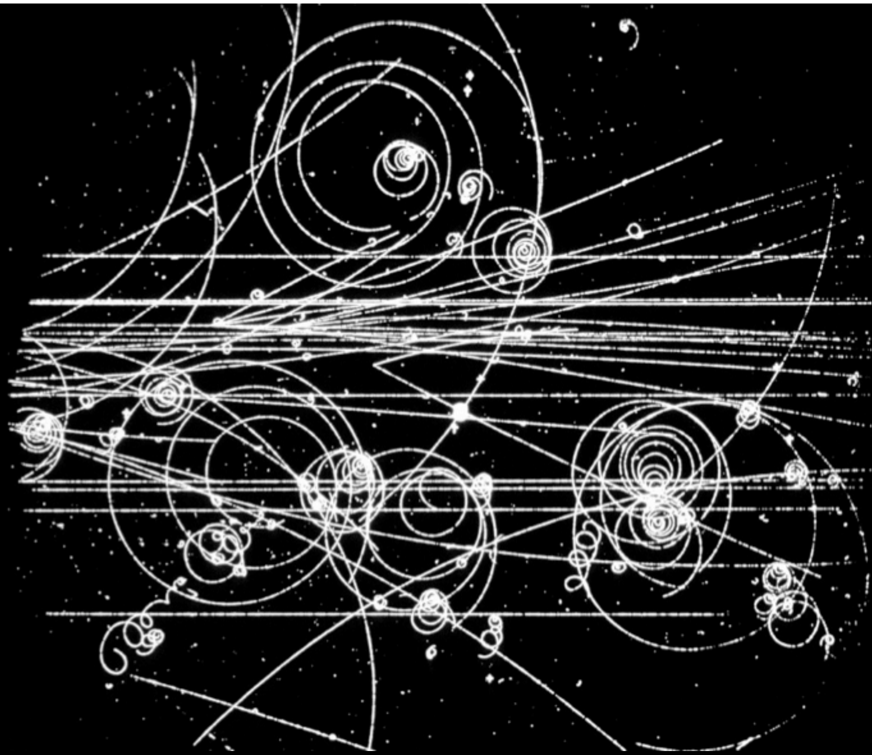
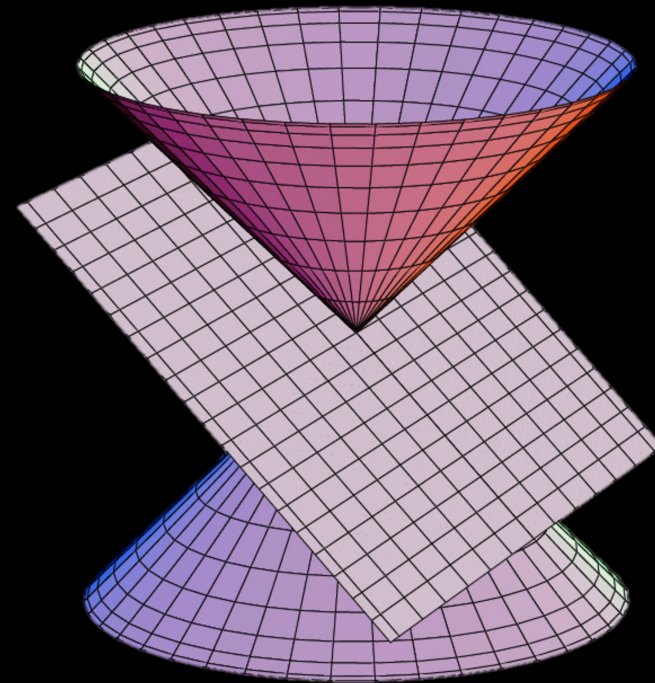
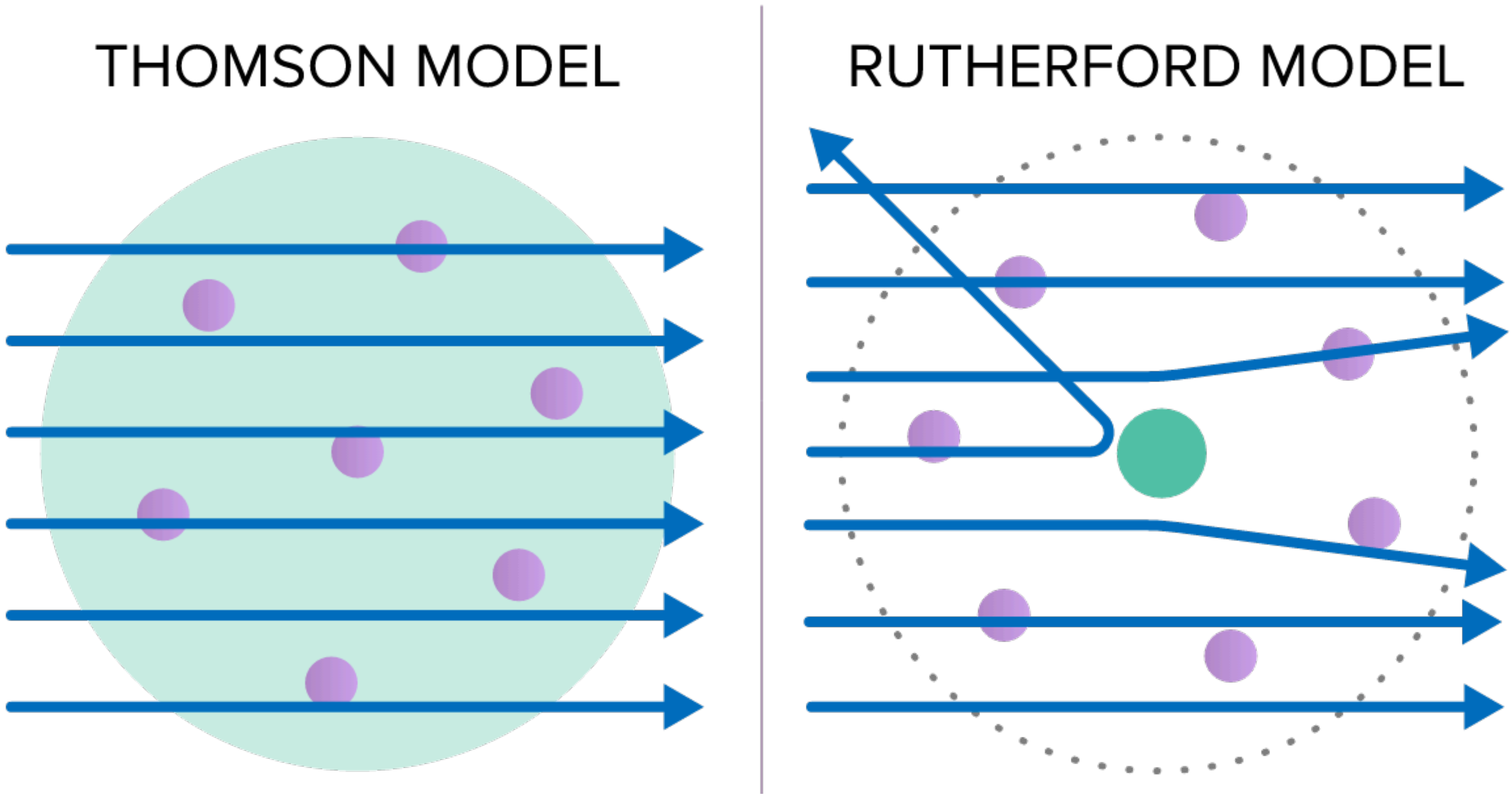




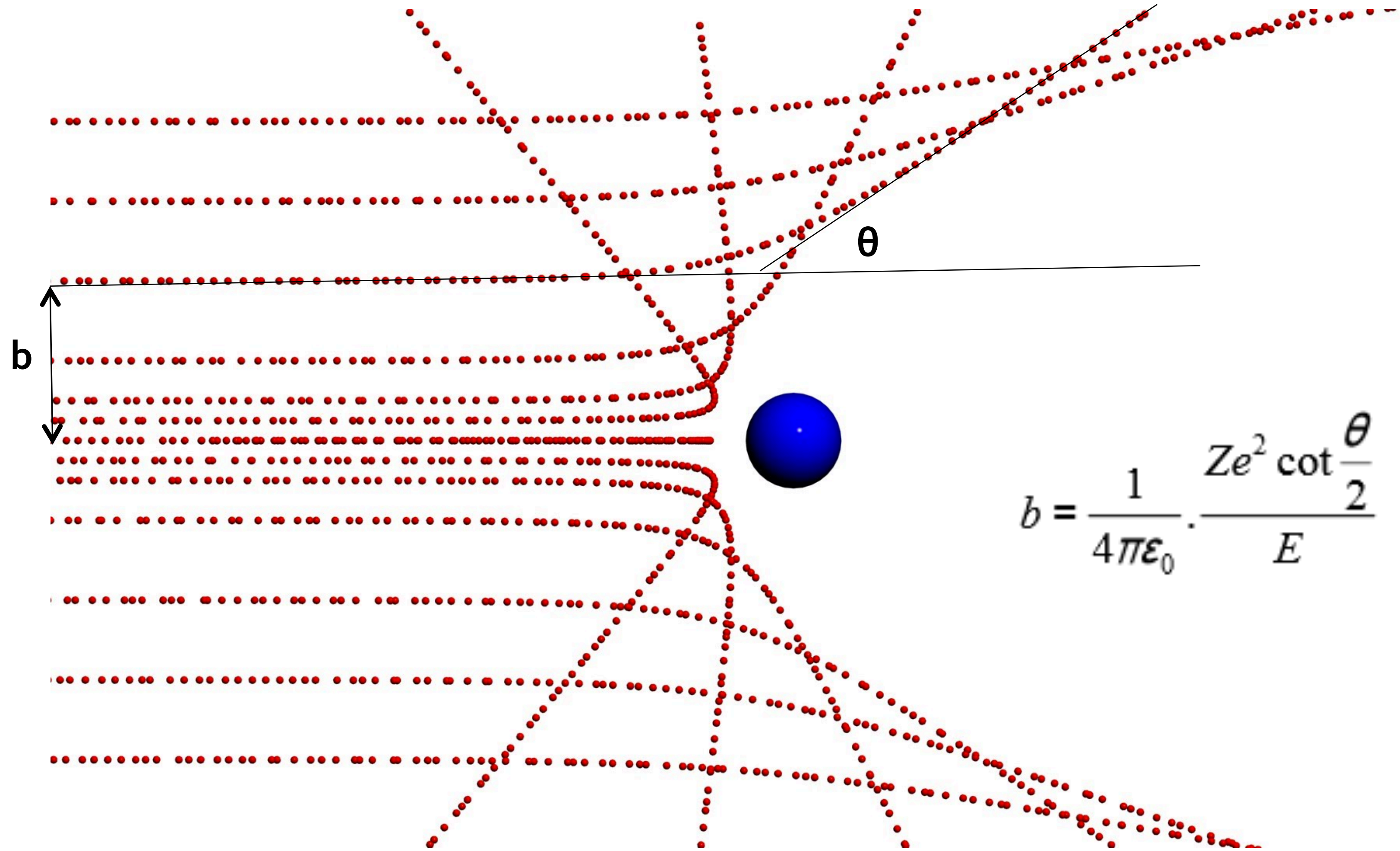
Modern Physics (Phys. IV): 2704

Professor Jasper Halekas
Van Allen 70
MWF 12:30-1:20 Lecture

Rutherford Scattering



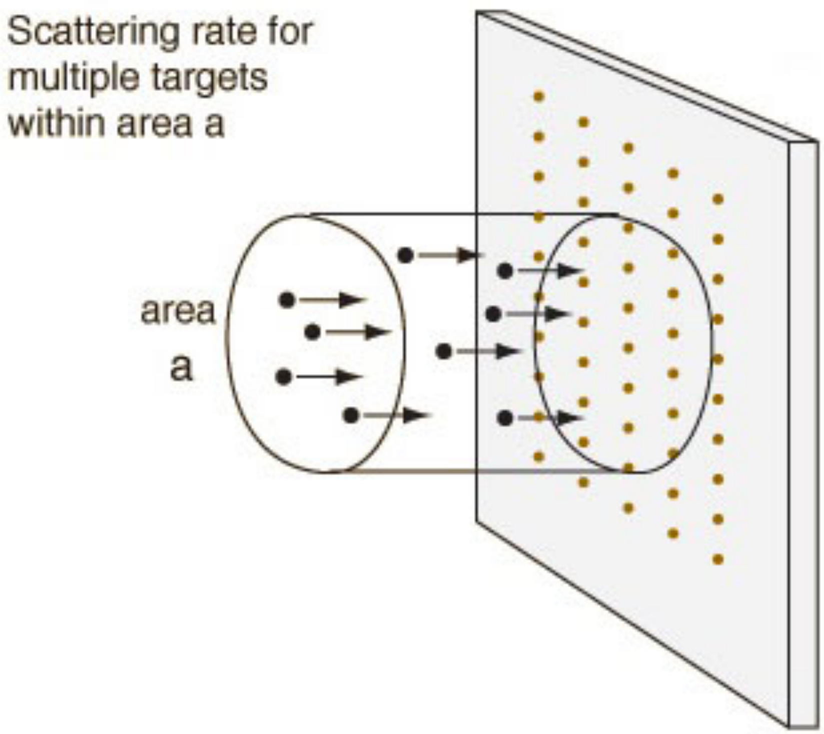
Rutherford Scattering

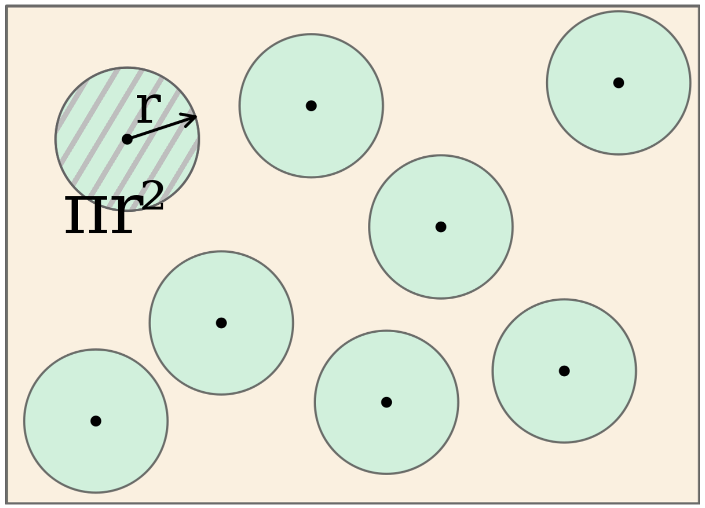


$$b = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2 \cot\frac{\theta}{2}}{E}$$

$$\frac{Z}{2}$$

Total Cross Section





Thickness T

Concept Check

- How should the probability of alpha scattering depend on the density of atoms and thickness of material in the target?

A. Linearly with density and not with thickness
B. Linearly with density and linearly with thickness
C. Not with density and not with thickness
D. Not with density and linearly with thickness

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Scattering Fraction

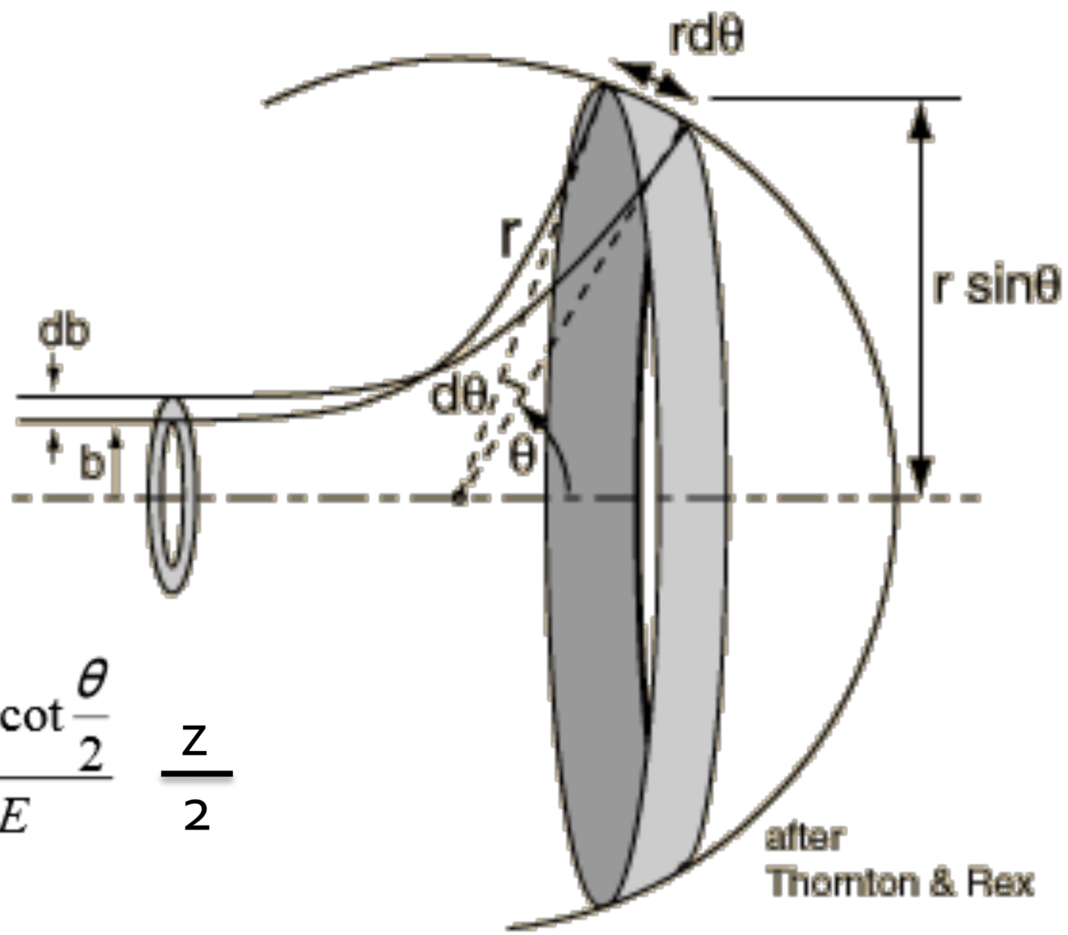
Fraction w/ scattering angle $> \theta$ same as fraction w/ impact parameter $< b$

$$f_{>\theta} = f_{<b} = n T \pi b^2$$

$$= \frac{\text{nuclei}}{\text{volume}} \cdot \frac{\text{volume}}{\text{area}} \cdot \text{scattering area}$$

$$= \text{nuclei} \cdot \frac{\text{scattering area}}{\text{Total area}}$$

Scattering Distribution Geometry



$$b = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2 \cot\frac{\theta}{2}}{E} \qquad \frac{z}{2}$$

Scattering Probability

$$df = nT \cdot d(\pi b^2)$$
$$= nT \cdot 2\pi b \, db$$

nT ↑ $\frac{\text{nuclei}}{\text{area}}$

$2\pi b \, db$ ↑ scattering area from b to $b+db$

$$db/d\theta = \frac{d}{d\theta} \frac{\alpha}{2K} \cot(\theta/2)$$
$$= \alpha/2K \cdot -\frac{1}{\sin^2(\theta/2)} \cdot 1/2$$

$$\Rightarrow df = \pi n T \cdot \left(\frac{\alpha}{2K}\right)^2 \frac{\cot(\theta/2)}{\sin^2(\theta/2)} \, d\theta$$

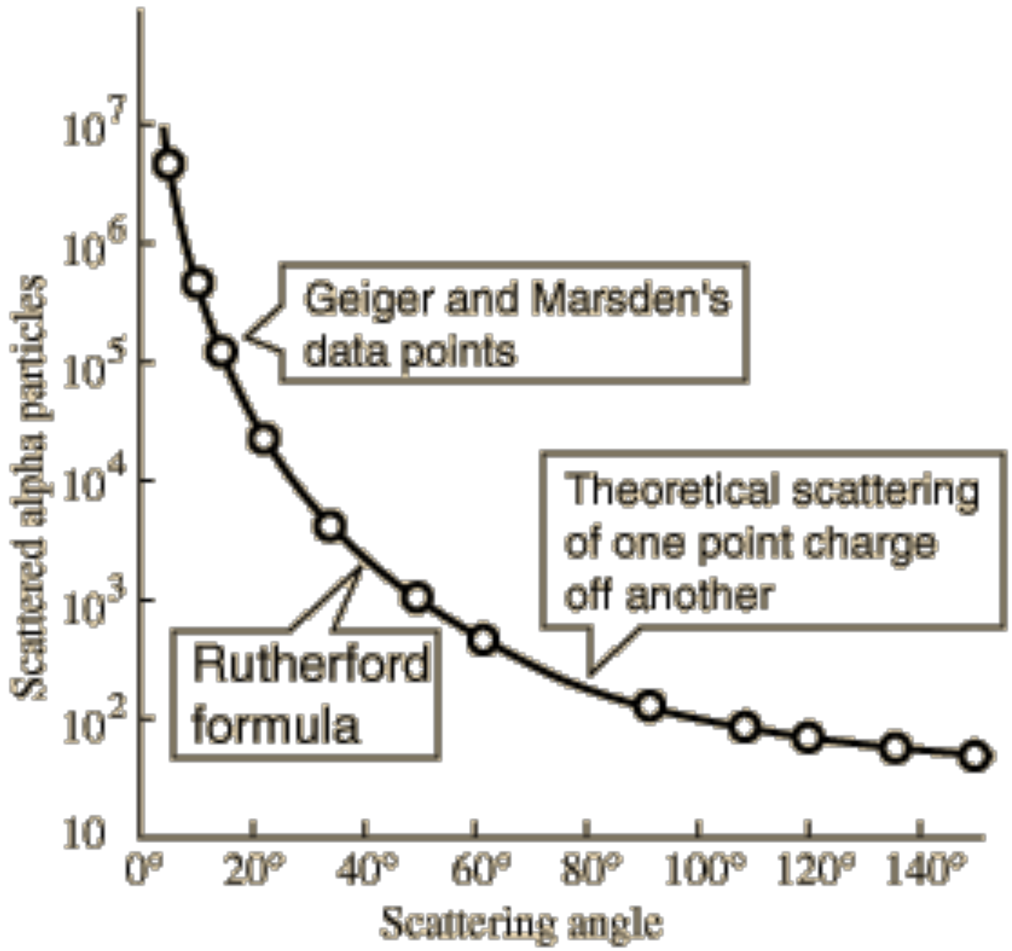
$$\text{Detector Area} = 2\pi r \sin\theta \cdot r \, d\theta$$

$r\sin\theta$ ↑ projected radius; $r\,d\theta$ ↑ thickness

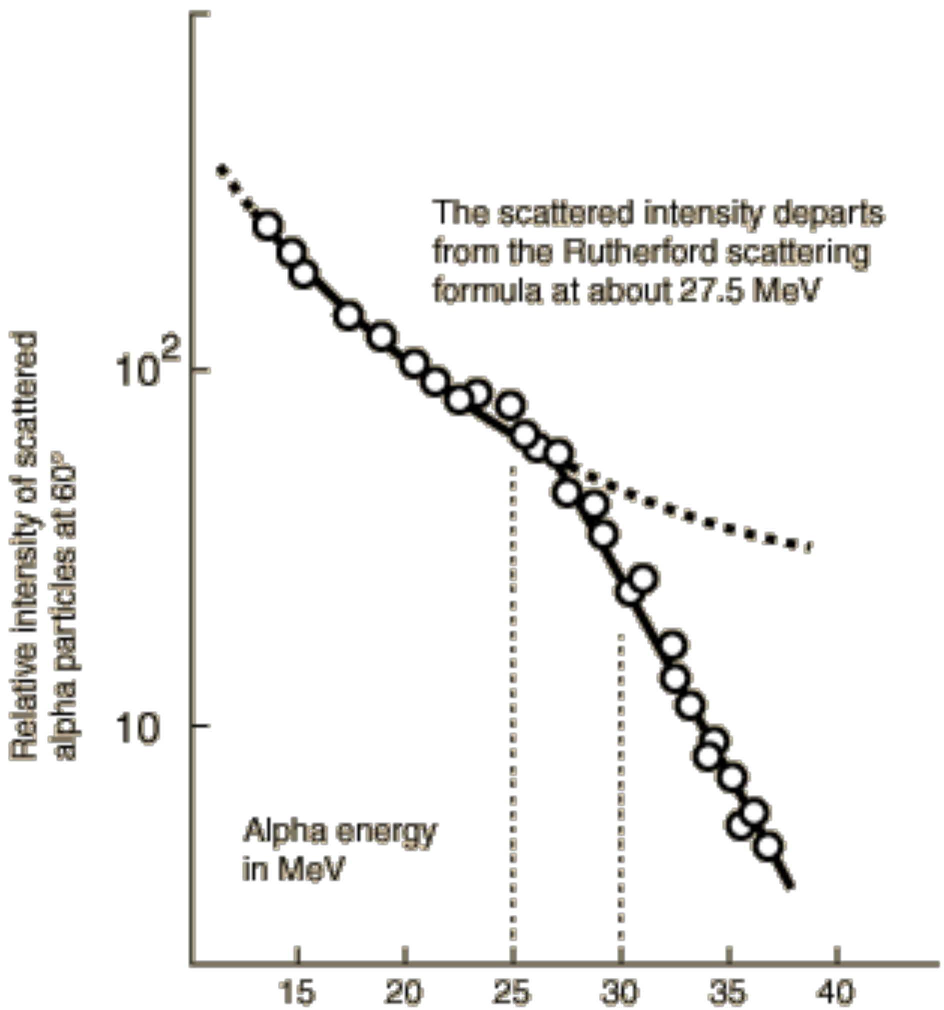
$$df/dA = \frac{nT}{2r^2} \left(\frac{\alpha}{2K}\right)^2 \frac{\cos(\theta/2)}{\sin^3(\theta/2) \sin\theta}$$

$$= \boxed{\frac{nT}{4r^2} \left(\frac{zZe^2}{8\pi\varepsilon_0 K}\right)^2 \cdot \frac{1}{\sin^4(\theta/2)}}$$

Rutherford Scattering



Rutherford Scattering Energy Dependence



Closest Approach

- If $b = 0$, $L = 0$
- r minimum when $\dot{r} = 0$

$$K = \frac{1}{2} m \dot{r}^2 + \frac{L^2}{2mr^2} + \alpha / r$$

$$\Rightarrow K = \alpha / r_{min}$$

$$\Rightarrow r_{min} \sim \alpha / K$$

$$= \frac{z Z e^2}{4\pi\varepsilon_0 K}$$

Gold $Z = 79$

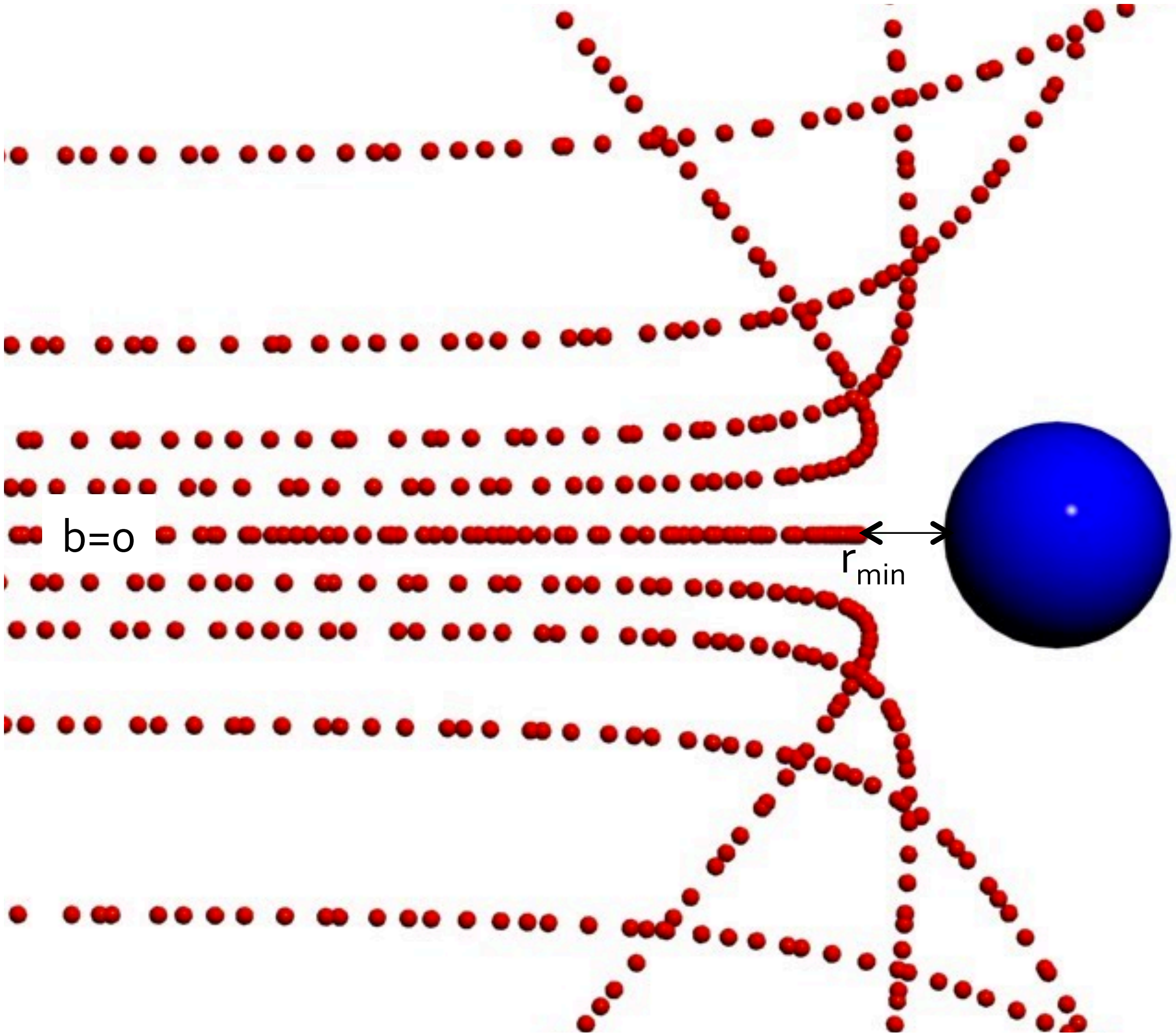
Alphas $z = 2$ $\quad K_{break} = 27.5$ MeV

$\Rightarrow r_{min} \sim 10$ fm

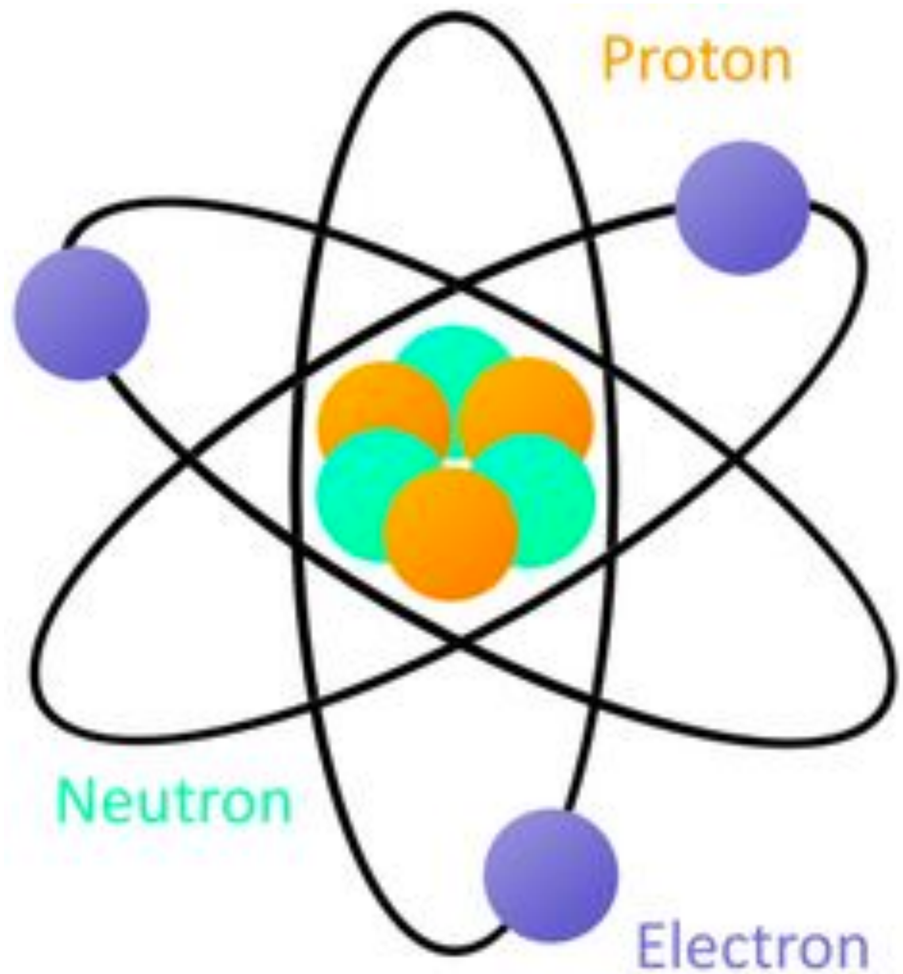
Actual gold nucleus

$r \sim 7$ fm

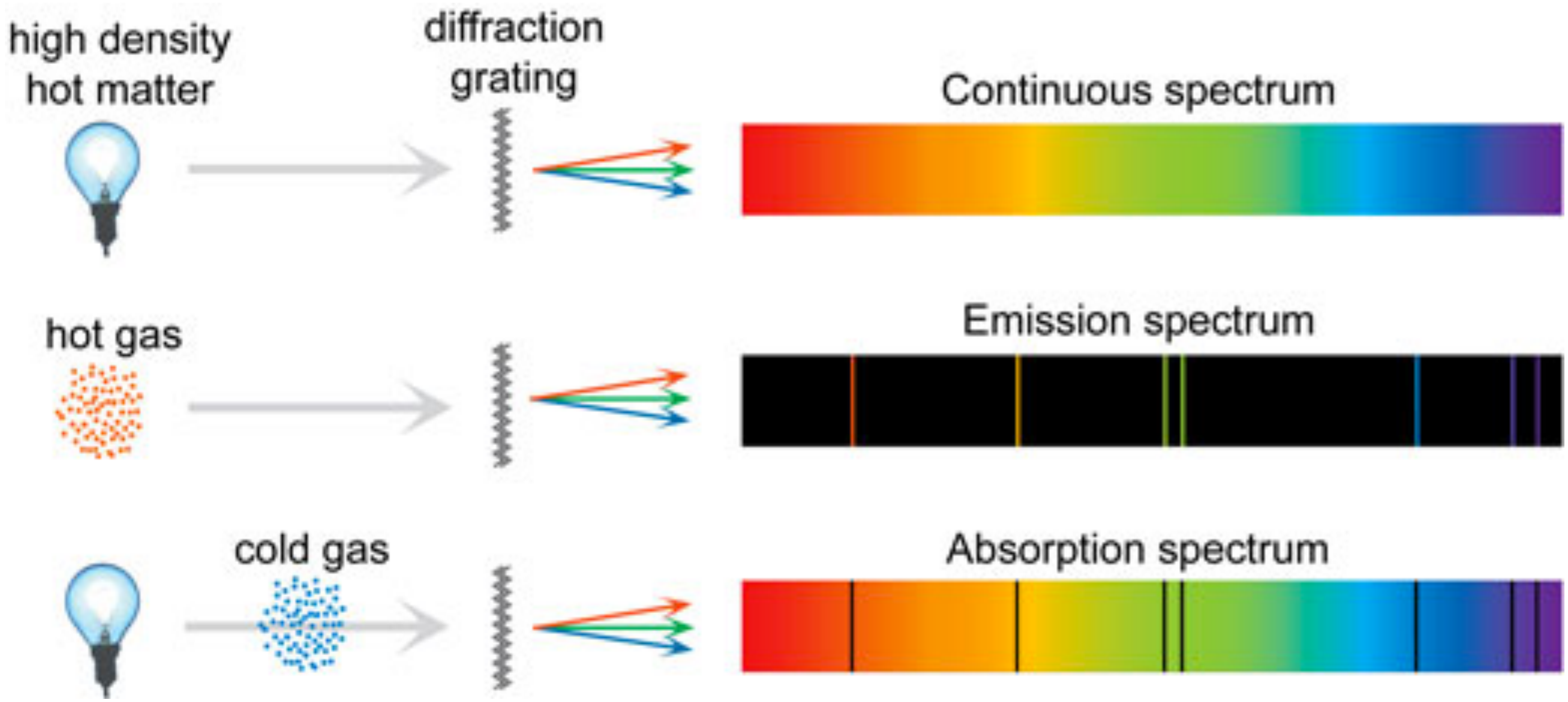
Rutherford Scattering



Rutherford Atom



Emission and Absorption



Concept Check

- Emission occurs when electrons orbiting the atom change their energy – the excess energy is carried away as a photon. What does the presence of discrete emission lines imply about electron energies?

A. Electrons in an atom can only have certain energies
B. Electrons in an atom can only change their energies by specific amounts
C. Both A and B
D. Neither A or B

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Empirical Rule

$$\frac{1}{\lambda} = R_H\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

n_1 and n_2 are integers and $n_2 > n_1$

Rydberg Constant for H = 1.0974×10^{-2} nm

Bohr Model Hydrogen

circular orbit

centripetal = Coulomb

$$\frac{m_e v^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

$$\Rightarrow K = \tfrac{1}{2} m_e v^2 = \frac{1}{8\pi\varepsilon_0} e^2/r = \frac{L^2}{2m_e r^2}$$

$$U = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

$$E = K + U = -\frac{1}{8\pi\varepsilon_0} e^2/r$$
$$= -K$$
$$= -L^2/2m_e r^2$$

Guess $L = n\hbar$ (no justification)

$$\Rightarrow E = \frac{-n^2\hbar^2}{2m_e r^2} = -\frac{e^2}{8\pi\varepsilon_0 r}$$

$$\Rightarrow \boxed{r_n = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} n^2}$$

$$= a_0 n^2$$

a_0 = Bohr radius

= 0.0529 nm

= 0.529 Å

$$E_n = -L^2/2m_e r_n^2$$

$$= \frac{-n^2\hbar^2}{2m}\left(\frac{m_e e^2}{4\pi\varepsilon_0 \hbar^2 n^2}\right)^2$$

$$= \boxed{\frac{-m_e e^4}{32\pi^2\varepsilon_0^2\hbar^2}\cdot\frac{1}{n^2}}$$

$$= \frac{-13.6\ eV}{n^2}$$

$$\Delta E = 13.6\left(1/n^2 - 1/m^2\right)$$

$$= h\nu$$

$$= hc/\lambda$$

$$\Rightarrow \boxed{\frac{1}{\lambda} = \frac{13.6\ eV}{hc}\cdot\left(\frac{1}{n^2} - 1/m^2\right)}$$

$$\frac{13.6}{hc} = 1.09 \times 10^7\ m^{-1}$$

$$= 1.09 \times 10^{-2}\ nm^{-1}$$

Matches Experiment!