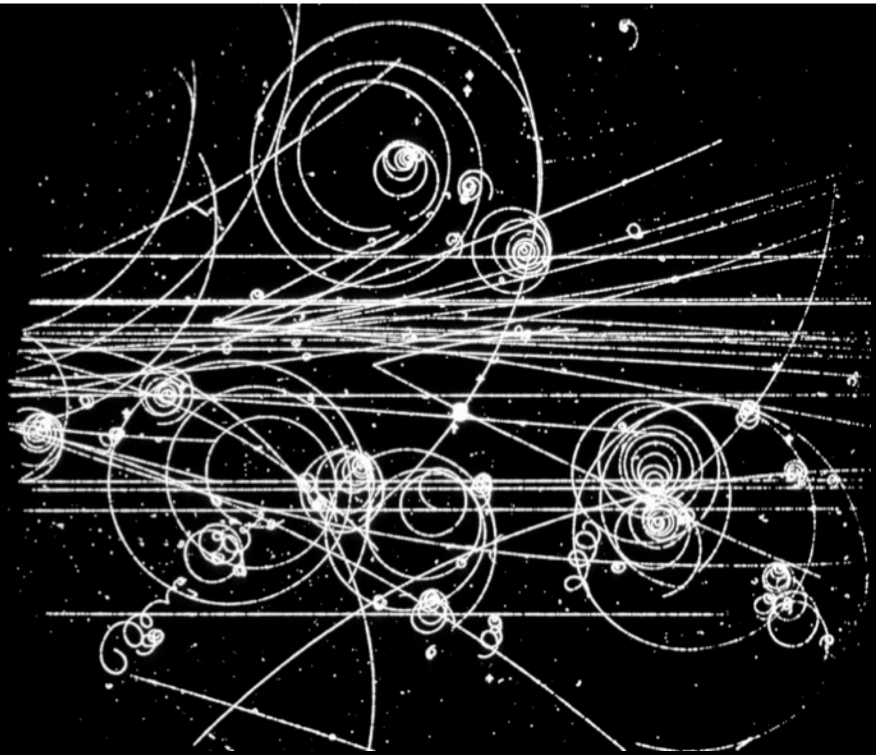
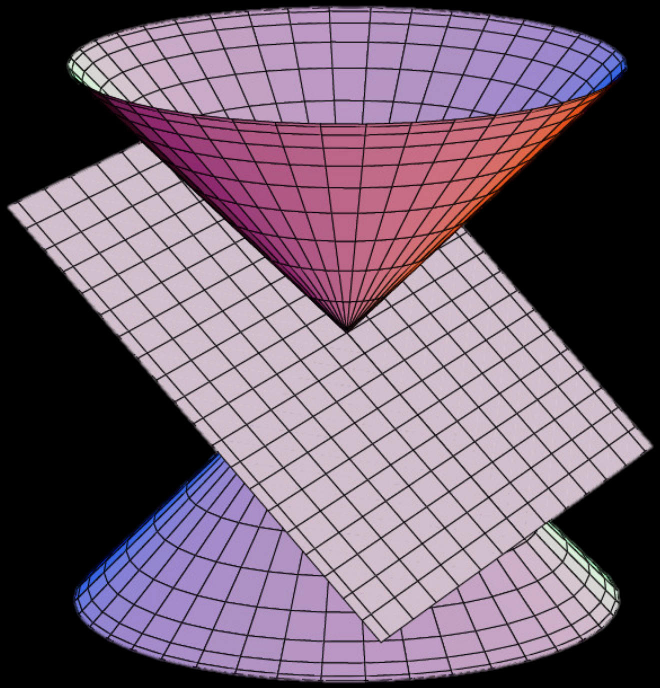




Modern Physics (Phys. IV): 2704

Professor Jasper Halekas
Van Allen 70
MWF 12:30-1:20 Lecture

Beat Frequency

$$y(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$$

say $A_1 = A_2 = A$

$$\cos(\theta_1 + \theta_2) = \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)$$
$$\sin(\theta_1 + \theta_2) = \sin(\theta_1)\cos(\theta_2) + \cos(\theta_1)\sin(\theta_2)$$

$$\cos(\theta_1)\cos(\theta_2) = \frac{1}{2}\left[\cos(\theta_1+\theta_2) + \cos(\theta_1-\theta_2)\right]$$
$$\sin(\theta_1)\cos(\theta_2) = \frac{1}{2}\left[\sin(\theta_1+\theta_2) + \sin(\theta_1-\theta_2)\right]$$

$$\cos(\theta_1) + \cos(\theta_2) = 2\cos\left(\frac{\theta_1+\theta_2}{2}\right)\cos\left(\frac{\theta_1-\theta_2}{2}\right)$$
$$\sin(\theta_1) + \sin(\theta_2) = 2\sin\left(\frac{\theta_1+\theta_2}{2}\right)\cos\left(\frac{\theta_1-\theta_2}{2}\right)$$

$$\Rightarrow A\left[\cos(\omega_1 t) + \cos(\omega_2 t)\right]$$

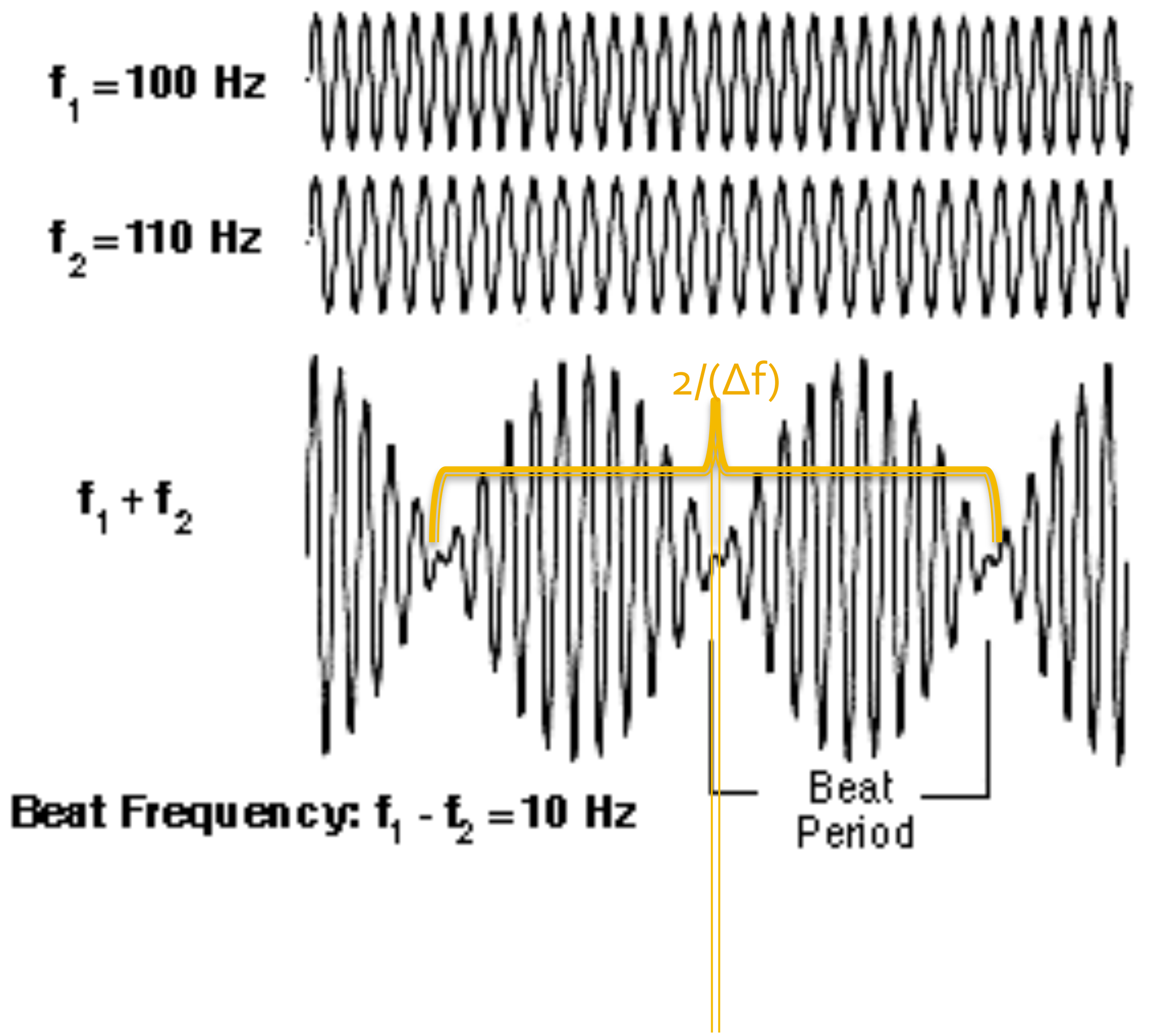
$$= 2A\cos\left(\frac{\omega_1 t + \omega_2 t}{2}\right)\cos\left(\frac{\omega_1 t - \omega_2 t}{2}\right)$$

$$= 2A\cos(\langle\omega\rangle t)\cos\left(\frac{\Delta\omega}{2} t\right)$$

Function w/ $\omega = \langle\omega\rangle$
modulated by $\omega_{beat} = \Delta\omega/2 \ll \langle\omega\rangle$
if $\omega_1 \sim \omega_2$

Sometimes said $\omega_{beat} = \Delta\omega$
since this gives angular frequency
of maxima/minima

Beat Frequencies



Traveling Wave

$$y(x,t) = A_1 \cos(k_1 x - \omega_1 t) + A_2 \cos(k_2 x - \omega_2 t)$$

$v = \omega/k$ = phase speed

$v_1 = \omega_1/k_1$

$v_2 = \omega_2/k_2$

For $A_1 = A_2 = A$

$$y(x,t) = 2A \cos\left(\frac{\Delta k}{2} x - \frac{\Delta \omega}{2} t\right) \cdot \cos(\langle k \rangle x - \langle \omega \rangle t)$$

velocity of modulation

$$v_g = \frac{\Delta \omega / 2}{\Delta k / 2} = \Delta \omega / \Delta k$$

$\Rightarrow d\omega/dk$ for continuous distribution of wave components

v_g = "Group Velocity"

Concept Check

- How are the phase and group velocity related?

A. Phase velocity is always faster
B. Group velocity is always faster
C. Either one could be faster
D. Phase and group velocity are equal

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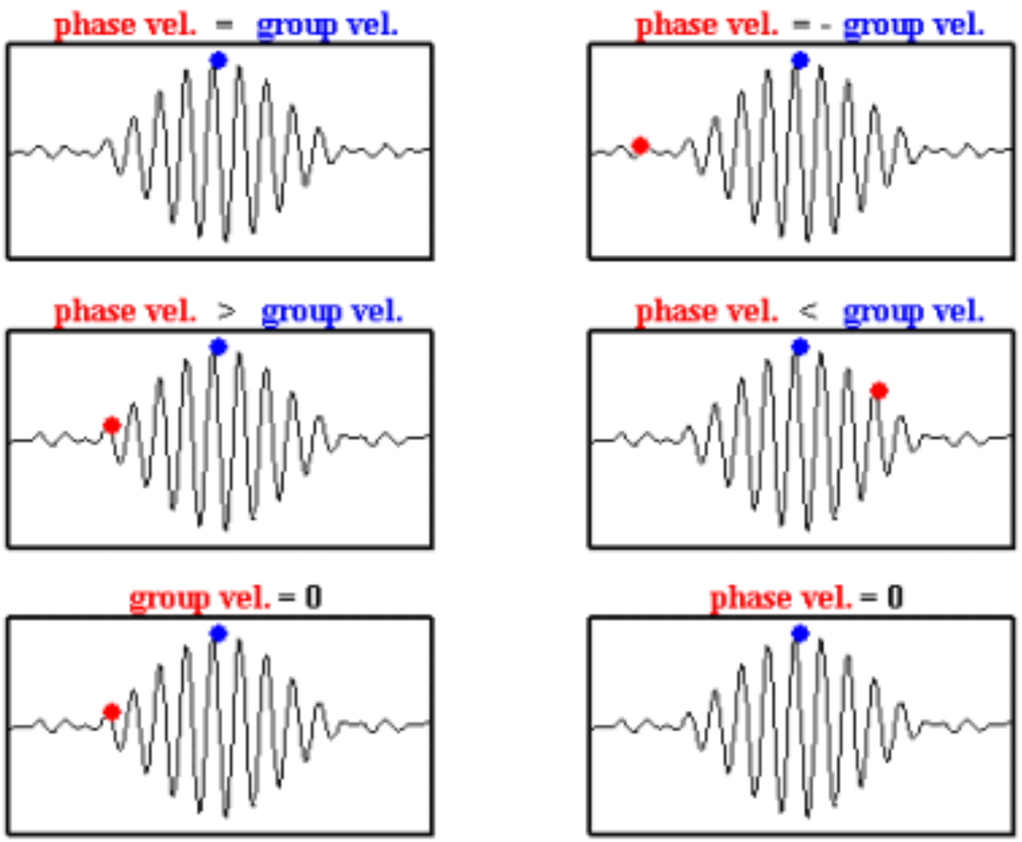
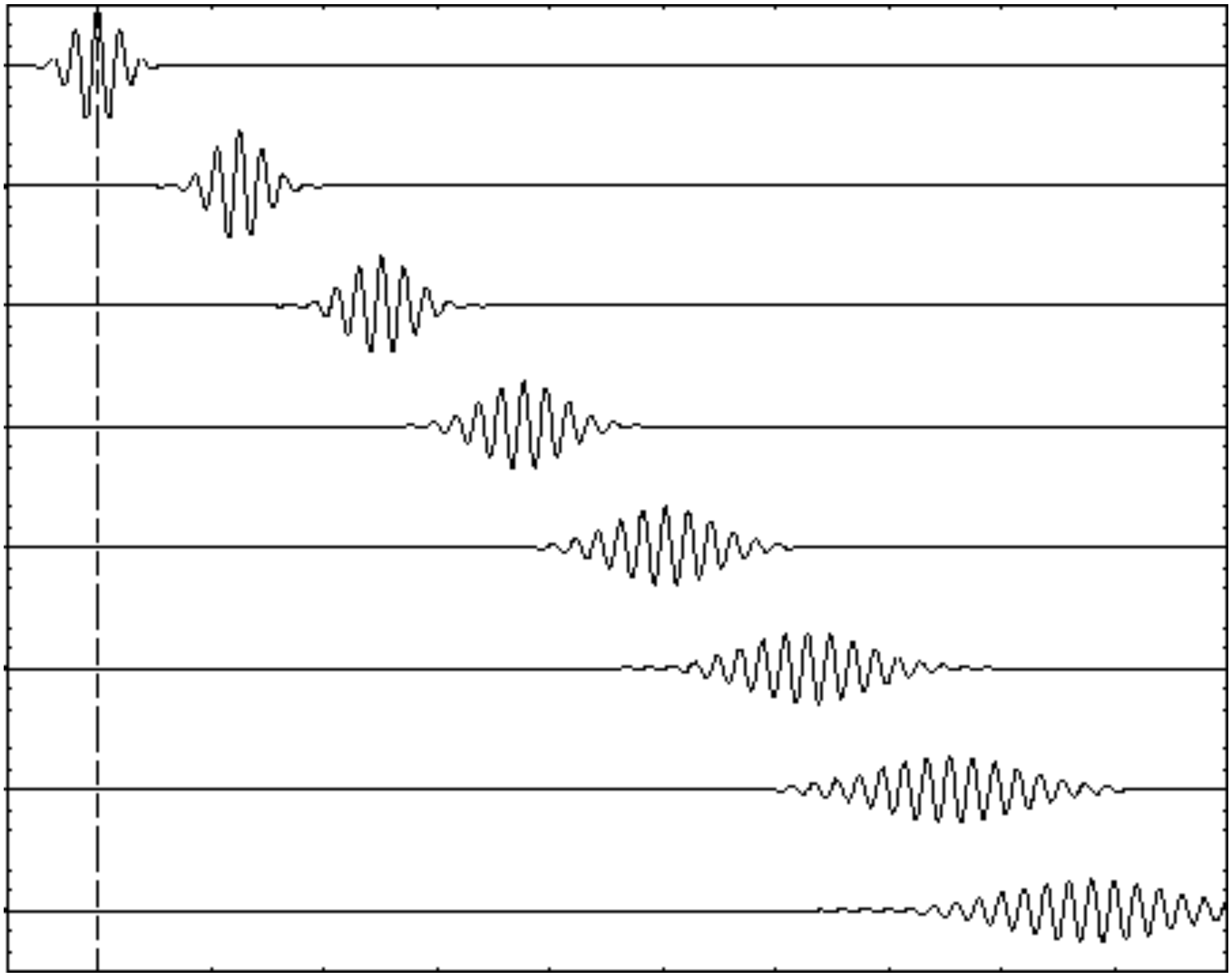


isvr

Wave Dispersion



Group vs. particle velocity

$v_g = d\omega / dk$

$\omega = 2\pi\nu = 2\pi E / h$

$k = 2\pi / \lambda \to 2\pi p / h$

$\Rightarrow d\omega / dk = dE / dp$

$$\frac{d}{dp}(E) = \frac{d}{dp}\sqrt{(mc^2)^2 + (pc)^2}$$

$$= \frac{1/2}{\sqrt{(mc^2)^2 + (pc)^2}} \cdot 2pc^2$$

$$= \frac{pc^2}{E} = \frac{\gamma m v c^2}{\gamma m c^2}$$

$$= v$$

so $V_{group} = V_{particle}$

Concept Check

- A particle wave packet is affected by a repulsive potential. What happens to the average wavelength of the wave packet?

- Increases
- Decreases
- Stays the Same

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Classical Energy

$$E = \frac{1}{2} m v^2 + U(x)$$

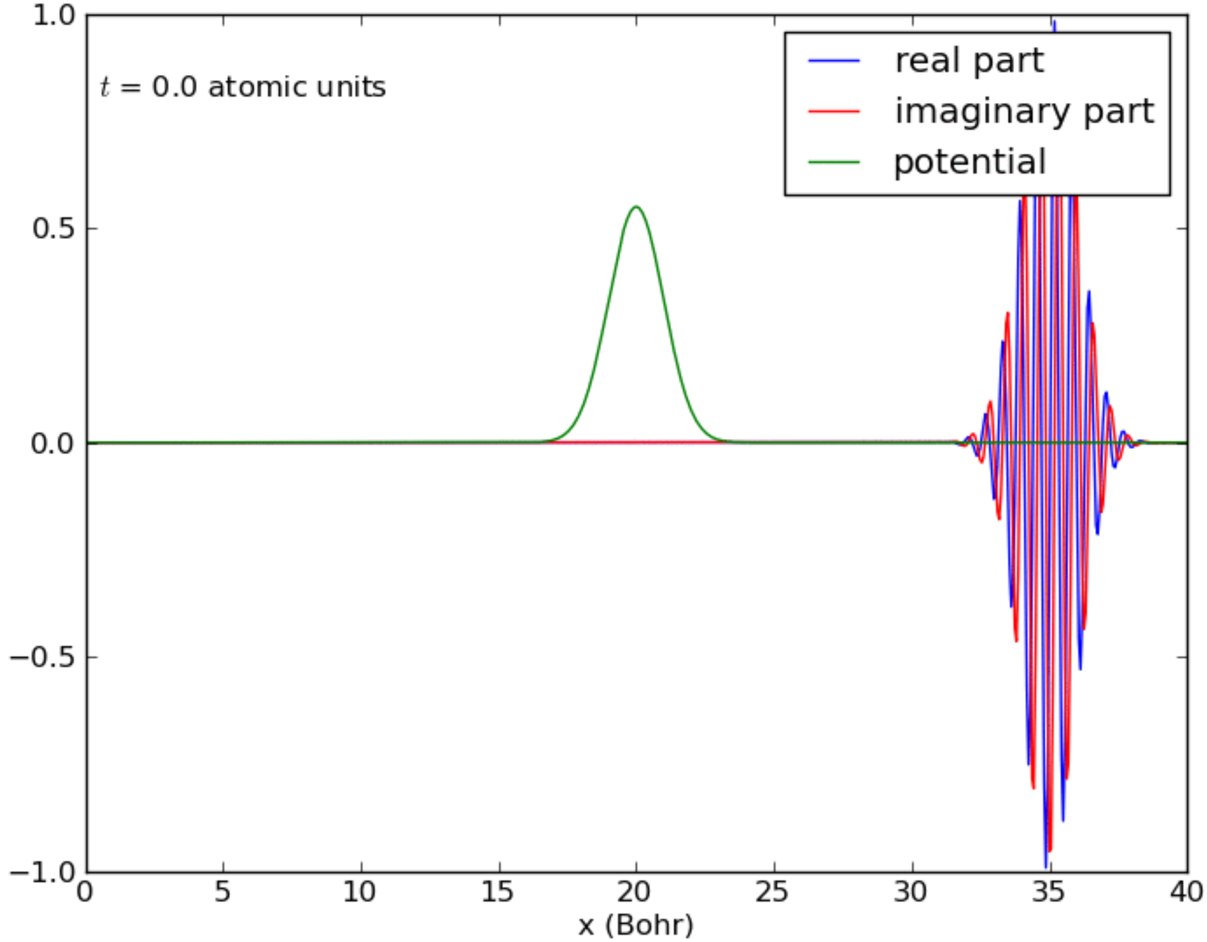
$$= p^2/2m + U(x)$$

$$= h^2/(2m\lambda^2) + U(x)$$

$$= \frac{\hbar^2 k^2}{2m} + U(x)$$

Basis for Schrodinger Eq.

Potential Barrier



Taller Potential Barrier