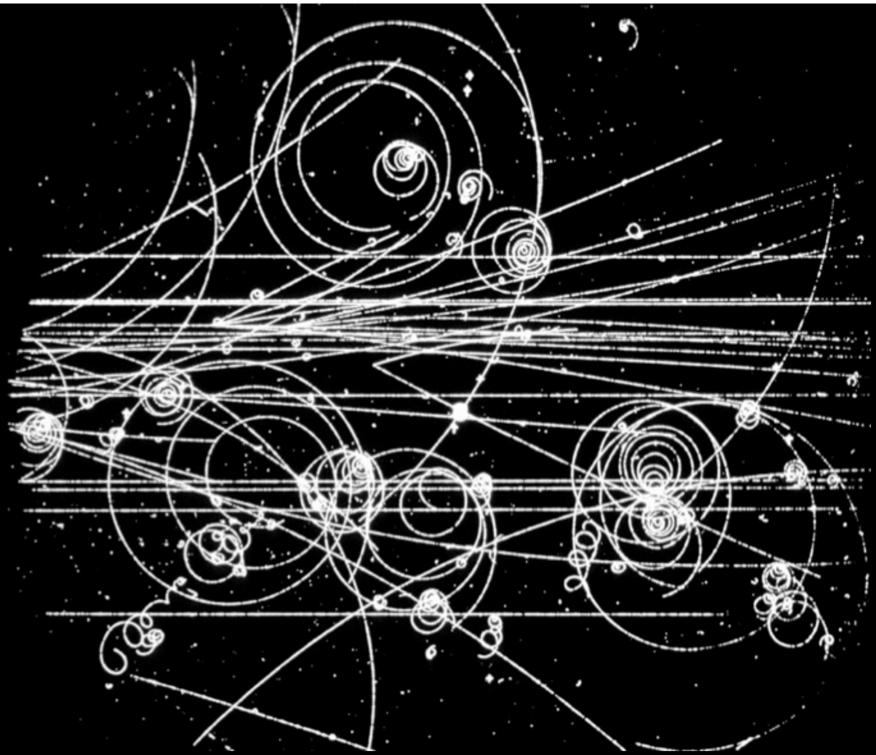
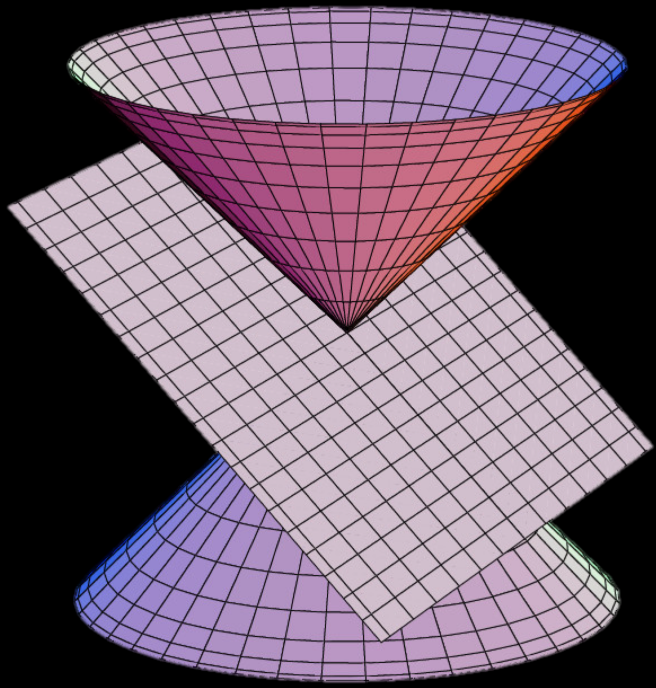
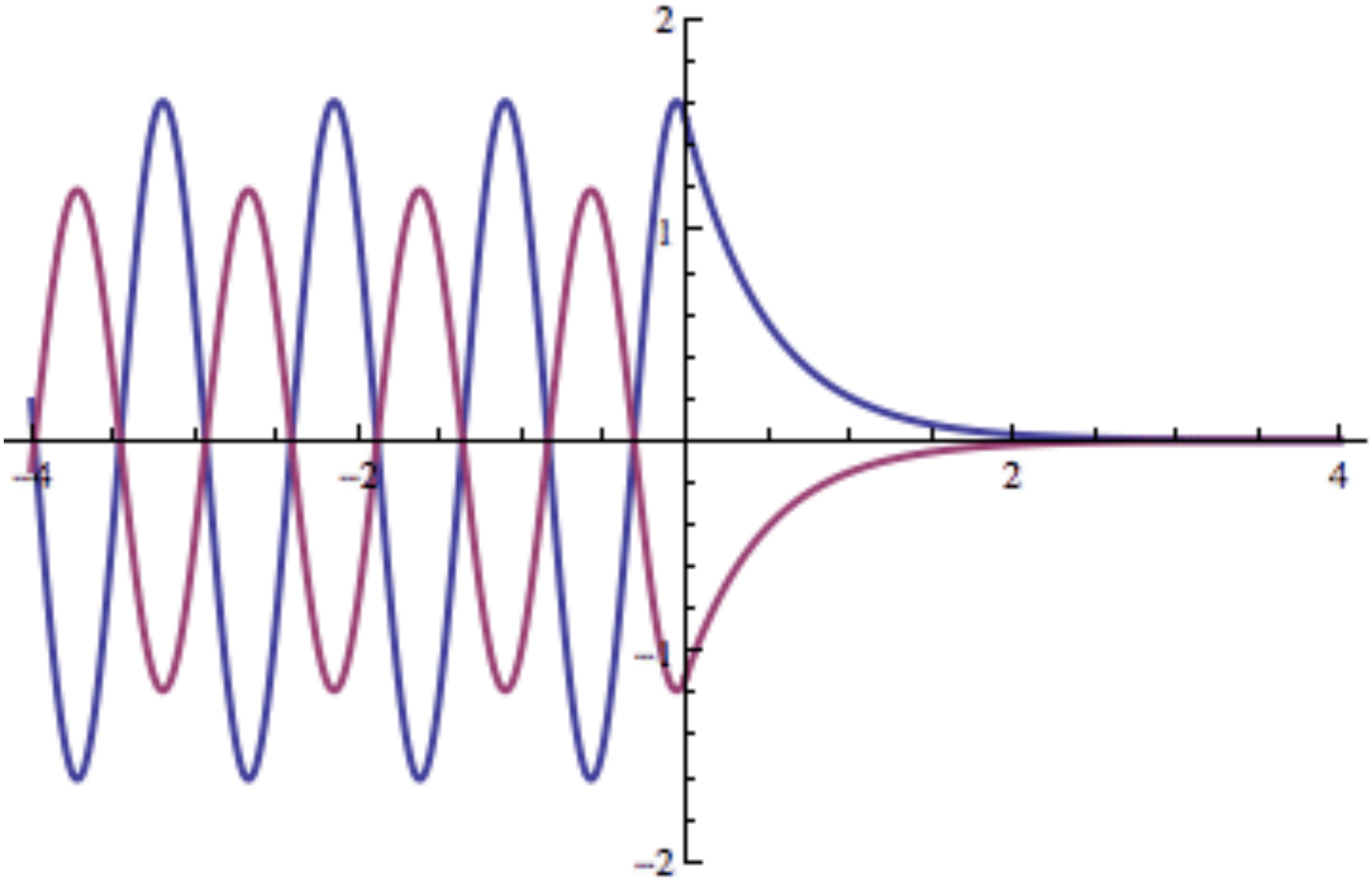




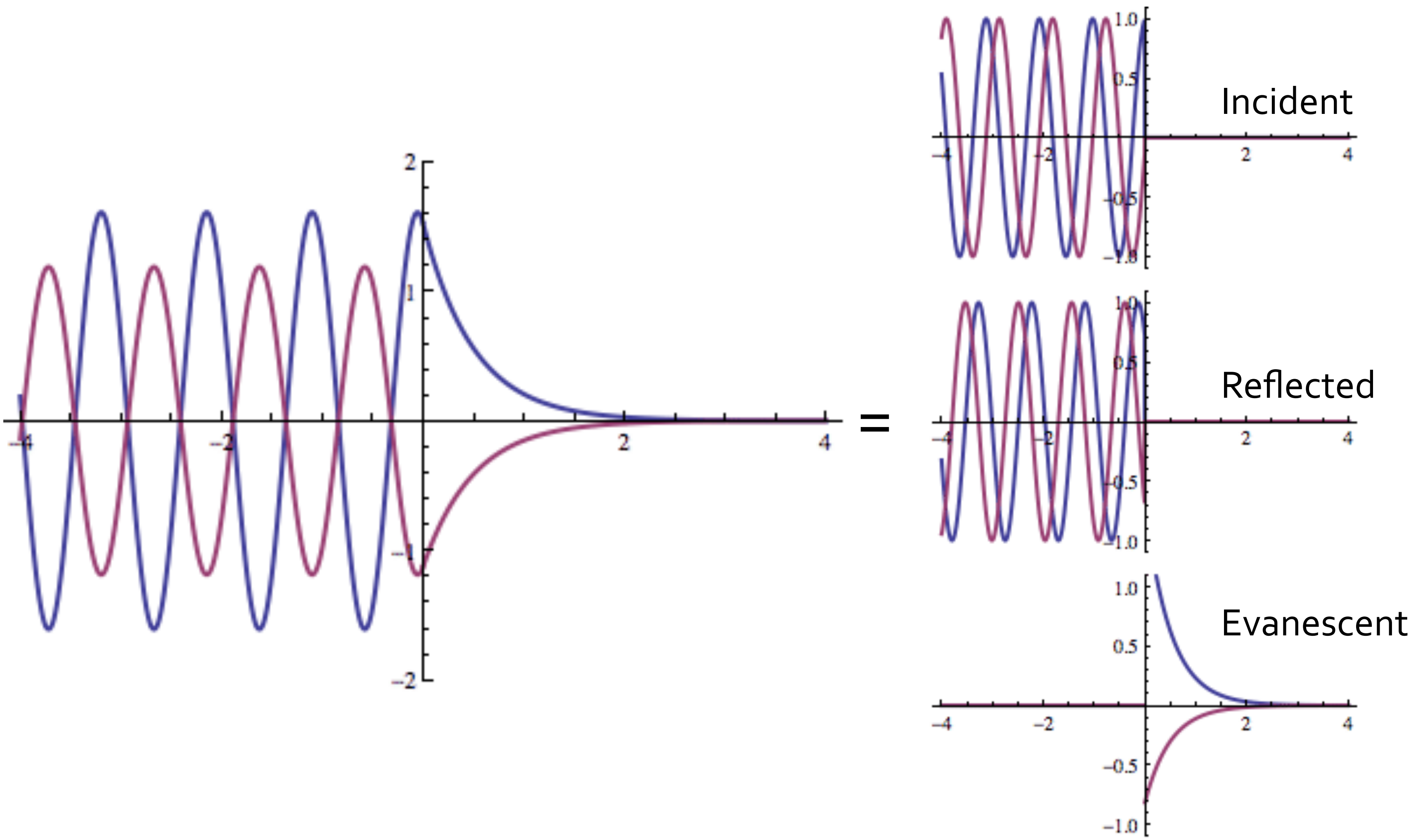
Modern Physics (Phys. IV): 2704

Professor Jasper Halekas
Van Allen 70
MWF 12:30-1:20 Lecture

Actual Wave Function



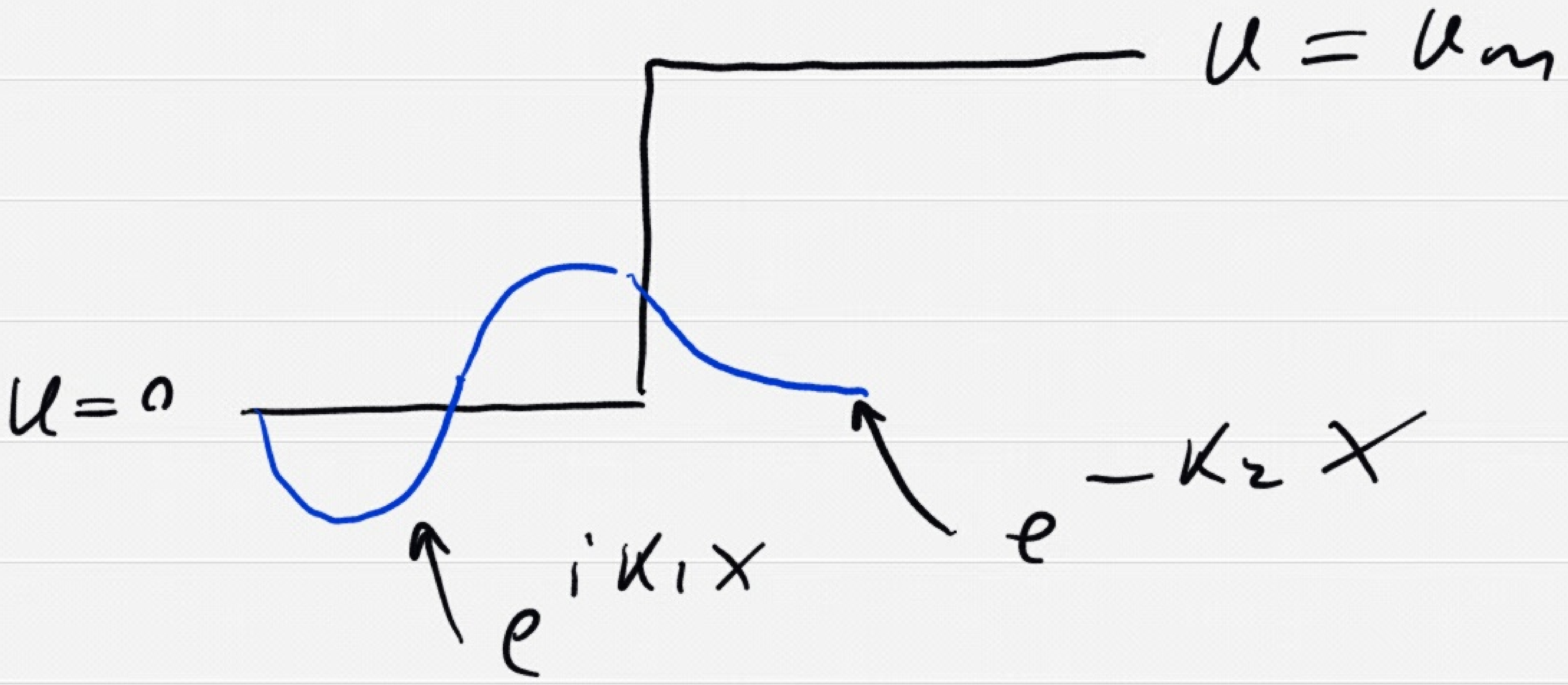
Wave Function Components



How Can This be So?

- Did we violate conservation of energy…?
- Did we violate conservation of momentum…?

Evanescent Wave



$$K_2 = \sqrt{\frac{2m}{\hbar^2}(U_m - E)}$$

$$\Delta x = \sigma_x \sim \frac{1}{2K_2} = \frac{\hbar}{2\sqrt{2m(U_m - E)}}$$

e-folding distance of $|\psi(x)|^2$

$$\Delta E \, \Delta t \sim \hbar$$

$$\Rightarrow \Delta t = \hbar / \Delta E$$

$$= \hbar / (U_m - E + K)$$

$-\Delta E$ to give energy K in "forbidden region"

$$v = \sqrt{2K/m} \qquad \xleftarrow{\Delta x}$$

$$\Delta x = \tfrac{1}{2} v \Delta t$$

$$\Delta x = \frac{1}{2}\sqrt{\frac{2K}{m}}\;\frac{\hbar}{U_m - E + K}$$

Find Δx_{max} by setting

$$d/dK\,(\Delta x) = 0$$

$$d/dK\,(\Delta x) = \frac{1}{\sqrt{2m}}\left(\frac{1}{2\sqrt{K}}\cdot\frac{\hbar}{U_m - E + K} - \sqrt{K}\,\frac{\hbar}{(U_m - E + K)^2}\right)$$

$$\Rightarrow \frac{1}{2\sqrt{K}}\cdot\frac{1}{U_m - E + K} = \sqrt{K}\cdot\frac{1}{(U_m - E + K)^2}$$

$$\Rightarrow U_m - E + K = 2K$$

$$\Rightarrow K = U_m - E$$

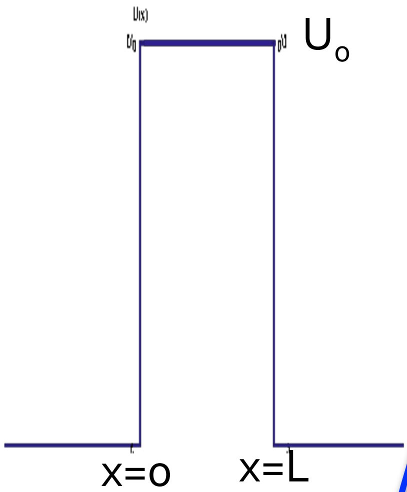
$$\Rightarrow \Delta x_{max} = \frac{1}{2}\sqrt{\frac{2(U_m - E)}{m}}\cdot\frac{\hbar}{2(U_m - E)}$$

$$= \hbar/2\cdot\frac{1}{\sqrt{2m(U_m - E)}}$$

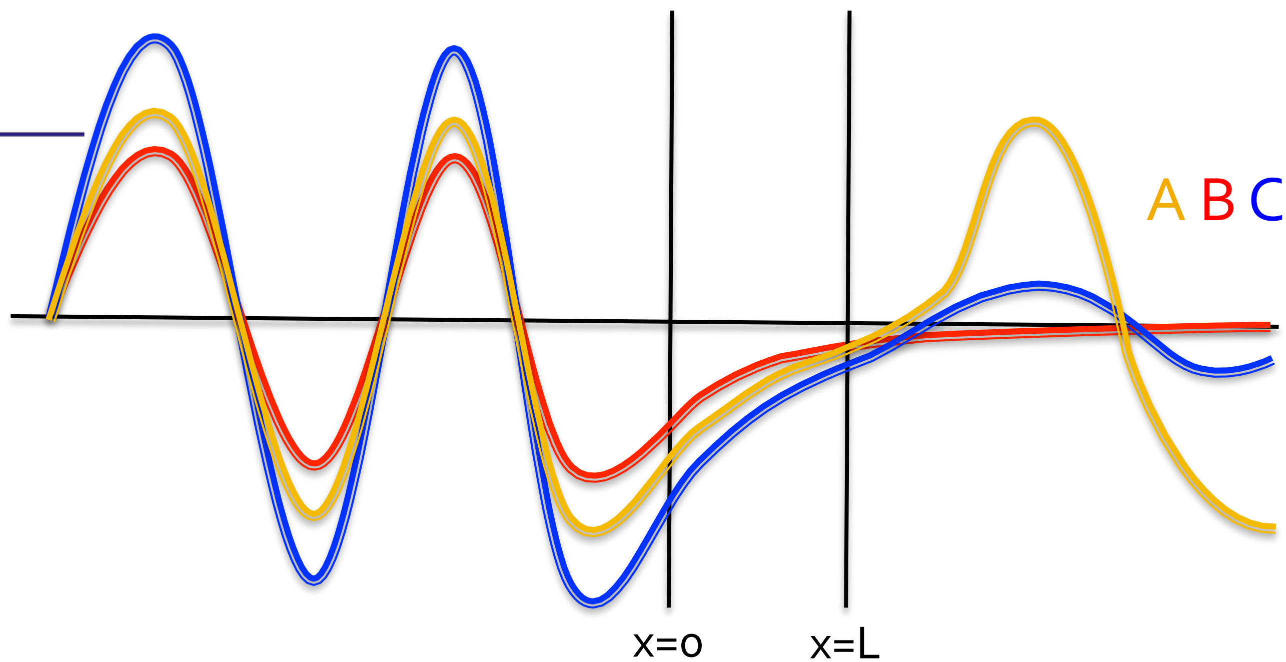
$$\sigma_x = \Delta x_{max}$$

- so uncertainty principle allows this amount of penetration into classically forbidden region

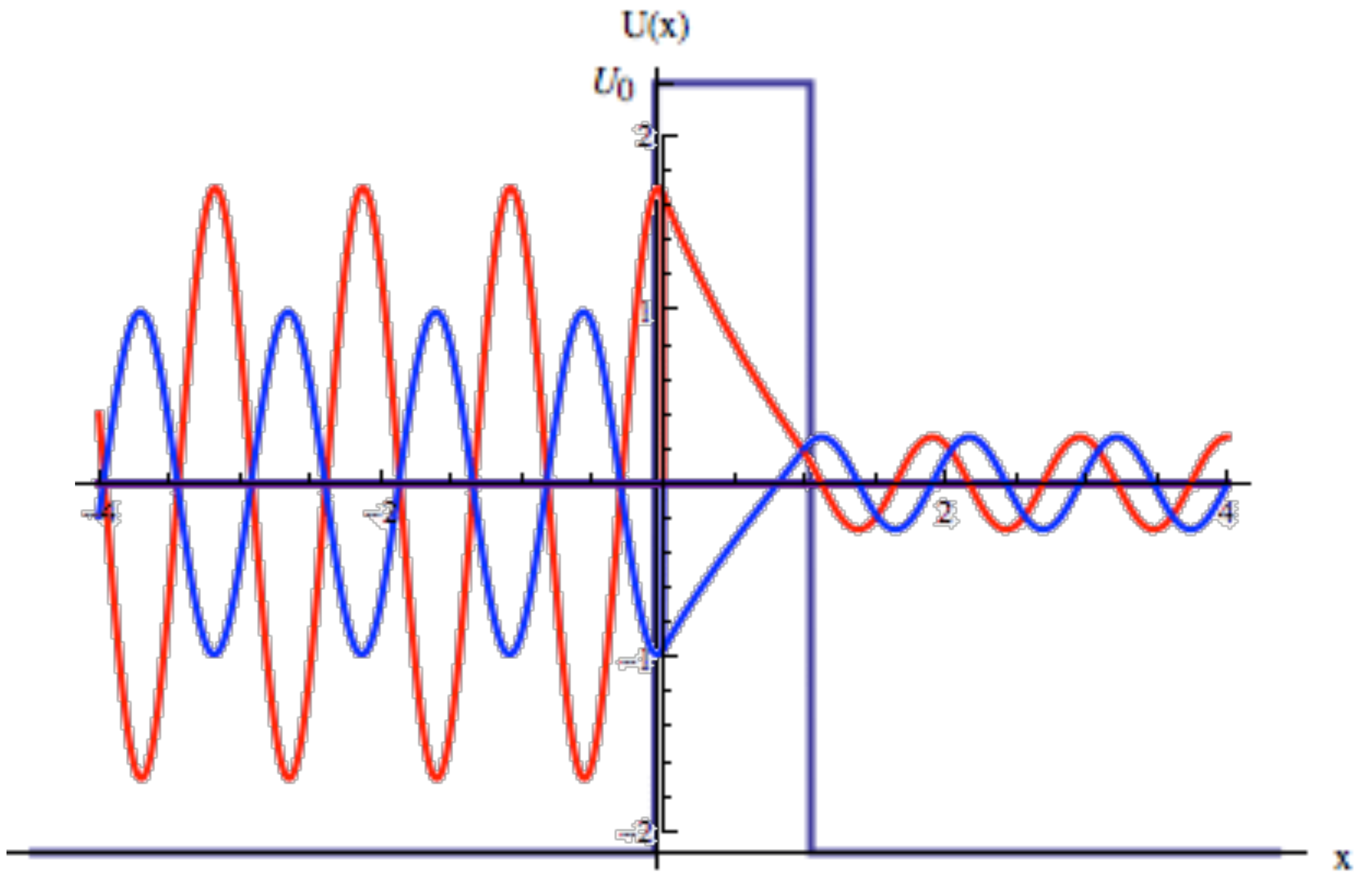
Wave Across a Potential Wall



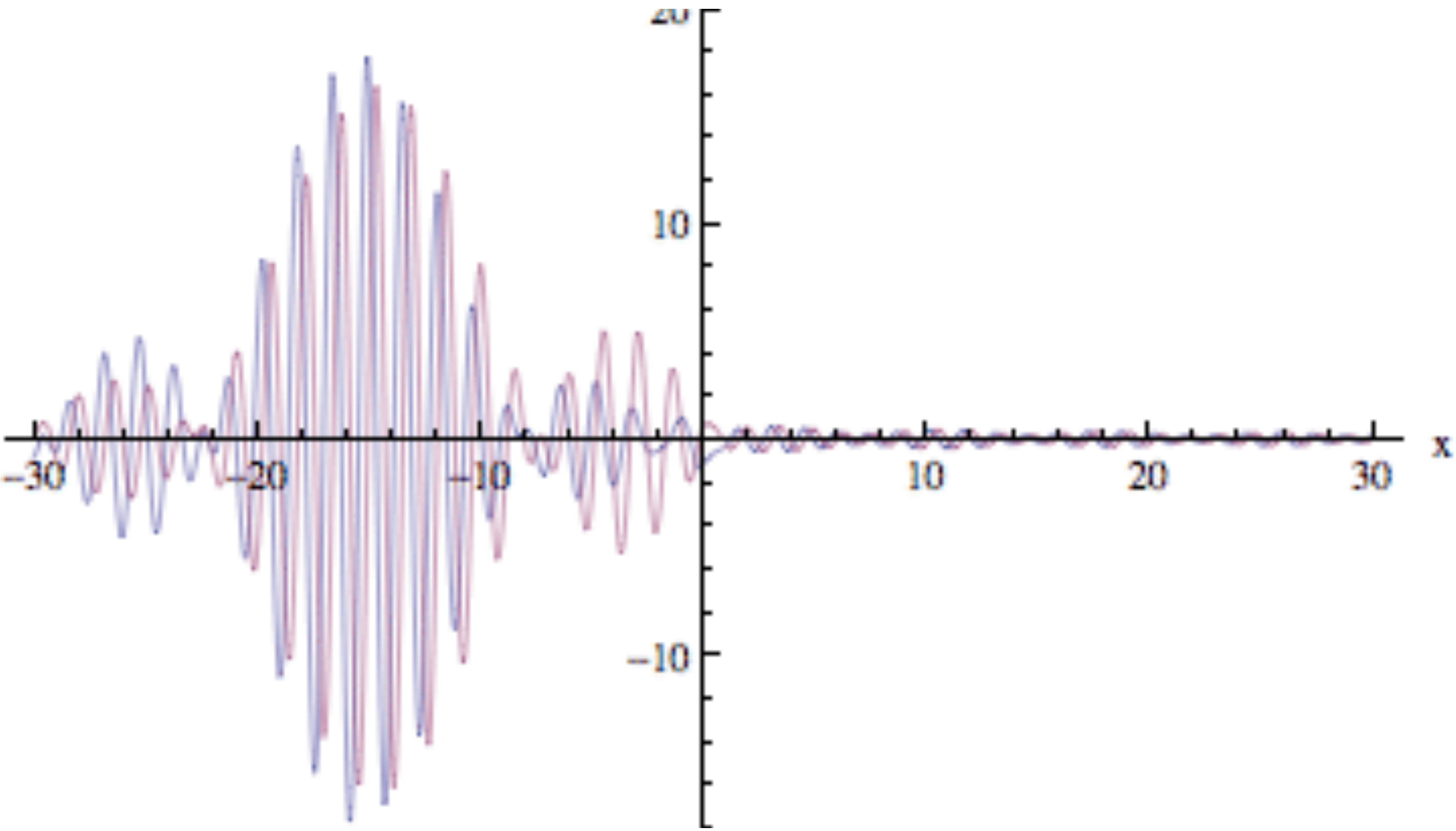
What does the wavefunction look like for an electron with $E < U_o$ (U_o = the height of the wall)?



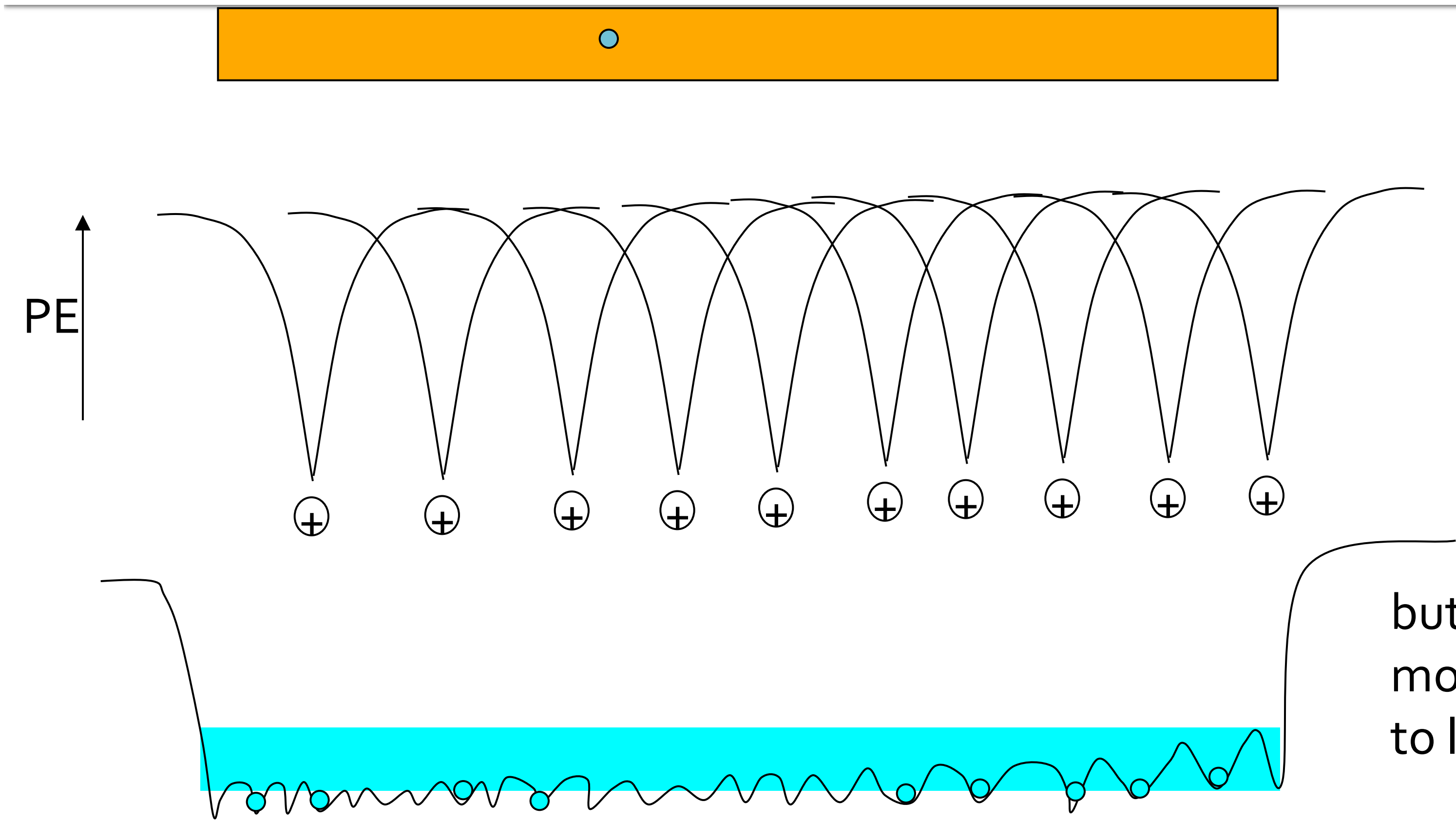
Tunneling!



Wave Packet Tunneling



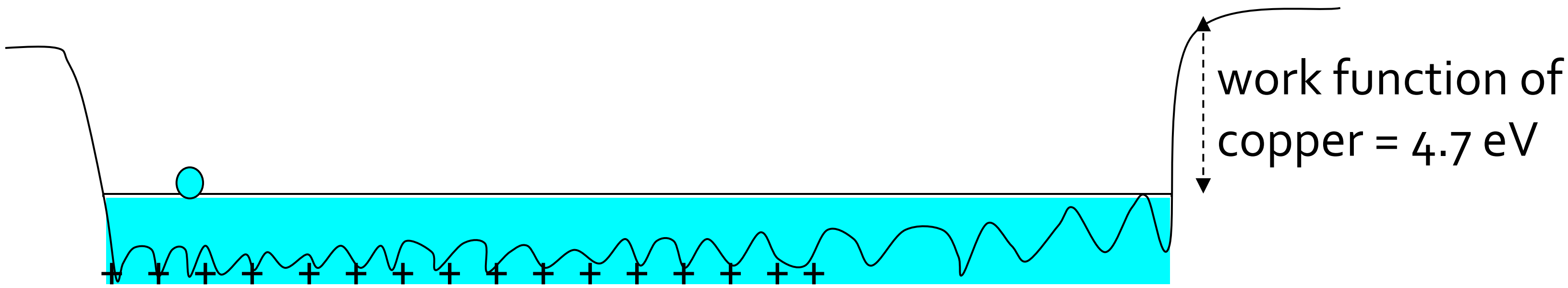
Electron in a Finite Wire



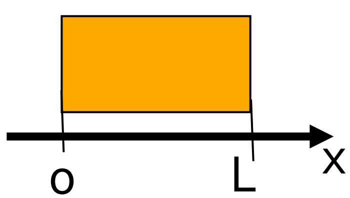
but lot of e's move around to lowest PE

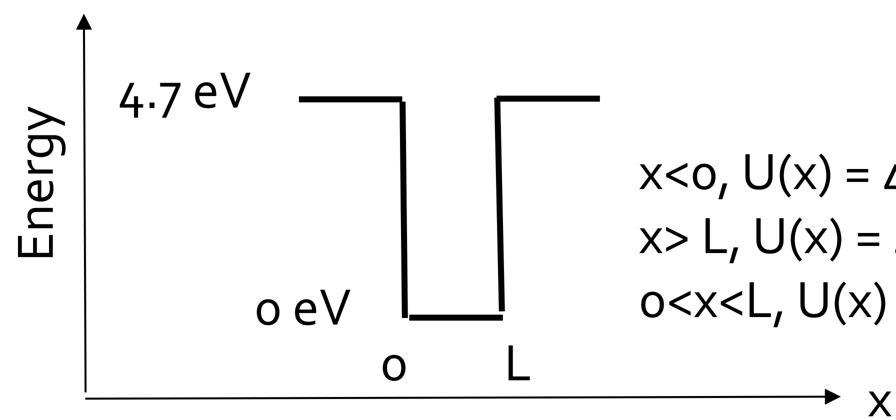
As more electrons fill in, potential energy for later ones gets flatter and flatter. For top ones, is VERY flat.

Electron in a Finite Wire



This is just the energy needed to remove them from the metal.
That is the work function!!



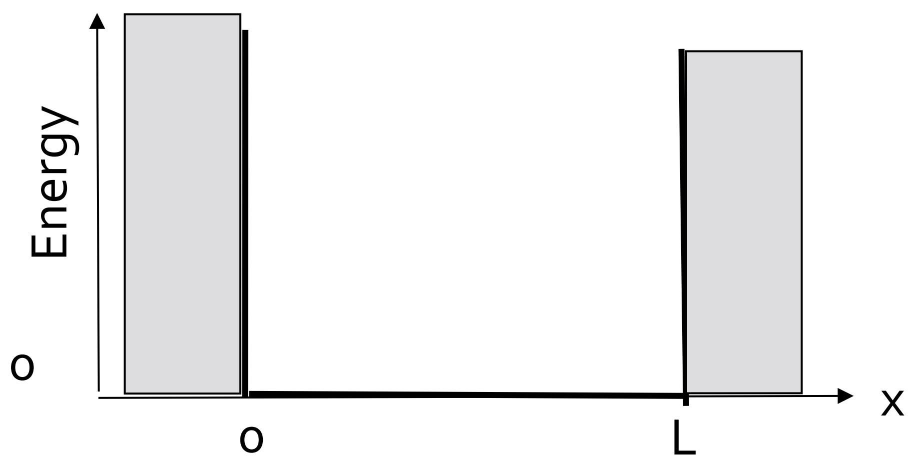


$x<0$, $U(x) = 4.7$ eV
$x> L$, $U(x) = 4.7$ eV
$0<x<L$, $U(x) = 0$

Finite Square Well

- kT ~0.025 eV << 4.7 eV so approximate 4.7 as ∞

x<0, V(x) ~ infinite
x> L, V(x) ~ infinite
0<x<L, V(x) =0

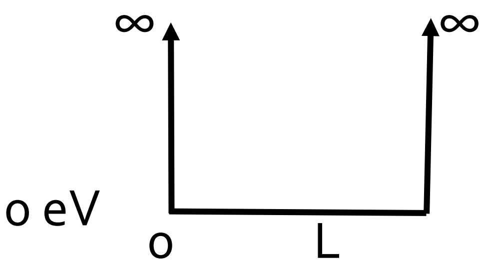


Simplified approach means just have to solve:

$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} = E\psi(x)}$$

with boundary conditions,
$\psi(0)=\psi(L)=0$

Infinite Square Well Solution

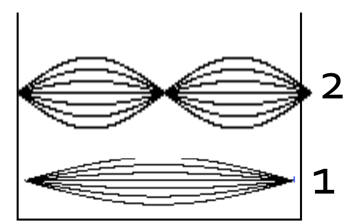


$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} = E\psi(x)}$$

functional form of solution: $\psi(x) = A\cos(kx) + B\sin(kx)$

x=0 → ? $\psi(0) = A$ → A=0

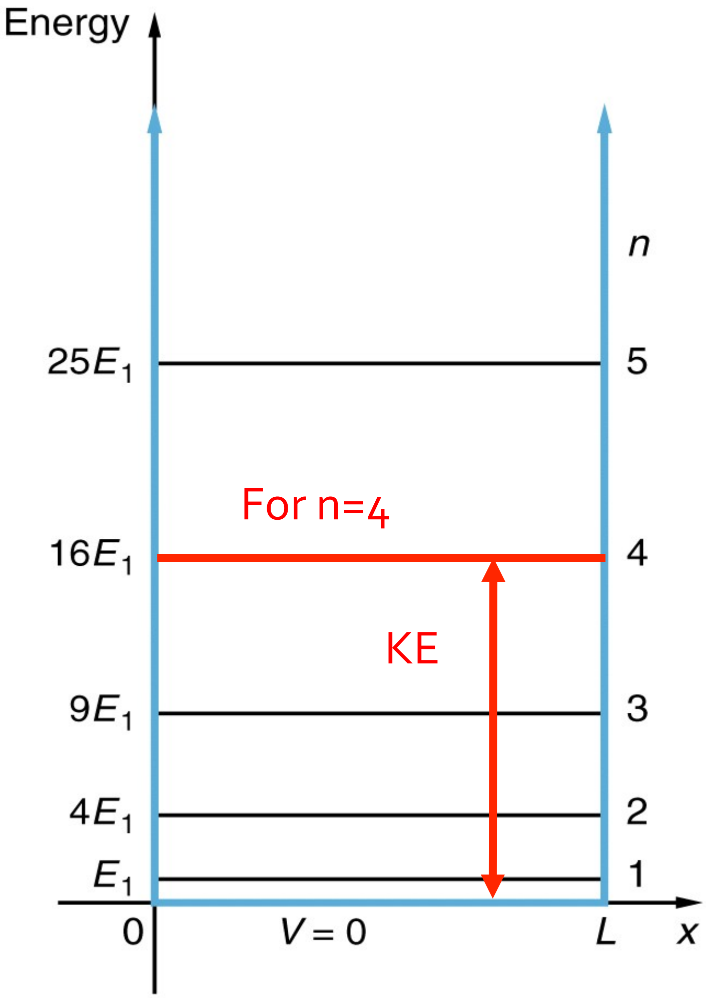
x=L → $\psi(L) = B\sin(kL) = 0$ kL=nπ (n=1,2,3,4 …)



$$p = \hbar k = \hbar(n\pi / L)$$

$$E = p^2 / 2m$$

Energy Level Diagrams



Square Well Energy Spacing

$E_1 = \pi^2 \hbar^2 / (2mL^2)$

$E_2 = 4\pi^2 \hbar^2 / (2mL^2)$

$\Delta E_{12} = 3\pi^2 \hbar^2 / (2mL^2)$

- Quantization important if $\Delta E \sim kT$

$$1.38 \times 10^{-23} \cdot 300 = \frac{3 \cdot 3.1^2 \cdot (1.1 \times 10^{-34})^2}{2 \cdot 10^{-30} \cdot L^2}$$

$\Rightarrow L^2 \sim 3 \times 10^{-17}\ m^2$

$L \sim 5 \times 10^{-8}\ m$

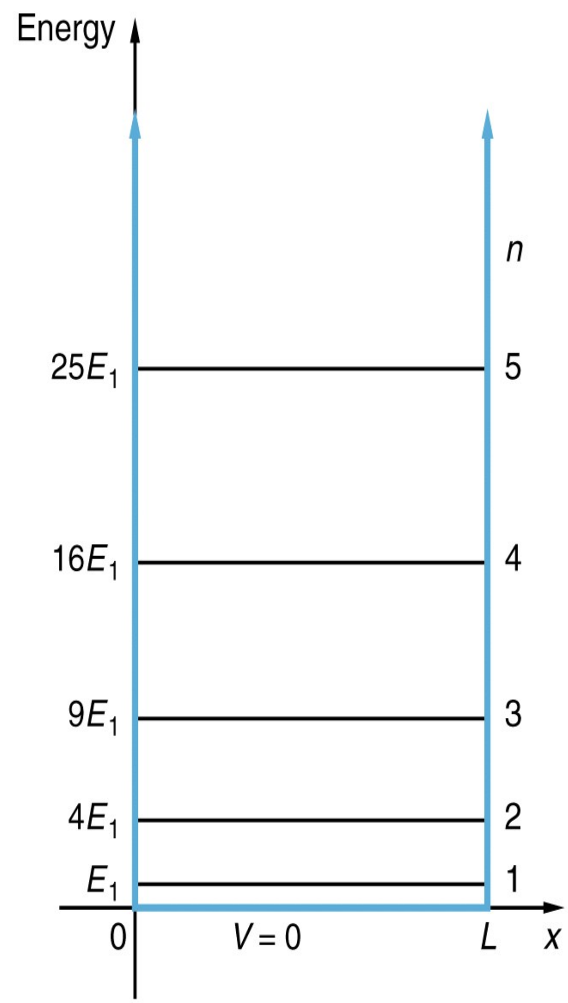
$= 50\ nm$

~ 250 atoms long

Concept Check

- How does the probability of finding an electron at x = L/2 for n =3 compare to the probability for n = 2?

A. Much more likely for n = 3
B. Much more likely for n = 2
C. Equally likely for n = 2 or n = 3



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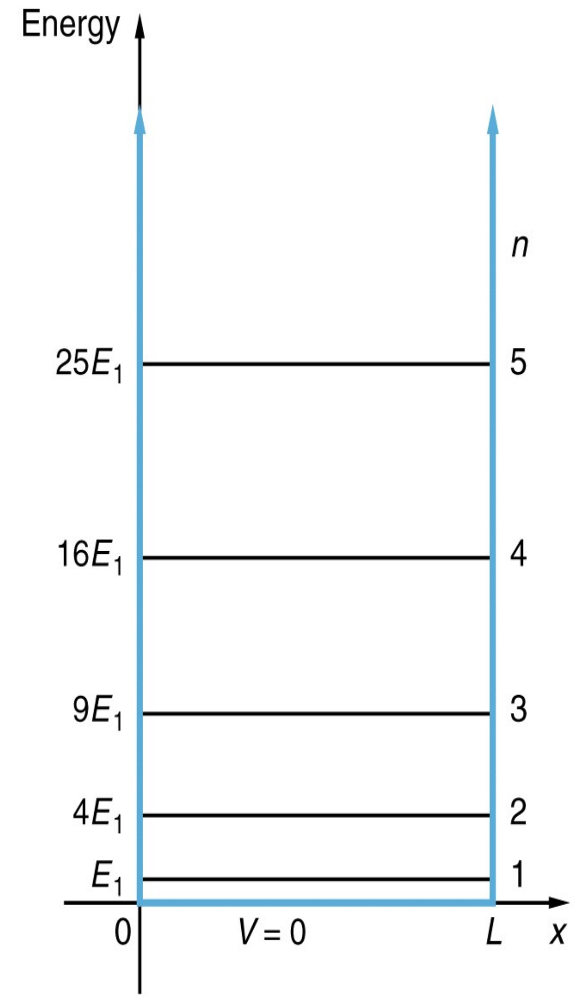

First three wave functions