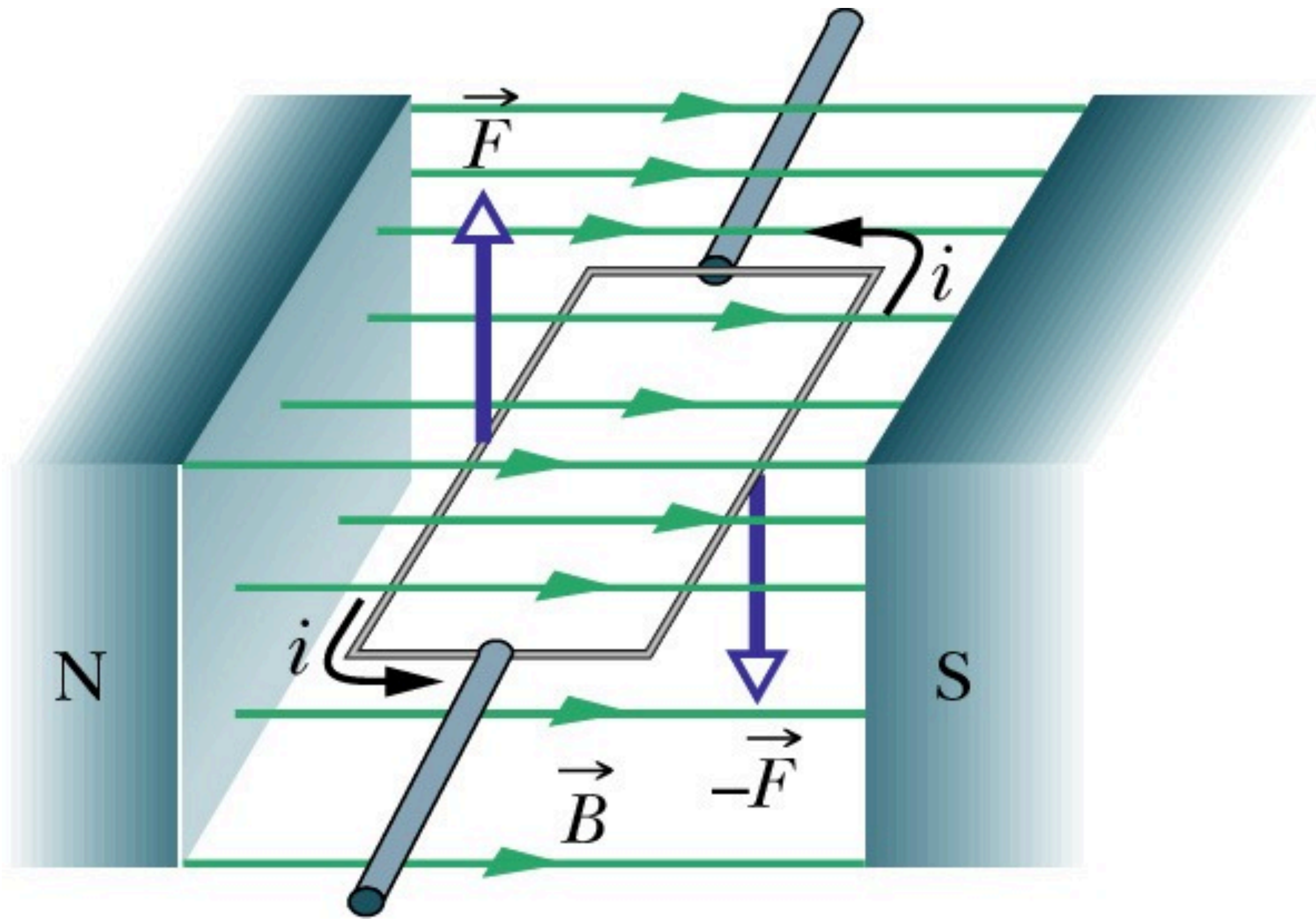
Physics II: 1702/029:028
Electricity and Magnetism

Professor Jasper Halekas
Van Allen 70 [Clicker Channel #18]
MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

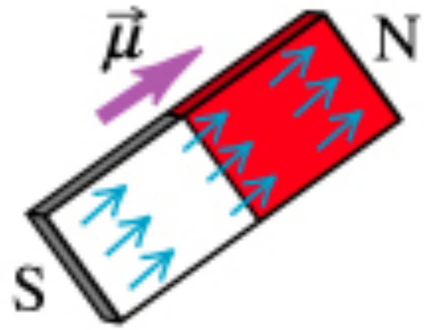
Announcements

- Next homework is of the hardcopy variety
- Available now on the “assignments” page on the main course web site

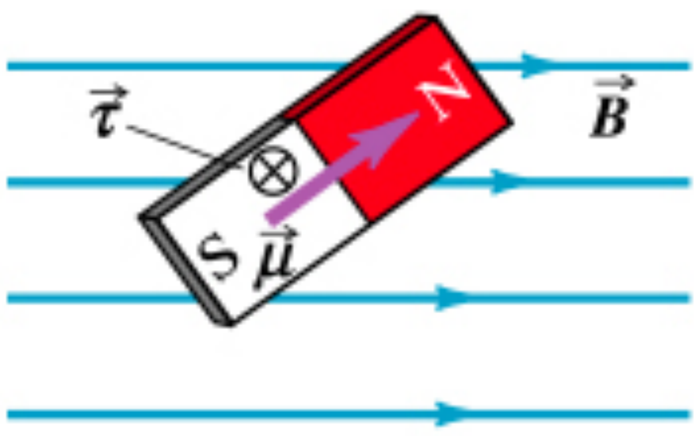
Magnetic Torque on a Current Loop



Torque on Magnetic Dipole



(b)



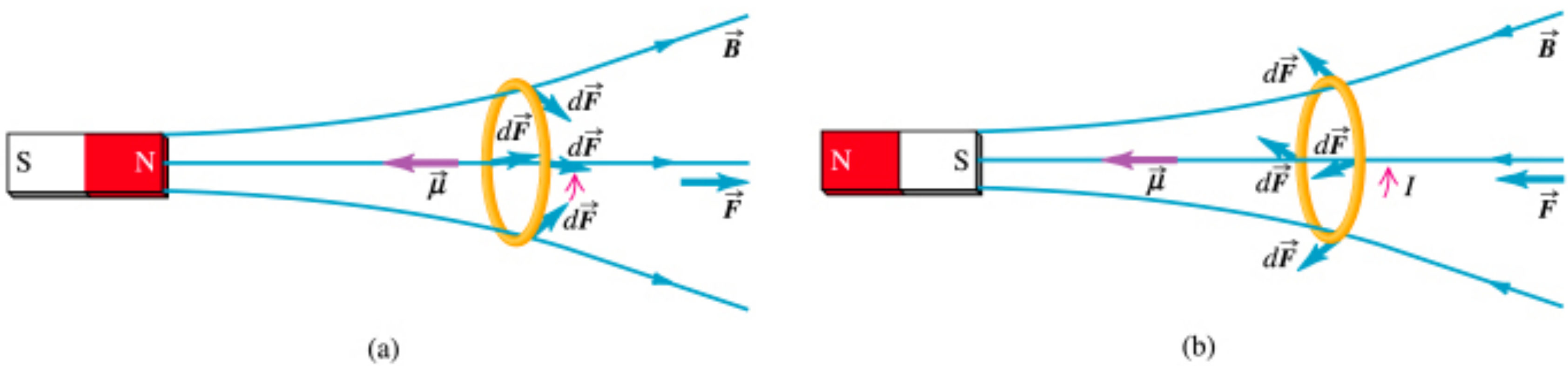
(c)

$$\tau = \mu \ x \ B$$

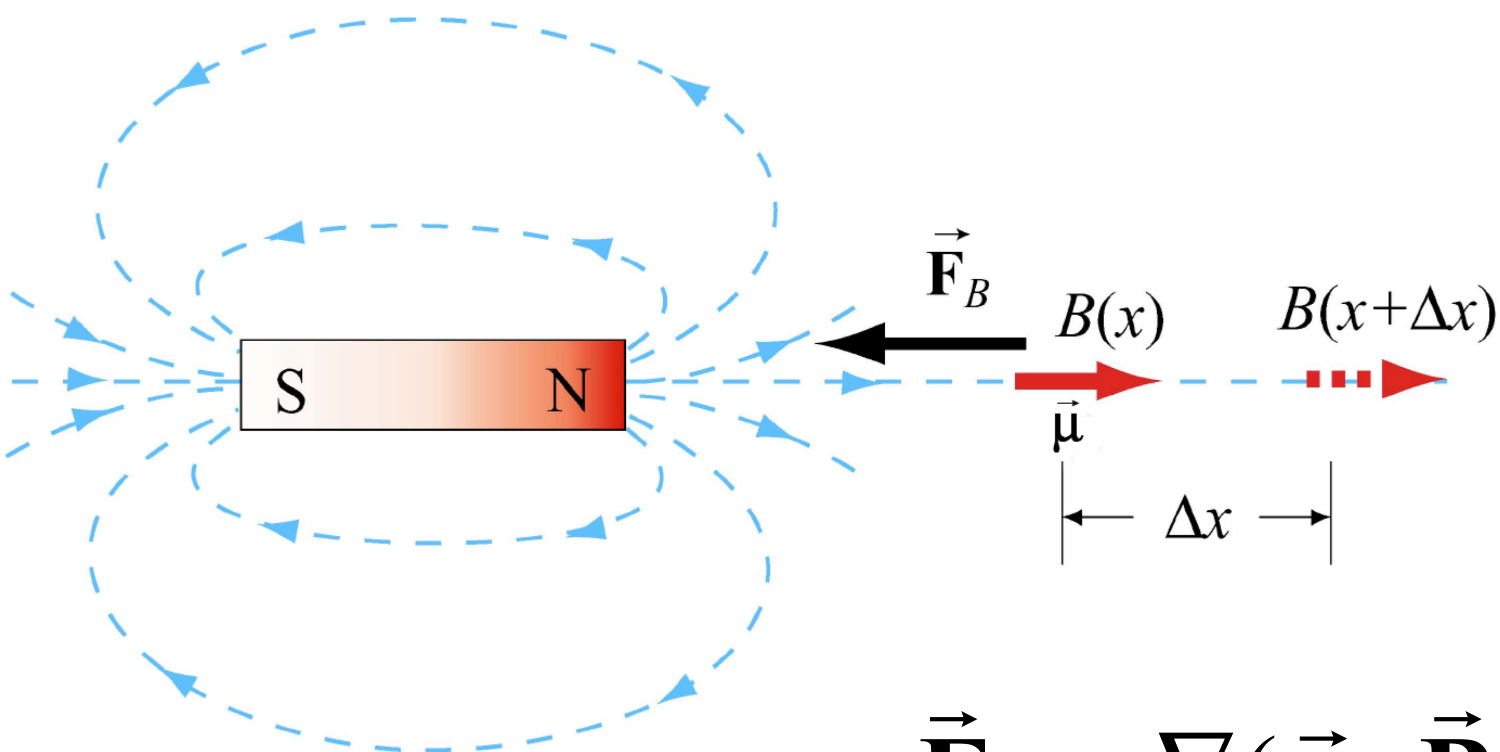
Magnetic Dipole Moment

$$\mu = IA$$

Magnetic Force on Current Loop

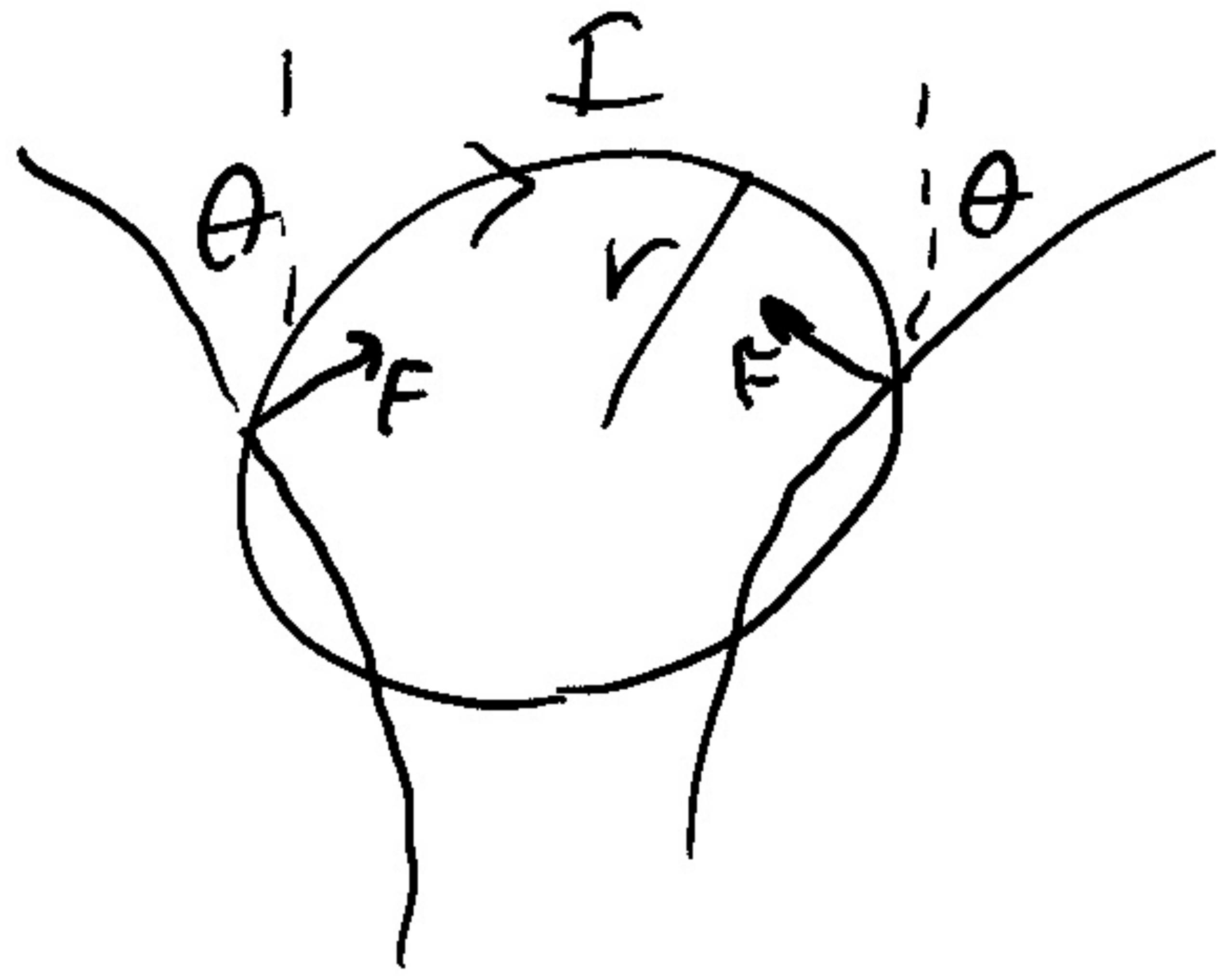


Force on Magnetic Dipole



$$\vec{\mathbf{F}}_B = \nabla(\vec{\mu} \cdot \vec{\mathbf{B}})$$

Force on current loop



$$d\vec{F} = I\, d\vec{L} \times \vec{B}$$

$$dF_z = dF \sin\theta$$

$$= I\,dL\, B \sin\theta$$

$$F_z = \int_{loop} dF_z$$

$$= I \cdot 2\pi r \cdot B \sin\theta$$

or

$$F = \nabla(\mu \cdot B)$$

$$= \mu_z\, \partial B_z / \partial z$$

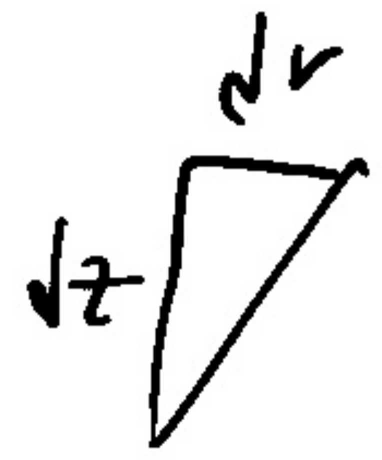
$$= -I \cdot A\, \partial B_z / \partial z$$

$$= -I \cdot \pi r^2\, \partial B / \partial z$$

what is $\partial B_z / \partial z$

know $\varphi_B = \int \vec{B} \cdot ds$
must be same @ two
close z-values

$$B(z)\cdot\pi r^2 = B(z+dz)\cdot\pi(r+dr)^2$$
$$= B(z+dz)\cdot\pi(r+dz\tan\theta)^2$$



so $$\partial B_z/\partial z = \frac{B(z+dz)-B(z)}{dz}$$

$$= \frac{B(z)\left[\frac{\pi r^2}{\pi(r+dz\tan\theta)^2}\right]-B(z)}{dz}$$

$$\cong B(z)\cdot\left(1-2\frac{dz}{r}\tan\theta-1\right)/dz$$

$$= B\cdot\left(-2\tan\theta/r\right)$$

$$\Rightarrow \nabla(\vec{\mu}\cdot\vec{B}) \sim -\pi r^2 I\cdot -\frac{2B\tan\theta}{r}$$

$$= 2\pi r I B\tan\theta$$

- For small angles $\tan\theta \sim \sin\theta$ so same if ring is small

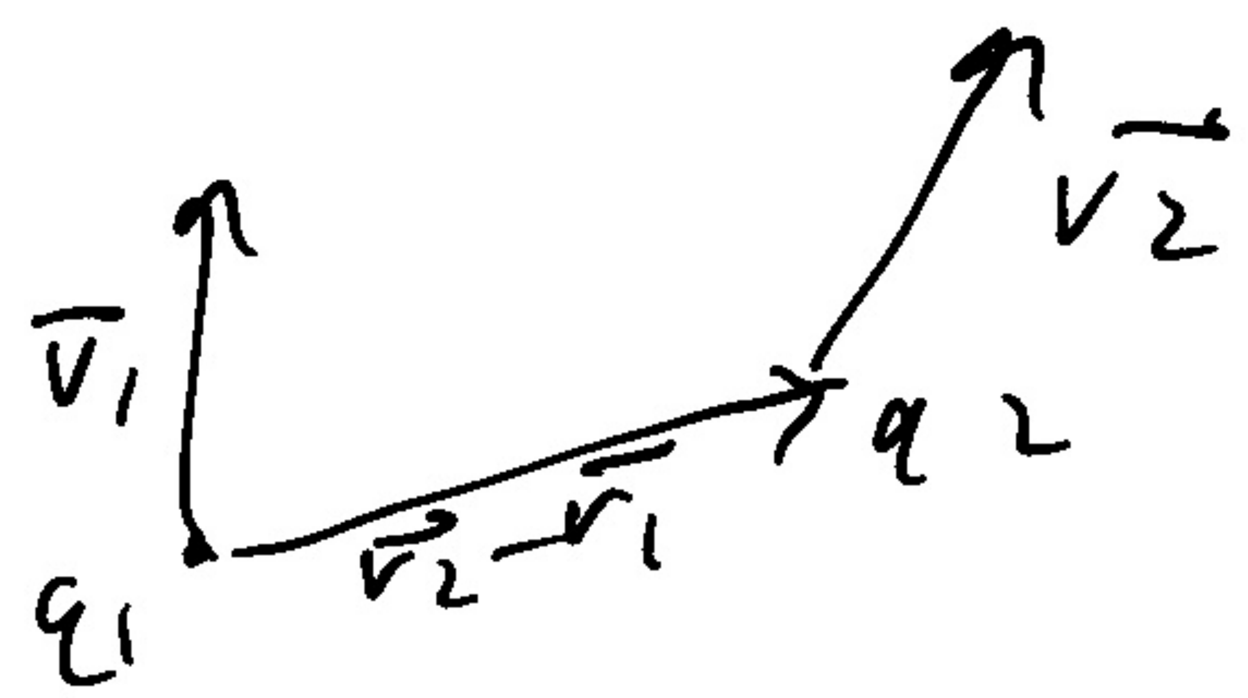
Magnetic Field of Point Charge

Magnetic Field created by a single moving charged particle.

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

Constant μ_o = 1.257 x 10^{-6} Tesla meter/Amp.

– Force between moving charges



$$\vec{B}_2 = \frac{\mu_0}{4\pi} \frac{q_2 \vec{v}_2 \times (\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3}$$

$$\vec{F}_{12} = q_1 \vec{v}_1 \times \vec{B}_2$$

$$= \frac{\mu_0 q_1 q_2}{4\pi} \vec{v}_1 \times \frac{\vec{v}_2 \times (\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3}$$

compare to $\vec{F}_E = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2 (\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3}$

$$|\vec{F}_B| / |\vec{F}_E| = \mu_0 \varepsilon_0 \vec{v}_1 \times \vec{v}_2 \times \hat{r}$$

$$\sim \mu_0 \varepsilon_0 v^2$$

$$\mu_0 \varepsilon_0 = 1/c^2$$

$$\Rightarrow F_B / F_E \sim v^2/c^2$$

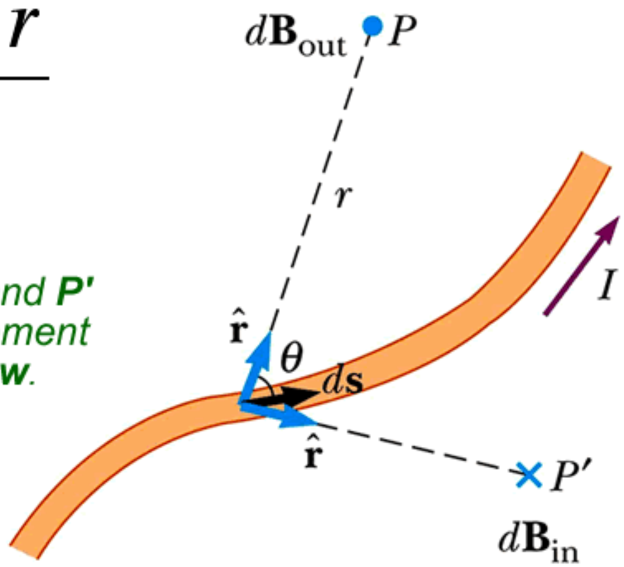
Electric dominates for $v \ll c$

Biot-Savart Law

The Biot-Savart Law

The Magnetic Field produced by the current in the wire

$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{Id\vec{s}\times\hat{r}}{r^2}$$



The magnetic field d**B** at a point **P** and **P'** due to the current ***I*** thru a length element d**s** is given by the **Biot-Savart law**.

This Law is based upon experimental observation

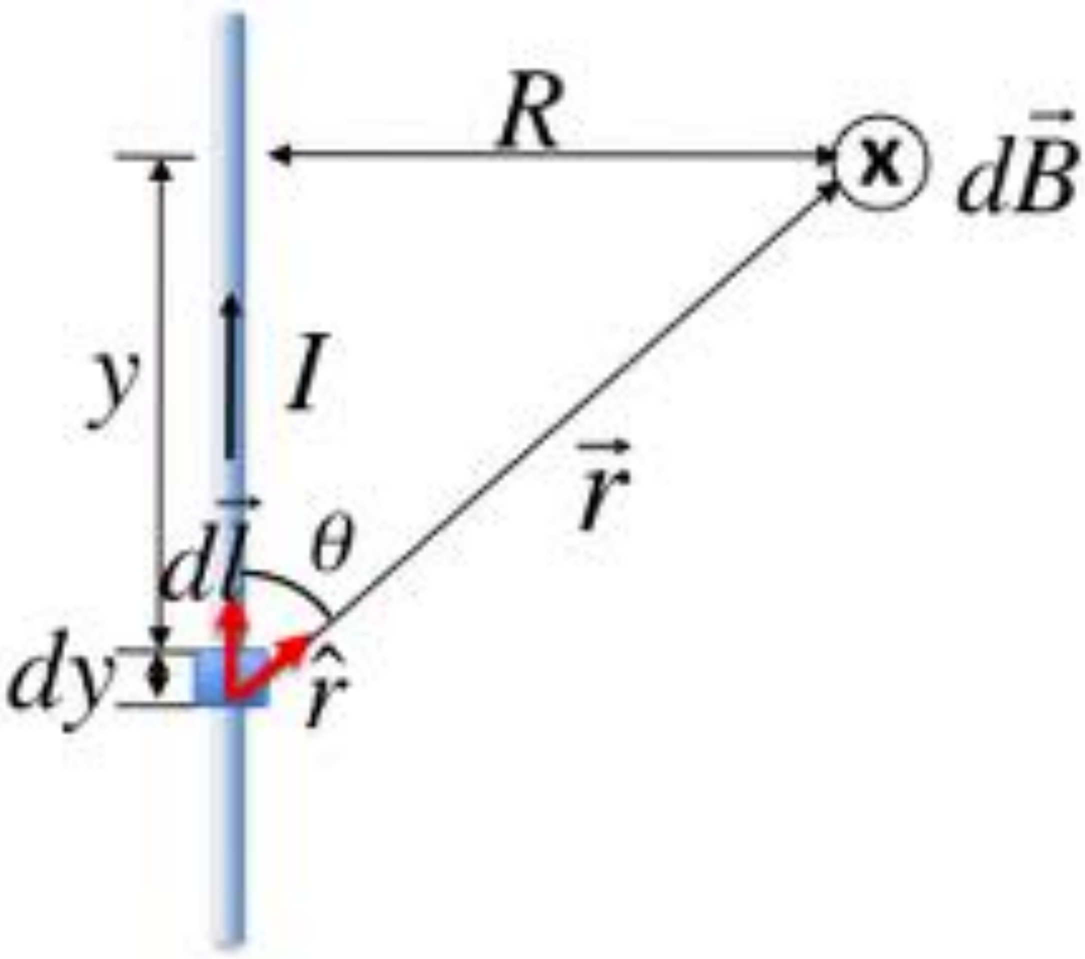
*The vector d**B** is perpendicular to both d**s** and to the unit vector **r** directed toward the point **P.***

*The magnitude of d**B** is proportional to the current **I**, to the length element d**s** and to the sine of the angle between d**s** and **r.** It is also inversely proportional to r^2.*

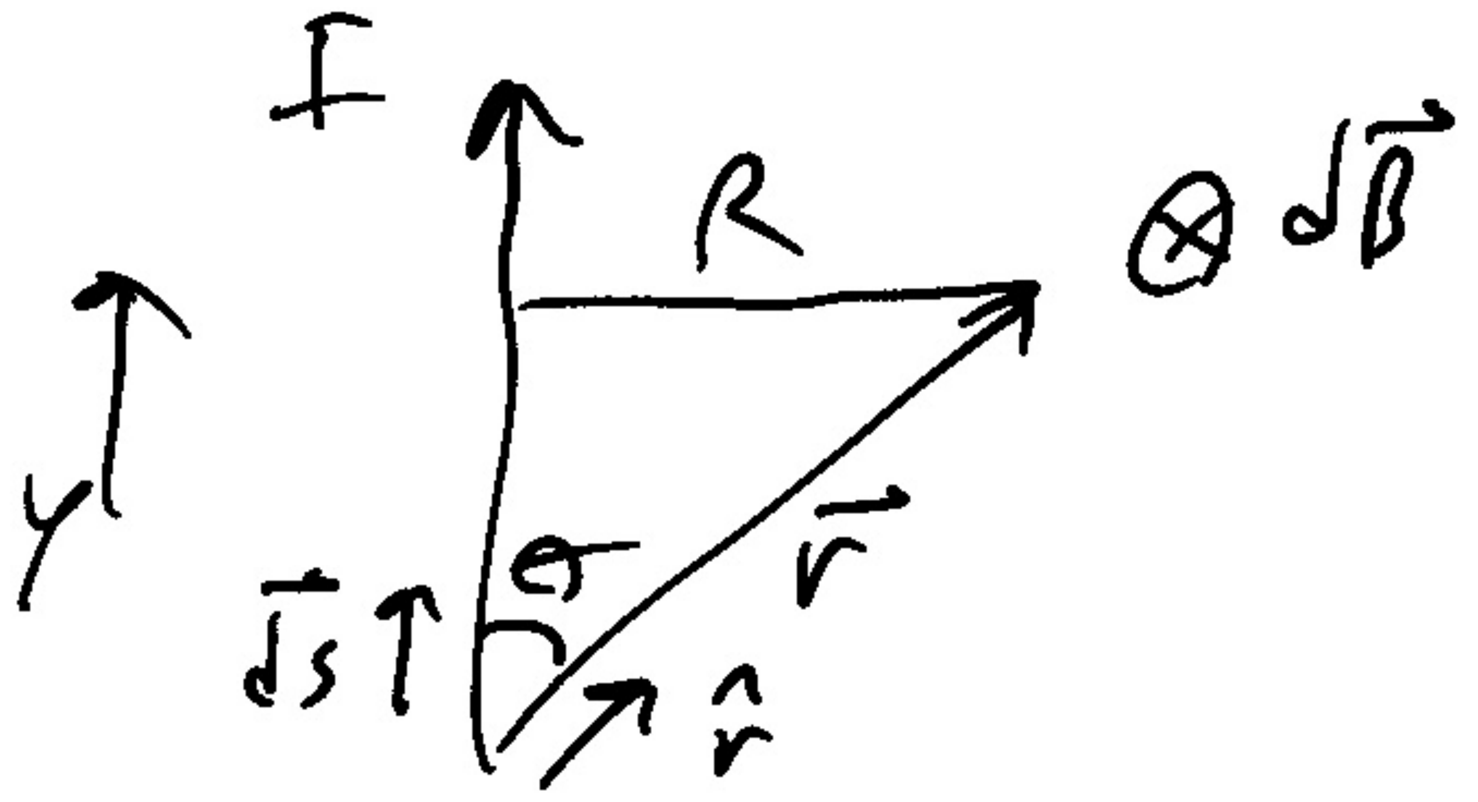
Obviously we will Integrate over the entire current distribution.

$$\vec{B} = \frac{\mu_o I}{4\pi}\int\frac{d\vec{s}\times\hat{r}}{r^2}$$

Field of an Infinite Wire



Infinite Wire



$$\sin\theta = R/r = R/\sqrt{R^2+y^2}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{s}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,dy\,\hat{j}\times\hat{r}}{r^2}$$

$$= \mu_0/4\pi \; \frac{I\,dy\cdot(-\sin\theta)}{r^2}$$

$$= \mu_0/4\pi \cdot \frac{-IR}{(R^2+y^2)^{3/2}}\,dy$$

$$\vec{B} = -\mu_0 I/4\pi \int_{-\infty}^{\infty} R/(R^2+y^2)^{3/2}\,dy$$

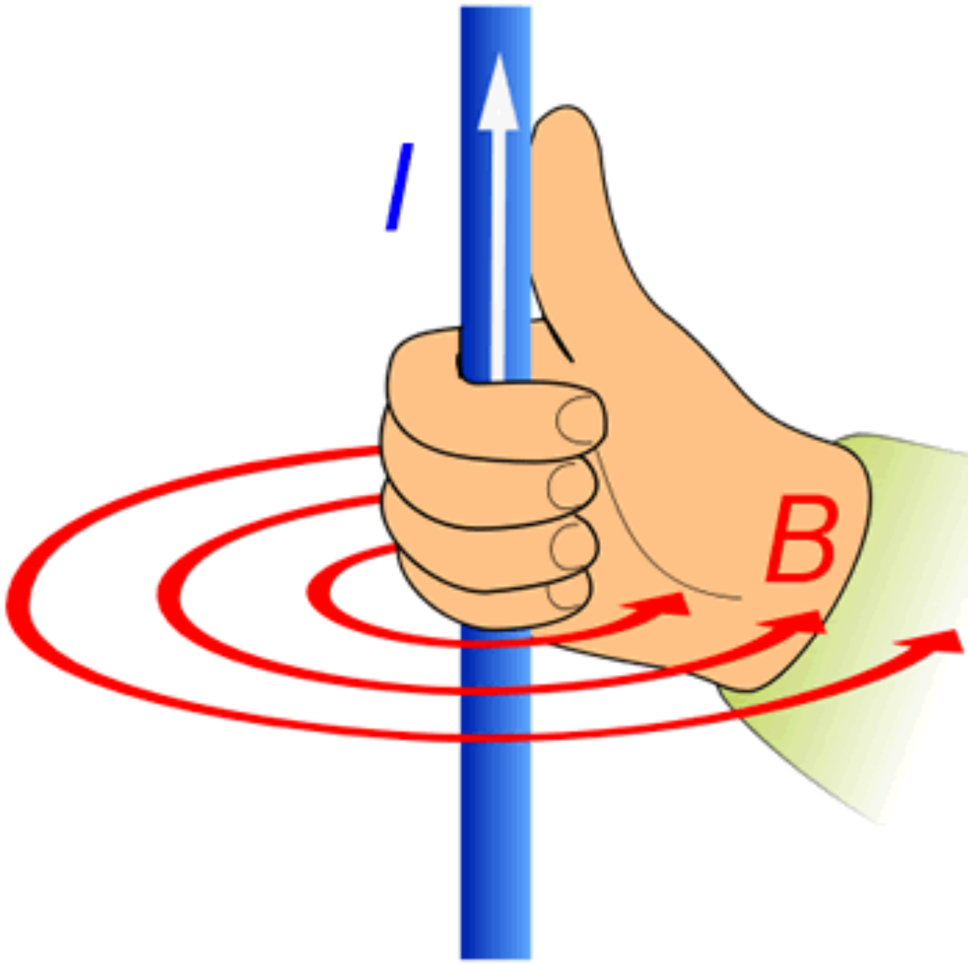
$$= -\mu_0 I/4\pi \cdot 2\int_0^{\infty} R/(R^2+y^2)^{3/2}\,dy$$

$$= -\mu_0 I/2\pi \cdot yR/R^2\sqrt{y^2+R^2}\,\Big|_0^{\infty}$$

$$= \boxed{-\mu_0 I/2\pi R}$$

- CCW around I
- right-handed

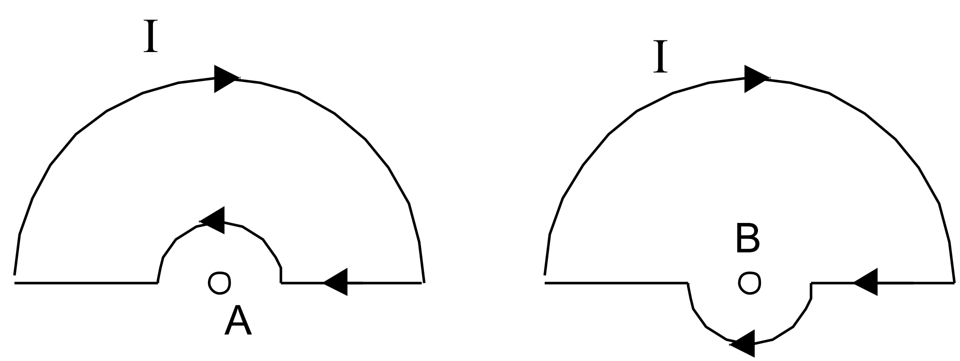
Magnetic Fields Generated by Wires



When do I get to use my left hand?

Concept Check

Which point A or B has the larger magnitude Magnetic Field?



$$d\vec{B} = \frac{\mu_0 i}{4\pi} \frac{d\vec{L} \times \hat{r}}{r^2}$$

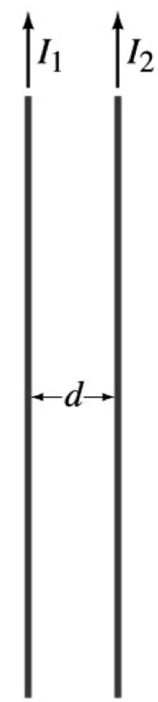
A B

C : The B-field is the same at A and B.

Answer: Case B has the larger magnetic field. Use the Biot-Savart Law to get the directions of the B-field due to the two semi-circular portions of the loop. In A the two fields oppose each other; in B they add.

Concept Check

Q13) The figure to the right shows two parallel wires carrying currents I_1 and I_2 that are in the same direction. What is the direction of the force on wire 2 because of the magnetic field produced by wire 1?



1) leftward
2) rightward
3) into page
4) out of page
5) none of the above

Forces Between Wires