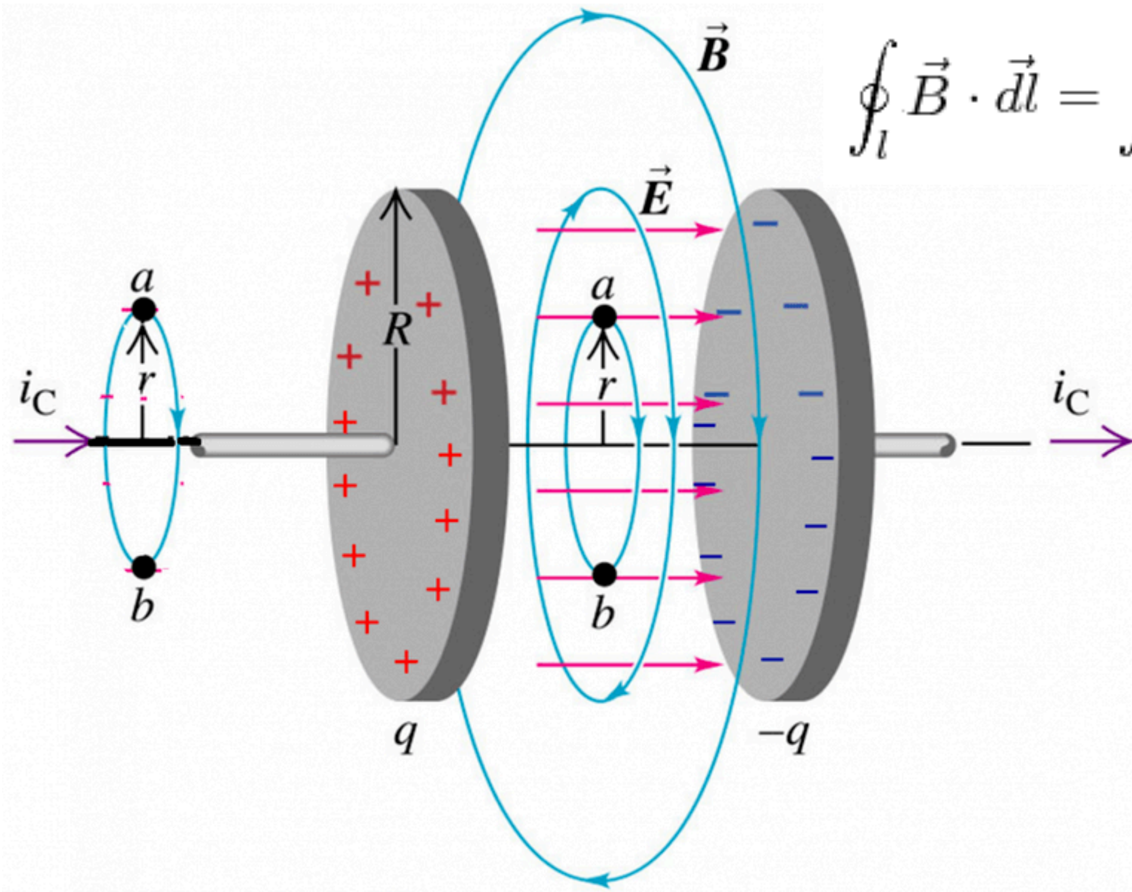
$$= \frac{V_0^2}{R} \cdot -\frac{L}{2R} e^{-\frac{2R}{L} t'} \Big|_0^t$$

$$= \frac{V_0^2 L}{2R^2} \left(1 - e^{-\frac{2R}{L} t}\right)$$

$$\boxed{U_R(\infty) = U_L(0) = \frac{V_0^2 L}{2R^2}}$$

- All energy starts in inductor
- All eventually dissipated in resistor

Check Your Understanding: Charging Capacitor



$$\oint_l \vec{B} \cdot \vec{dl} = \int_S \left(\mu_o \vec{J} + \mu_o \varepsilon_o \frac{\partial \vec{E}}{\partial t} \right) \cdot \vec{ds}$$

Use the integral form of Ampere's Law to find B(r) in the region between the two capacitor plates, given that $E = q/(\pi R^2 \varepsilon_o)$ in the region between the plates.

Q4:

$$\oint \vec{B}\cdot d\vec{l} = \int \mu_0 \varepsilon_0 \, \partial \vec{E}/\partial t \cdot d\vec{a}$$

$$E = q/\pi R^2 \varepsilon_0$$

$$\partial E/\partial t = i_c/\pi R^2 \varepsilon_0$$

$$\oint \vec{B}\cdot d\vec{l} = B \cdot 2\pi r$$

$$\int \mu_0 \varepsilon_0 \, \partial \vec{E}/\partial t \cdot d\vec{a} = \frac{\mu_0 \varepsilon_0 i_c}{\pi R^2 \varepsilon_0} \cdot \pi r^2$$

$$= \mu_0 i_c \frac{r^2}{R^2} \quad r < R$$

$$\Rightarrow \boxed{B = \frac{\mu_0 i_c r}{2\pi R^2} \quad r < R}$$

$$\boxed{= \frac{\mu_0 i_c}{2\pi r} \quad r > R}$$

same as around wire for $r > R$